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Methods of Capacitated Vehicle Routing Problem based on Constraints

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Abstract

The Vehicle Routing Problem (VRP) is a combinatorial optimization and integer programming problem which asks "What is the optimal set of routes for a fleet of vehicles to traverse in order to deliver to a given set of customers?" In our paper to describe the Methods of Capacitated Vehicle Routing Problem we implement using c code the Clarke and Wright's savings algorithm to solve our problem where the constraints are to find a solution which minimizes the total transportation costs, demand and capacity of a vehicle. Furthermore, the solution must satisfy the restrictions that every customer is visited exactly once, where the demanded quantities are delivered, and the total demand on every route must be within the vehicle's capacity. The transportation costs are specified as the cost of driving from any city to any other city. The costs are identical in the two directions between two given points. We use this above methodology in solving both Single and Multi depot vehicle routing problem and compare their results.

Keywords: C&W, CVRP, MD, SD, TSP, VRP

1. Introduction

The Vehicle Routing Problem (VRP) dates back to the end of the fifties of the last century when Dantzig and Ramser set the mathematical programming formulation and algorithmic approach to solve the problem of delivering gasoline to service stations[1-4]. Since then the interest in VRP evolved from a small group of mathematicians to the broad range of researchers and practitioners, from different disciplines, involved in this field today.

In[1] published an algorithm for the solution of that kind of vehicle routing problem, which is often called the classical vehicle routing problem. This algorithm is based on a so-called savings concept[8].

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The VRP definition states that m vehicles initially located at a depot are to deliver discrete quantities of goods to n customers. Determining the optimal route used by a group of vehicles when serving a group of users represents a VRP problem[10]. The objective is to minimize the overall transportation cost. The solution of the classical VRP problem is a set of routes which all begin and end in the depot, and which satisfies the constraint that all the customers are served only once[2-6]. The transportation cost can be improved by reducing the total travelled distance and by reducing the number of the required vehicles. The majority of the real world problems are often much more complex than the classical VRP[3-7]. Therefore in practice, the classical VRP problem is augmented by constraints, such as vehicle capacity or time interval in which each customer has to be served, revealing the Capacitated Vehicle Routing Problem (CVRP) and the Vehicle Routing Problem with Time Windows (VRPTW), respectively. In the last fifty years many real-world problems have required extended formulation that resulted in the multiple depot VRP, periodic VRP, split delivery VRP, stochastic VRP, VRP with backhauls, VRP with pickup and delivering and many others.

The Vehicle Routing Problem (VRP) is an important problem in the distribution network and has a significant role in cost reduction and service improvement[9]. The problem is one of visiting a set of customers using a fleet of vehicles, respecting constraints on the vehicles, customers, drivers etc. The goal is to produce a minimum cost routing plan specified for each vehicle.

The problem of vehicle scheduling may be stated as a set of customers, each with a known location and a known requirement for some commodity, that is to be supplied from a single depot by delivery vehicles, subject to the following conditions and constraints:

- The demands of all customers must be met.
- Each customer is served by only one vehicle.
- The capacity of the vehicles may not be violated (for each route the total demands must not exceed the capacity).

The objective of a solution may be stated, in general terms, as that of minimizing the total cost of delivery, namely the costs associated with the fleet size and the cost of completing the delivery routes. The problem frequently arises in many diverse physical distribution situations. For example bus routing, preventive maintenance inspection tours, salesmen routing and the delivery of any commodity such as mail, food or newspapers. Section 2: It provides methods for single and multi depot vehicle routing problem along with the respective algorithm and flow-charts. Section 3: Result Analysis of single and multi depot. Section 4: Limitations of the Approach Section 5: Future scope.

2. Methods for Single and Multi Depot CVRP

2.1 Single Depot

In single depot vehicle routing problem, a vehicle is assigned from only one depot to deliver goods to n number of customers or cities based on their demands. The vehicle is capacitated. After delivering the goods the vehicle returns back to the depot from where it started. Here we take cities/customers, demand of each customer/cities and cost acquired by the vehicle to travel as input[11].

Algorithm:

Step 1. Each of the n vehicles serves one customer.

Step 2. For all pairs of nodes i,j, i…j, we have to calculate the savings for joining the cycles using edge [i,j]: S(ij)=C(0i)+C(0j)-C(ij)

Step 3. The saving is sorted in decreasing order.

Step 4. Edge [i,j] is taken from the top of the savings list. Two separate cycles are joined with edge [i,j], if 1. The nodes belong to

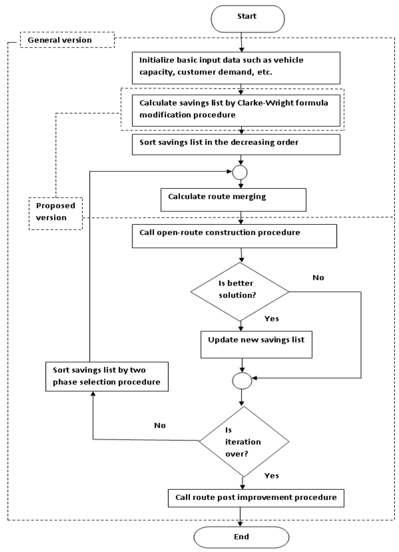

Figure 1. Flowchart for method of single depot CVRP.

separate cycles 2. The maximum capacity of the vehicle is not exceeded 3. i and j are first or last customer on their cycles.

Step 5. Step 4 is repeated until the savings list is handled or the capacities don't allow more merging. The cycles if i and j are NOT united in step 4, if the nodes belong to the same cycle OR the capacity is exceeded OR either node is an interior node of the cycle.

2.2 Multi Depot

In multi depot vehicle routing problem, there may be any number of depots. From each depot a vehicle is assigned to deliver goods to n number of customers or cities based on their demands. The vehicle is capacitated. After delivering the goods the vehicle returns back to the depot from where it started. Here we take number of cities/customers, distance between each cities and depots, demand of each customer/cities and cost acquired by the vehicle to travel as input[12].

Algorithm:

Step 1. As a number of depots are available, a group or cluster is formed between a depot and the nodes based on their distance. That is taking one depot at a time, a group is formed between the particular depot and the nodes nearest to it.

Step 2. Now a vehicle is assigned to each depot.

Step 3. For all pairs of nodes i,j, i…j, the savings for joining the cycles using edge [i,j]: S(ij)=C(0i)+C(0j)-C(ij).

Step 4. The savings is sorted in decreasing order.

Step 5. Edge [i,j] is taken from the top of the savings list. Two separate cycles are joined with edge [i,j], if 1. The nodes belong to separate cycles 2. The maximum capacity of the vehicle is not exceeded 3. i and j are first or last customer on their cycles.

Step 6. Step 4 is repeated until the savings list is handled or the capacities don't allow more merging. The cycles if i and j are NOT united in step 4, if the nodes belong to the same cycle OR the capacity is exceeded OR either node is an interior node of the cycle.

Step 7. The above mentioned steps from 3 to 6 are repeated for the remaining number of depots.

3. Result Analysis of Single and Multi Depot CVRP

The performance of the methods of capacitated vehicle routing problem using Clarke and Wright savings algorithm has been investigated by (N) no. of cities with constraints Demand (D) for each city,

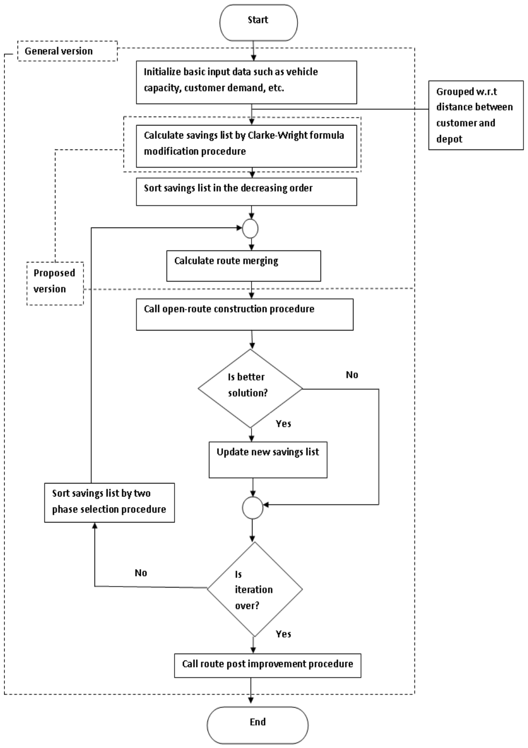

Figure 2. Flowchart for method of multi depot CVRP.

Capacity (K) of each vehicle, Symmetric travel cost for each city (C_{ij}). Experimental results shown for both Single and Multi depot.

3.1 Single Depot

We implement using c code Single Depot CVRP by taking number of cities (N) =20 including depot. *Constraints*- Capacity of vehicle (K) = 40, N*N symmetric travel cost matrix **(C_{ij})** and Demand (D). The cost thus entered is saved in a n*n matrix format where n refers to the different cities. Cost matrix is represented in the form $\mathbf{C}_{ij}$, where i and j represent two cities. For example, the cost incurred while traversing from city 1 to city 4 is stored as C_{14}=4. We cannot have negative cost between two cities, it will give negative result which is not feasibly possible.

Routes	Load	Cost
0-2-5-6-8-0	35	45
0-10-3-4-0	38	40
0-13-17-18-7-0	36	42
0-9-11-14-12-0	32	36
0-15-16-19-20-0	30	31

Total cost f= 194

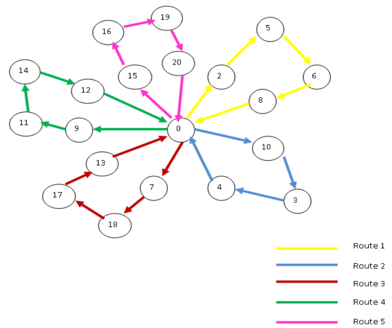

Figure 3. The Solution for single depot CVRP with 20 cities and 40 capacity of vehicle.

Demand: We take demands as input from the user for each city. We cannot take negative inputs as it is not feasibly possible.

Nodes: The circle are the called the nodes. Node 0 is the depot from which vehicle starts and ends.

Arcs: The line segments connecting the nodes are called the arcs. Arcs may describe the time, cost or distance required to travel from one node to another.

Tour: Route of the vehicle.

3.2 Multi Depot

Here like previously we implement using c code Multi Depot CVRP by taking number of cities (N) = 20 and number of depots (Dpt) = 2. *Constraints*- Capacity of vehicle (K)=40, N*N symmetric travel cost matrix (**C_{ij}**), Demand (D) and the Distance (M) from each depot to every city. The cities closest to their depot are thus clustered together as a group. The distance (M) entered cannot be negative value as it is not feasibly possible. The cost entered is saved in a n*n matrix format where n refers to the different cities. Cost matrix is represented in the form **C_{ij}**, where i and j represent two cities. For example, the cost incurred while traversing from city 1 to city 4 is stored as $C_{14} = 4$. We cannot have negative cost between two cities, it will give negative result which is not feasibly possible.

Demand: We take demands as input from the user for each city. We cannot take negative inputs as it is not feasibly possible.

Grouping with respect to least distance between the depots and cities

- Cities near depot 1: [1,3,5,6,7,11,13,15,16,17,19,20].
- Cities near depot 2: [2,4,8,9,10,12,14,18].

In single depot while implementing with many number of cities we have some disadvantages as the distance covered from 1 depot

Routes for Depot 1

Routes	Load	Cost
D1-3-1-6-11-D1	33	46
D1-5-15-19-20-D1	35	48
D1-7-20-13-D1	38	34
D1-16-17-D1	15	18

Total Cost f_1= 146

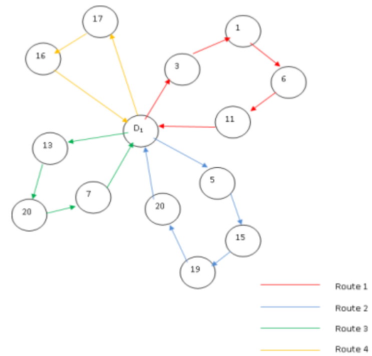

Figure 4. The solution for depot 1 with 13 cities and 40 vehicle capacity.

Routes for Depot 2

Routes	Load	Cost
D_2-2-4-8-D_2	34	46
D_2-9-10-12-D_2	36	38
D_2-14-18-D_2	15	18

Total Cost f_2= 102

increases when number of cities increases. However in multi depot we can choose another depot which is nearest to a cluster of cities, thus ensuring distance covered is lesser than that of a single depot and saving travel cost. Since multiple depots are working simultaneously across various regions, it reduces the total time consumption, thus the delivery system becomes more efficient.

4. Limitations of the Approach

Minimum-cost solution or any other criterion like time or distance traveled is subject to the tour being feasible. Feasibility implies that a

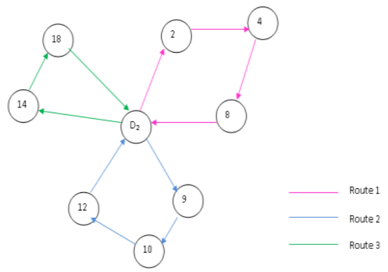

Figure 5. The solution for depot 2 with 8 cities and 40 vehicle capacity.

tour must include all nodes. A node must be visited only once, A tour must begin and end at a depot, Here the cost matrix that we have taken as input for both single and multi depot problem is symmetric. But such kind of symmetric matrix may not be feasible in real life scenario.

5. Future Scope

The Future scope is to c code implement Green Vehicle Routing Problem based on the constraints of Single depot and multi depot.

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