Throughput Optimization in Multiple Antenna based Cognitive Radio using an Improved Energy Detector

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Abstract

In this paper, the performance of a Cognitive Radio (CR) regarding cooperative spectrum sensing is analyzed. The CR usually has multiple antennas and calculates decision statistics for all antennas using an improved energy detector. Upon calculation the CR makes a binary decision on whether Primary User (PU) is absent or present. However, in this paper we have considered a single antenna at the CR. The energy detector uses amplitudes of samples of the PU's signal, raised to an arbitrary positive power (p). Expressions for false alarm probability and missed detection probability are derived. The effects of fading have also been included in the expression of the error probabilities. Through calculation the optimal value of *p* for optimum throughput has been calculated. Also through simulation the minimized values of false alarm probabilities and missed detection probabilities have been found.

Keywords: Cognitive Radio, Detection Probability, False Alarm Probability, Missed Detection Probability

1. Introduction

Cognitive Radio (CR) has now been proposed as a promising technology to meet the spectrum scarcity problem due to static frequency band allocation by exploiting dynamic spectrum access[1,2]. It is proposed as an intelligent device that can adaptively modify its various parameters in order to facilitate unutilized frequency band access[3]. In[4], it is proposed that the performance of CR can be improved by using a modified energy detector. In the modified version, instead of using a squaring operation on the received signal amplitude, an arbitrary positive power (p) is used. Using multiple antennas at each CR, as opposed to only one, helps improve reliability. Hence in this paper, we are considering cooperative spectrum sensing with multiple antennas at each CR and an improved energy detector. We are also considering fading

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effects and imperfect transmission channels when calculating the error probabilities (false alarm and missed detection). Our purpose here is the optimization of

the cooperative spectrum sensing scheme by minimizing the probability of false alarm (P_f), keeping the probability of missed detection (P_m) under a fixed threshold (β).

1.1 Related works

In (2012)[4] introduced the improved energy detector for better throughput. They replaced the square operation of the received signal amplitude in the convenient energy detector with and arbitrary positive power p. They also showed that reliability of a CR can be improved by using multiple antennas at each CR. They took into consideration a multiple antenna based cooperative Cognitive Radio network with Rayleigh fading PU-CR links. They showed that for a given value of threshold power (λ), SNR (γ), number of antennas at each Cognitive Radio (M), the optimum value of p varies with number of Cognitive Radios (N). They also showed that it is possible to decrease Total Error Rate (TER) significantly with the joint optimization of p, λ and N. The authors also wrote that by using the Total Error Rate minimization criterion and multiple antennas at each CR it is possible to achieve significantly low values of the probabilities of false alarm and missed detection at very low SNR.

1.2 Contributions of the Present Work

In this paper, we applied energy detection technique in the context of spectrum hole search for a CR. However, instead of using a conventional energy detector with a squaring operation, we use an improved energy detector with an arbitrary positive power p. We maximize throughput by minimizing the probability of false alarm, while the probability of missed detection is under a fixed threshold.

In this way we obtain an optimum point where error probability is optimized.

The rest of the paper is organized as follows. Section 2 focuses on the System model. Performance analysis of multiple antenna based Cognitive Radio using improved energy detector are calculated in Section 3. Numerical results are shown in Section 4. Finally the paper is concluded in Section 5.

2. System Model

We consider a Cognitive Radio network containing one CR, one PU. It is assumed that the PU contains a single antenna and the CR contains M antennas. we have considered two hypotheses H_0 and H_1 regarding the signal received in the ith antenna at the CR. H_0 holds true if the received signal contains noise signal only and H_1 holds true if the received signal contains the noise signal along with the PU signal.

$$H_0 : y_i(t) = v_i(t) \qquad \text{if PU is absent}$$
$$H_1 : y_i(t) = h_i(t)s(t) + v_i(t) \qquad \text{if PU is present} \qquad (1)$$

Where i is the antenna index ($i = 1, 2, \ldots., M$) at the CR, $s(t)$ denotes the signal transmitted by the PU at the time instant t with energy E_s, $v_i(t) \sim \text{CN}(0, \sigma_n^2)$ is circularly symmetrical complex AWGN, and $h_i(t) \sim \text{CN}(0, \sigma_h^2)$ are independent and identically distributed complex normal circularly symmetrical channel gains implying Rayleigh fading. It is assumed that the CR does not have any information about the channels of the PU-CR link. Further it is assumed that the CR contains the improved energy detector, the statistic at the ith antenna for deciding the presence or absence of the PU is given by:

$$W_i = |y_i|^p, \, p > 0 \qquad (2)$$

3. Performance Analysis of Multiple Antenna based Cognitive Radio with Improved Energy Detector

The cumulative distribution function (c.f.d) of the improved energy detector can be written as:

$$P_{W_i}(x) = \Pr(|y_i|^p \le x), \tag{3}$$

Where Pr(.) denotes the probability. By using the conditional probability density function (p.d.f) of $|y_i|^2$ in (3) and after some algebra, we get the conditional p.d.f of W_i under hypotheses H_0 and H_1, respectively, as:

$$f_{w_i|H_0}(y) = \frac{2y^{\frac{2-p}{p}} \exp\left(\dfrac{y^{\frac{2}{p}}}{\sigma_n^2}\right)}{p\sigma_n^2} \tag{4}$$

$$f_{w_i|H_0}(y) = \frac{2y^{\frac{2-p}{p}} exp\left(-\dfrac{y^{\frac{2}{p}}}{E_s\sigma_n^2 + \sigma_n^2}\right)}{p(E_s\sigma_h^2 + \sigma_n^2)} \tag{5}$$

Maximal-ratio combining scheme is not considered since it has spectrum sensing overhead due to channel estimation. Moreover, a combining scheme based on the sum of the decision statistics of all antennas in the CR is not analytically tractable. Therefore, we assume that the CR contains a Selection Combiner (SC) that outputs the maximum value out of M decision statistics calculated for different diversity branches as $Z = \max(W_1, W_2, W_3, \ldots, W_M)$. Hence, the c.d.f of the SC under hypothesis H_0 is:

$$Pz(z|H_0) = \Pr[\max(W_1, W_2, W_3, \ldots, W_M) \le z \,|\, H_9]$$

$$= \left[1 - exp\left(-\frac{z^{\frac{2}{p}}}{\sigma_n^2}\right)\right]^M \tag{6}$$

It can be seen that for $p = 2$, the SC and square-law combiner perform almost similarly if the channels of the PU-CR link are independent of each other. The conditional p.d.f. $fz|H_0|(z)$ of the SC can be obtained by differentiating (6) w.r.t z.

The output of the SC is applied to a one-bit hard detector which takes decision of a spectrum hole as:

$$Z \underset{0}{\overset{<}{\gtrless}} \lambda \tag{7}$$

Where λ is the decision threshold in each CR and binary bits 1 and 0 correspond to the decision about presence and absence, respectively, of the PU. From (6),(7),and after many algebraic manipulations, the probability of false alarm P_f in each CR can be obtained as:

$$P_f = \frac{1}{M} - \frac{1}{M}\left[1 - \exp\left(-\frac{\lambda^{\frac{2}{p}}}{\sigma_n^2}\right)\right]^M \tag{8}$$

From (6), (7) and after many algebraic manipulations, the probability of missed detection P_m in each CR can be obtained as:

$$P_m = \frac{1}{M}\left[1 - exp\left(-\frac{\lambda^{\frac{2}{p}}}{(1+\gamma)\sigma_n^2}\right)\right]^M \tag{9}$$

We know that the maximum throughput equation is given by[5]:

$$T = R(1 - P_f) \tag{10}$$

Where R = achievable throughput.

In order to achieve the maximum throughput we need to solve for p, the optimization problem with a constraint in missed detection probability given as follows:

Maximize T;
Subject to $P_m \leq \beta$ (11)

Where β is a fixed threshold

Considering R as constant, this immediately turns into the following problems as follows:

Minimize P_f;
Subject to $P_m \leq \beta$ (12)

This problem can be solved from the following Lagrangian equation:

$$f = P_f + C(P_m - \beta)$$

$$= \frac{1}{M} - \frac{1}{M}\left[1 - exp\left(\frac{\lambda^{\frac{2}{p}}}{\sigma_n^2}\right)\right] + C\left(\left\{\frac{1}{M}\left[1 - \exp\left(\frac{\lambda^{\frac{2}{p}}}{(1+\gamma)\sigma_n^2}\right)\right]\right\} - \beta\right) \quad (13)$$

Where C is a constant. From Equation (13), considering some minor approximations we find the expression for the optimum value of p.

$$p = \frac{2M\,ln\lambda}{\sqrt{2 \times \left[\frac{(1 - Mln(1+\gamma)}{exp(1+\gamma)} + 1\right]}} \quad (14)$$

We consider the values as $\lambda = 30$, $M = 1$, $\gamma = 10$ and calculate the value of p to be 4.81.

4. Numerical Results

Figure 1 shows the probability of false alarm versus probability of missed detection for $M = 2$, normalized threshold $\lambda_n = \lambda/\sigma_n^2 = 30$,

SNR = 10 dB and a single CR. It can be seen from Figure 1 that there exists a unique point for which P_f and P_m are both minimized. At this point, we find the value of P_m to be 7.692×10^{-2} and the value of P_f to be 4.172×10^{-3}. Similarly from Figure 2, for $M = 4$, the values of P_m and P_f are 5.916×10^{-3} and 4.155×10^{-3} respectively.

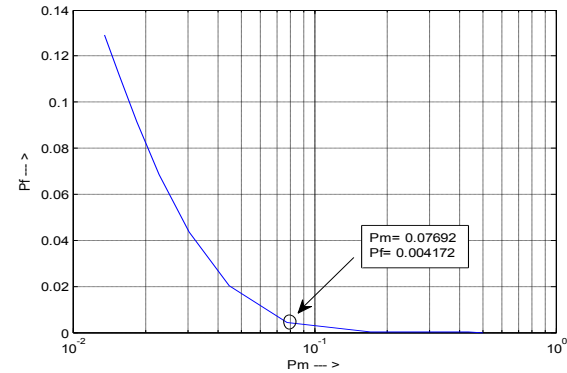**Figure 1.** Probability of false alarm of the proposed Cognitive Radio vs. probability of missed detection for M = 2, λ = 30, SNR = 10 dB.
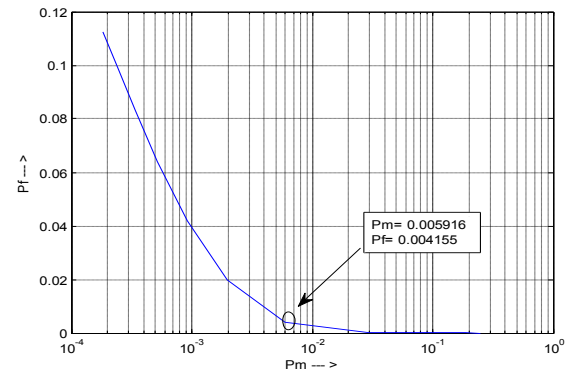

Figure 2. Probability of false alarm of the proposed Cognitive Radio vs. probability of missed detection for M = 4, λ = 30, SNR = 10 dB.

5. Conclusion

Maximizing the throughput of a Cognitive Radio with an improved energy detector, for a single antenna, has been discussed. It is shown that by limiting the value of probability of missed detection to a certain threshold, we can minimize the probability of false alarm and obtain the optimum throughput.

6. References

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