Effective estimation of GAGAN signals with adaptive equalization

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The main objective of this study is to compensate channel distortions in GPS-Aided GEO Augmented Navigation (GAGAN) applications by means of adaptive equalization. Positional accuracy of GAGAN system is basically dependent on ranging errors and satellite constellation geometry. Further, this study focuses on enhancing the phenomena of instrumental biases and GAGAN augmentation.

Keywords: Adaptive equalization, GAGAN, LOS, GPS, LMS, RLS, EKF, BER.

LINE of sight ionospheric measurements, which are derived from GPS observables, are corrupted by instrumental biases. These biases are present in both GPS satellites and receivers. Filter coefficients are being adopted for channel compensation; this is a key aspect in the design of an adaptive filter. Flat frequency response and linear phase are offered by an adaptive filter, which is the combination of a filter and channel. Here the bit error rate (BER) performance of an equalizer can be compared for both cases of with and without using adaptive equalization. In general to evaluate the least mean square (LMS) and recursive laser square (RLS) estimators, the details of receiver decision quality are used, which in turn are used to strengthen their robustness towards the hard decision errors and channel noise. Methods of adaptive equalization by using LMS, RLS and extended Kalman filter (EKF) schemes are compared to each other based on their performance parameters. The tradeoff between different performance factors such as convergence rate, computation cost and minimum squared errors is to be analysed for the selection of an optimum adaptive equalizer. This multipath channel estimation technique with adaptive equalization is then verified using simulations. To provide position, velocity and time information the GPS signals are processed by a receiver. In general, the critical air navigation requirements are not met by GPS. Different phases of a flight are the non-precision approach and precision approach. For CAT-I Precision Approach (PA), both horizontal and vertical accuracies are required to be 16 m and 6-4 m respectively. The idea of GPS-based augmentation system evolved such that GPS can be made applicable for all phases of the flight. Based on the data received from a number of ground reference stations, the satellite-based augmentation system can approximate the error corrections. These error corrections are further transmitted to the users through geostationary (GEO) satellites.

Satellite-based augmentation system

In satellite-based augmentation system (SBAS), dual frequency GPS receivers are located at different places and these places are known as wide area reference stations (WRS). All the GPS satellites are being monitored constantly by GPS receivers. GPS measurements collected of WRS are processed at wide area master stations (WMS). For each satellite, wide area differential (WAD) corrections are being generated using the measurements of master stations. These corrections include the ionospheric delay and satellite's clock and position. Apart from these corrections, the WMS offers different integrity checks for validation of the satellite signals. Using C-band signals, information regarding integrity and differential corrections is transmitted to the geostationary satellite. These satellites in turn relay the information to the users, using L-band signals.

In view of performance enhancement, SBAS provides critical information in respect of differential corrections which improves the positional accuracy. Geo-stationary satellites transmit GPS-like signals to improve the availability and continuity, thereby providing an additional ranging signal. Any indication of malfunction cautions the users instantly, this refers to the safety concern of SBAS. SBAS utilizes the GPS as a prime navigational aid for civil aviation.

Apart from civil aviation application, SBAS can also be useful for all other modes of transportation, which include maritime, highways, railways, etc. At this stage, a number of SBAS in different parts of the globe are in the process of implementation. In a similar fashion, the GPS-Aided GEO Augmented Navigation (GAGAN) technology is being developed in India².

GAGAN/SBAS - errors and their correction

An SBAS system is required to nullify most of the GNSS errors with a provision of needful corrections. Leftover

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errors can be minimized to a maximum extent by transmitting them with bounding information. The concept of providing integrity, accuracy, availability and continuity for a GPS is a prime concern of the SBAS.

The accuracy of satellite signals will be improved by means of SBAS corrections. The integrity information of SBAS confirms that the leftover errors are bounded. Various types of random errors and biases affect the GNSS measurements. Among these, some of the errors can be eliminated and some can be minimized. Errors related to the propagation medium and the errors generated due to the satellite and the receiver are the noteworthy errors³.

Multipath effects

A multipath issue arises because of the reflected radio signals. These reflections may occur due to the surrounding terrain; buildings, walls, hard ground, etc. Multipath issue is also a major concern with GPS signals. These delayed signals because of the multipath issue are the cause for inaccuracy. Minimization of these multipath errors can be obtained with implementation of a narrow correlator spacing technique. In case of longer delay multipath, it can be eliminated by the receiver itself; whereas the shorter delay multipath can be reduced by using a specialized antenna. Owing to their interference with true signals, it is difficult to discard short delay reflections. Further, these reflections may cause almost irreparable effects because of routine fluctuations in atmospheric delay. In case of moving vehicles, multipath effects are of less importance. In case of moving GPS antennas, the false solutions because of the reflected signals fail to converge quickly, whereas the direct signal results in stable solutions⁴.

Simulation of EKF for GAGAN positioning

Consider EKF function with the application of GAGAN. The pseudorange and satellite position of a GPS receiver at fixed location for a period of 25 sec is provided. Least squares and EKF are used for this task. The following is a brief illustration of the principles of GPS. GPS provides the user with proper equipment to access positioning information. The most commonly used approaches for GPS positioning are the iterative least square (ILS) and the Kalman filtering (KF) methods. Both of them are based on the pseudorange equation

$$rho = ||X_s - X|| + b + v, (1)$$

in which X_s and X represent the positions of the satellite and the receiver respectively, and $||X_s - X||$ represents the distance between them; b represents the clock bias of receiver, and it needs to be solved along with the position

of receiver. rho is a measurement given by receiver for each satellite, and v is the pseudorange measurement noise modelled as white noise.

There are four unknowns: the coordinate of receiver position *X* and the clock bias *b*. The ILS can be used to calculate these unknowns and is implemented in this example as a comparison. To deal with the nonlinearity of the pseudorange equation, the Kalman filter solution uses EKF, and a constant velocity (CV) model as the process model⁵. Filters designed are tested with simulated GPS data by adding error models into the predicted pseudorange. Adaptive EKF can also be employed to estimate the ephemeris parameters of the orbiting satellites. Apart from this, an adaptive unscented Kalman filter can be used. Thereby, a high-precision kinematics satellite, aided with inertial navigation will be developed. Further to obtain carrier phase smoothing and ambiguity resolution^{6,7}, a modern receiver can be included.

Estimation of channel SNR for GAGAN applications

In order to compensate channel distortions, adaptive equalization is the best proposed technique. A flat frequency response and linear phase can be offered when a filter is associated with a communication channel. In case of the adaptive filters, filter coefficients are being adapted to compensate the channel. Most of the times, the adaptive equalization approach can be implemented by using RLS and LMS algorithms. These techniques should be compared with each other in terms of bit error rate performance. There will be a continuous analysis of various performance factors such as convergence rate, computation cost instability and ensemble averaged minimum squared errors, to select an optimum adaptive equalizer⁸. The basic communication system model is shown in Figure 1.

General operation mode of adaptive equalizer

Training and tracking are the general operations of an adaptive equalizer. The training sequence is a shifted version of the original transmitted symbols. Following this training sequence, the user data is immediately sent and the adaptive equalizer uses specific algorithms to evaluate the time dispersive channel. Filter coefficients are estimated further. Thereby, the channel distortion can be compensated. The function of training sequence is to allow receiver to obtain stable state when the worst case happens such as long time delay spread or fast velocity, so that after iteration of data have been sent, the filter coefficients are almost optimal values⁹.

As the adaptive equalizer is continuously changing its filter characteristics over time, the varying channel will be tracked by the algorithm. This process will be observed soon after receiving the user data. To obtain fastest

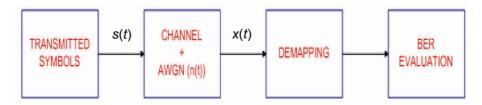


Figure 1. GAGAN-based communication system model.

convergence, RLS is the optimal algorithm. Faster convergence means less sequence is used during the training to obtain the minimum error power. Each time when a new data block has been received, the same training sequence is used. As the baseband complex envelope expression is used in representing band pass waveform, the equalizer is implemented at the baseband. Usually at baseband, the impulse response of the channel, adaptive equalizer algorithm and the demodulated signal are simulated and implemented.

Consider the equation y(t) = x(t) * h(t) + n(t), in which the transmitted signal is x(t), and the received signal y(t)is fed into equalizer. Assume the complex baseband impulse response of a transversal filter equalizer is given by

$$f(t) = \sum_{n} c_n \delta(t - pT), \tag{2}$$

in which c_n are the equalizer coefficients; T the equalization delay; the adaptive equalizer has p taps which are complex multipliers, called weights. So, the output of the equalizer is $\hat{g}(t)$.

$$\hat{g}(t) = x(t) * h(t) * f(t) + f(t) * n(t).$$
(3)

Adaptive equalization algorithms

To initiate linear equalizer coefficients, a number of adaptive equalization algorithms are introduced. They are used to track the channel variations. To identify the best performance of a filter various factors are to be considered:

The number of iterations required for an algorithm is defined as the rate of convergence with respect to a stationary input, where the number of iterations are needed to obtain an optimum solution. In addition, these algorithms adapt quickly to a stationary environment of unknown statistics with a faster convergence rate. Further, this allows the algorithm to track statistical variations during the operation of a non-stationary environment.

It is understood that small estimation errors are the resultant of tiny disturbances only. These disturbances may occur inside or outside the filter because of simple factors. The block diagram representing adaptive channel

equalization is shown in Figure 2. An adaptive algorithm, for its use in digital implementation, is observed to be insensitive to the variations of a word size. The number of bits used for the numerical representation, of data samples and filter coefficients are required in the computation of numerical accuracy, of an adaptive algorithm.

Two classic equalizer algorithms such as LMS and RLS are discussed here. They are primitive according to most of today's wireless standards; they provide optimum performance of the algorithm design and operation¹⁰.

Least mean square algorithm

For an adaptive filter, the filter tap weights, in respect of LMS algorithm for each iteration, are updated based on the following expression

$$t(p=1) = t(p) + 2\mu e(\hat{p})x(p).$$
 (4)

Here, x(p) is the time delayed input value of the vector

$$x(p) = \{x(p)x(p-1)\cdots x(p-P+1)\}^{T}.$$
 (5)

The vector $t(n) = \{t_0(p)t_1(p)\cdots t_{P-1}(p)\}^T$ represents coefficients of the adaptive FIR filter. The parameter μ is a step size parameter, and it is a small positive constant.

For each iteration, this algorithm is required to be implemented in the following sequence.

(1) y(p), output of the FIR filter is calculated using the equation

$$y(p) = \sum_{i=0}^{P-1} t(p)x(p-i) = t^{T}(p)x(p).$$
 (6)

(2) Using the following equation error estimation value is computed

$$e(p) = d(p) - y(p)$$

MSE

$$\varepsilon(p) = E\{|e(p)|^2\}. \tag{7}$$

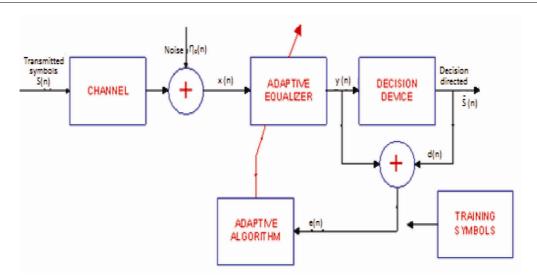


Figure 2. Adaptive channel equalization in GAGAN systems.

(3) As a part of preparation for the next iteration, the FIR filter's, tap weights are updated by using the expression

$$t(p+1) = t(p) + 2\mu e(p)x(p).$$
 (8)

The ease of implementation and simplicity of computation made the LMS algorithm a most commonly used technique for the adaptive filtering process.

Recursive least squares algorithm

All the past input samples of a RLS algorithm are used for the estimation of the inverse autocorrelation matrix of the input vector. This feature is not applicable in the case of LMS algorithm. Mean square errors (MSE) are estimated using instantaneous values, this approach may not provide sufficiently rapid rate of convergence.

If r(c) is representing a matrix with c previous input column vectors, till present time, then s(c) can be represented as,

$$R(c) = [r(1), r(2), ..., r(c)]$$

$$s(c) = R^{T}(c)t(p).$$
(9)

Alternatively, to consider error measure that does not include expectations, MSE is replaced by LSE.

Where λ is an exponential forgetting factor, which decreases the influence of the past input samples. The error signal k(i), calculated for all times such that $1 \le i \le p$, using the current filter coefficient tp

$$k(i) = d(i) - tp^{T} x(i).$$

$$(10)$$

The error is estimated at time i using the latest filter coefficient set, which assumes that the weights are held con-

stant over the entire observation interval (0, p). When $0 < \lambda < 1$, there will be an exponential decrement for all past error values.

Minimizing least square error (LSE) in RLS is different from minimizing MSE. However, minimizing MSE produces the same filter coefficient set for all the sequences that have the same statistics, which means that the coefficients depend on ensemble average of the incoming data. On the other hand, minimizing LSE depends on specific value of input, which means for different signals, different filters are obtained. Different realizations of x(n) and d(n) will lead to different solutions. The filter coefficients which minimize LSE are found to be optimal for a given data; whereas for a particular class of process, they are observed to be statistically optimal. Although exponential weighting improves the tracking characteristics of RLS, it is not quite clear to choose forgetting factor. In the result part, BER versus performance shows how to choose forgetting factor.

Algorithm for a *n*th order RLS filter can be shown as parameters which are as follows: n is filter order; λ forgetting factor; δ value of initialize, N(0) and initialization tp is 0.

In case of RLS algorithm, the memory is limited to the finite number of values. This number is indicated with reference to the order of the filter tap weight vector. It is required to realize the two basic factors related to RLS implementation. They are: (i) Implementation of RLS algorithm reduces the computational complexity to a greater extent. This happens because matrix inversion is essential in the derivation of the RLS algorithm. Further, it does not require any matrix inversion calculations for its implementation. (ii) Values of the previous iteration can be used for updating the current variables, within the present iterations which are ready to be used.

The steps for RLS implementation are as follows

 Filter out is computed based on the filter tap weights from the previous iteration and the current input vector,

$$\overline{s}_{c-1}(c) = \overline{t}^T(c-1)r(c). \tag{11}$$

• Using the following equation, the intermediate gain vector can be calculated

$$u(c) = \tilde{\psi}_{\lambda}^{-1}(c-1)r(c).$$

$$k(c) = \frac{1}{\lambda + r^{T}(c)u(c)}u(c).$$
(12)

By using the following equation, the estimation error value can be calculated

$$\overline{e}_{c-1}(c) = d(c) - \overline{y}_{c-1}(c).$$
 (13)

Updating of the filter tap weight vector can be done using the equation in step (3) and the gain vector is calculated in the following equation

$$t(p) = \overline{t}^{T}(p-1) + k(p)\overline{e}_{p-1}(p). \tag{14}$$

Using eq. (15), the inverse matrix can be calculated

$$\tilde{\psi}_{\lambda}^{-1}(p) = \lambda^{-1}(\tilde{\psi}_{\lambda}^{-1}(p-1) - k(p)[r^{T}(c)\tilde{\psi}_{\lambda}^{-1}(p-1)]). \quad (15)$$

Figure 3 shows simulated result of channel equalization. In this result, SNR forgetting factor and BER are plotted on X axis and Y axis respectively. Observations were made using RLS scheme such that different values of E_bN_0 channel equalization will be done. Based on the

same purpose to update the weight coefficients to the optimal values, they act in a reverse way but have different results. Even though they are all ranged from 0 to 1, the errors decrease as step size decreases and the errors decrease as forgetting factor increases.

RLS algorithm has higher advantage as it gives lower BER. This can be explained as LMS algorithm considers only the current error value, which is the total weighted error from the beginning to the current data point for RLS. Additionally, an exponential weighting factor plays a significant role in determining how to treat the past data input with the algorithm. So in the end, an exponential weighted LSE is estimated among the desired signals and output.

Another essential parameter for determining how to choose an algorithm is the learning curve, which depicts the speed of convergence to the MSE. Figure 4 shows simulated results representing the performance of GAGAN signals, which are plotted across MSEs to number of iterations. These results are plotted for both LMS and RLS algorithms. Observations were made for the learning curve of two algorithms with low and high values.

For low values on the right side of Figure 5, it is difficult to sort out which algorithm gives the faster convergence. As for the rate of convergence is concerned, it can be seen that RLS and LMS perform almost in the same manner. However, for high values on the left side of the figure, it is obvious that RLS consistently achieves faster rate of convergence with smaller ensemble average squares error. The convergence of RLS is attained in about 70 iterations, approximately twice the number of equalizer taps. The convergence for LMS is about 700 iterations. Hence, it is observed that the magnitude of convergence is faster in case of RLS than LMS. It is because RLS algorithm in the process of initialization uses the inverse of the correlation matrix. Here, inverse of the correlation matrix is obtained from the input vector. So RLS has the effect of whitening the tap input by

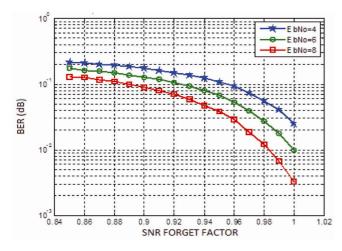


Figure 3. BER performance of channel equalization using RLS scheme.

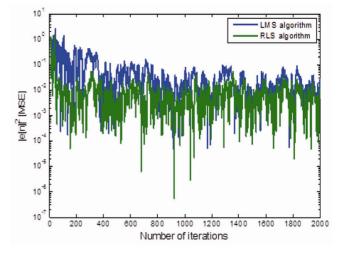


Figure 4. MSE performance of GAGAN signals (LMS versus RLS).

zero mean, and it uses less iteration time to lower the error power. For LMS, the applied step size is independent of the input data. However, this advantage of faster convergence is lost when the signal-to-noise ratio is of small values. The equalizer performance is estimated according to different factors, such as convergence rate and computation complexity¹¹.

Comparison of computation cost between RLS and LMS

RLS filter whitens the input data; thereby, its convergence rate becomes much faster than LMS algorithm. This could happen by using the inverse correlation matrix of the input data. Hence the input becomes white with zero mean. Owing to the expensive computational complexity, RLS algorithm exhibits the faster convergence rate. For LMS, the computation cost is 2p + 3 multiplications and 2p + 2 additions; whereas, the computation cost of RLS requires the order of operations p^2 instead of p operations, i.e. totally $3(p+1)^2 + 3(p+1)$ times. Thereby, the computational complexity for the RLS algorithm is found to be more than the LMS algorithm.

So far, fading effects, channel parameters, signal parameters, channel modelling and modulation skills in GAGAN channel are studied and verified by simulated results. This is followed by a description of different adaptive equalization techniques combating noise, channel distortion and interference to improve the BER performance. Further, two types of applied algorithms such as RLS and LMS are compared through simulated results in terms of rate of convergence, computation complexity and stability. In this case, MATLAB simulation will use modulation technique and combine channel with the equalizer to mitigate channel distortion. The bi-phase shift keyed (BPSK) and quadrature phase shift keyed

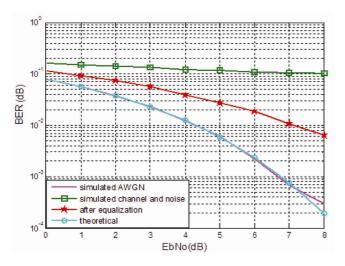


Figure 5. BER performance of GAGAN signal estimation using RLS scheme.

(QPSK) modulation procedure give symbols after generating random bits. After that, upsampling is applied to the symbols. Upsampling and oversampling are both effective ways to increase sampling rate, with slight difference in the implementation method. Upsampling inserts zero in the original sample stream. Upsampling factor is equal to four means inserting three zeros between all samples, which means the number of samples per symbol is 4. The symbol period is 16 ns.

Equalization using training sequence and tracking unknown channel characteristics does improve the BER without changing the data rate. However, QPSK transmits two bits per symbol, which carries more information. Hence, the QPSK modulation is chosen for the next equalization part. Based on upsampling factor which is equal to 4, the received symbols will be sent to the equalizer to check the equalizer's performance. The training sequence uses the shift version of the whole transmitted sequence by 10 symbols delay. The equalizer length has to be larger than the channel characterization length.

Simulated results for BER performance of GAGAN signal estimation using RLS scheme are shown in Figure 5. These results are plotted for BER versus SNR to obtain channel equalization. It is observed that after equalization, the BER curve is closer to the ideal curve which means distortion is reduced. It is further observed that if equalization curve gets more closer to the theoretical curve, there will be a better equalizer performance. Apparently, the RLS equalizer offers smaller error than the LMS equalizer. However, it involves two factors to be considered: the weight number and algorithm factors. About RLS algorithm, it works best when the weight number is 35 and forgetting factor is equal to 0.99.

Analysis and comparison of BER performance among LMS, RLS and EKF algorithms

Extended Kalman filter

The EKF enhances the property of a linear Kalman filter approach for nonlinear systems, which offers better estimated results. This filter is the most used nonlinear filter in the earlier decades^{5,12}. It is based directly on the linear Kalman filter. In addition to this concept, EKF is used to linearize the nonlinear system. Both the time and measurement updates are implemented in their non-linear forms; but for the covariance matrix, linearization is needed. EKF offers a simple and practical approach to deal with essential nonlinear dynamics. The algorithm for computing the extended sequential estimate can be summarized as follows.

The EKF will yield a new state estimate at each observation, which is of value when a real-time solution is desired as the filter processes data. By adding the corrections to the state at each observation, the effects of the

non-linearities in the equations of motion are not as severe, as the attitude parameters are being updated with each measurement. Also, the partials of the system dynamic function are recomputed at each time step at the given updated state. This allows for a more accurate state transition matrix.

The algorithm for implementing EKF can be summarized as follows.

The algorithm for EKF requires

$$p_{k+1} = f(p_k, q_k, r_k)$$
 and $s_k = t_k(p_k, u_k)$.

Initialize with

$$\hat{p}_0 = E(p_0), \hat{V}_0 = E[(p_0 - \hat{p}_0)(p_0 - \hat{p}_0)^T],$$

where p is state vector; s_k the observation vector and q_k is the control vector.

1. for
$$k = 1, 2, ...$$
 do
2. if time update, then
3.
$$F_k = \frac{\delta f}{\delta p^T} \text{ at } p = \hat{p}$$
4.
$$\Gamma_k = \frac{\delta f}{\delta v^T} \text{ at } p = \hat{p}$$
5.
$$\bar{p}_{k+1} = f(\bar{p}_k, q_k)$$
6.
$$V_{k+1} = F_k \hat{V}_k F_k^T + \Gamma_k Q_k \Gamma_k$$
7. end if
8. if measurement update, then
9.
$$H_k = \frac{\delta t}{\delta p^T} \text{ at } p = \bar{p}$$
10.
$$\hat{K}_k = \bar{V}_k H_k^T (H_k \bar{V} H_k^T + R_k)^{-1}$$
11.
$$\bar{x}_k = \bar{x}_k + K_k (z_k - h(\bar{x}_k))$$
12.
$$\hat{V}_k (I - K_k H_k) \bar{V}_k$$
13. end if
14 end for

where r_k is the process noise of covariance matrix Q; w_k the measurement noise of covariance matrix R; Λ_k binds the inputs to the system states; Γ_k binds the process noise to the process state vector; Φ_k state transition matrix; K_k the Kalman gain; and H_k is the observation matrix.

The performance of the two filters RLS and LMS is observed to be poor when lower signal-to-noise ratios were used. These signals may have a Doppler shift. By adding another state, additional Doppler effects can be accounted for, in future research. EKF algorithm is found to be more efficient since it takes the statistics of the transmitted sequence into consideration. It seems to be more robust with respect to local minima, and channel drifting problem. With the availability of several fully

operational satellite navigation systems, it has been recognized that an optimal combination of the output of one or more satellite navigation systems with the output of an inertial navigation system has a number of advantages over a stand-alone inertial or satellite navigation system. The use of adaptation facilitates inter-operable mixing of the outputs of any satellite navigation system with the output of an inertial navigation system.

Table 1 shows the results of GAGAN signal processing, for BER values of approximately 10^{-2} and 10^{-3} with QPSK modulation. The SNR values observed for the EKF and LMS schemes are found to be 9.55 and 11.25 dB respectively. These observations are taken for BER value of 10^{-2} with QPSK modulation. It shows an improvement of 1.7 dB in the performance of GAGAN signal processing with EKF. Similarly for BER values of 10^{-3} using QPSK modulation, the GAGAN signal processing with EKF shows SNR value of 11.35 dB compared to that of LMS where the SNR is 13.85 dB. An improvement of 2.5 dB in the performance of GAGAN signal processing with EKF is observed compared to that of LMS.

A simulated result shown in Figure 6 illustrates the BER performance analysis and comparison among LMS, RLS and EKF algorithms. It is observed that after equalization, the RLS equalizer offers smaller error than the LMS equalizer and the EKF equalizer offers smaller error than the RLS equalizer. However, it involves two factors to be considered: the weight number and algorithm factors.

From Figure 6, it is observed that as the value of SNR increases, the BER will decrease. As SNR increases QPSK_BER curve will lean downward indicating reduction in BER. As SNR increases, so does the accuracy. It is further observed that, for the same values of BER and QPSK modulation, the STBC shows an approximate SNR value of 18 dB compared to the SNR value of 26.5 dB for OFDM. It is also found that for the same BER values 16-QAM (quadrature amplitude modulation), the spacetime block coding (STBC) and orthogonal frequency division multiplexing (OFDM) show the SNR values of 19 and 28.5 dB respectively. Among all the three cases of digital modulation schemes, discussed so far, the SNR values for OFDM multiplexing are found to be high, indicating a large improvement, of >13 dB.

However, it is observed that the efficiency of data collection has gone down by setting a minimum value of SNR. On the other side, the efficiency of data collection will go up by setting the higher values of SNR. To attain this condition, the GPS receiver needs to be positioned for a longer time, till it receives stronger signals.

Conclusion

It is concluded that the adaptive equalization is the most effective technique, that can be implemented for compensating channel distortions. This is the most commonly

Table 1. Improvement of BER for required SNR with QPSK modulation

Method	LMS (dB)	RLS (dB)	Gain of RLS versus LMS (dB)	EKF (dB)	Gain of EKF versus RLS (dB)
Required SNR for BER of 10^{-2}	11.25	9.75	1.5	9.55	0.2
Required SNR for BER of 10^{-3}	13.85	11.65	2.2	11.35	0.3

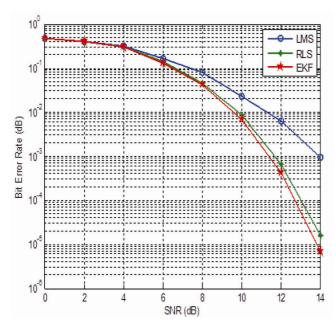


Figure 6. BER performance analysis and comparison between LMS, RLS and EKF.

used technique for GAGAN applications. BER performance of an adaptive equalizer is compared, with and without using the equalizer. GAGAN signal processing for BER values of approximately 10^{-2} and 10^{-3} with QPSK modulation is compared for various algorithms. The SNR values observed for EKF and LMS schemes are found to be 9.55 and 11.25 dB respectively. These observations are taken for BER value of 10^{-2} with QPSK modulation. It shows an improvement of 1.7 dB in the performance of GAGAN signal processing with EKF. Similarly for BER values of 10⁻³ using QPSK modulation, the GAGAN signal processing with EKF shows SNR value of 11.35 dB compared to that of LMS, where the SNR is 13.85 dB. An improvement of 2.5 dB in the performance of GAGAN signal processing with EKF is observed compared to that of LMS. The convergence of RLS is attained in about 70 iterations, approximately twice the number of equalizer taps. The convergence for LMS is about 700 iterations. Hence, it is observed that the magnitude of convergence is faster in case of RLS than LMS.

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