A new approach for tracking moving objects in underwater environment

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Obstacle avoidance and navigation is a demanding task for an autonomous underwater vehicle (AUV) due to the complex nature of the underwater environment. However, an automatic detection and tracking system is the primary element for an AUV or an aqueous surveillance network. Tracking underwater objects in an active context, represents an ongoing challenge in the field of signal processing. In order to detect the target's presence under water, the echoes reflected by the target are analysed by the receiver. Track accuracy is one of the paramount performance measures of a tracking system. Towards this, various methods such as Kalman filter (KF), extended Kalman filter (EKF) and least squares (LS) have been explored. However, all these methods have their own drawbacks. In this study, a new approach called modified gain EKF has been implemented on the simulated data for tracking of underwater moving object using bearing and elevation measurements. AUV fitted with a single sonar is used for validating the proposed bearing and elevation only tracking (BEOT) algorithm. The performance of the algorithm is evaluated in Monte Carlo simulations and results are presented in stipulated geometries.

Keywords: AUV, BEOT, EKF, obstacle avoidance, tracking.

In underwater environment, automatic detection and tracking are very much important for safe navigation of autonomous underwater vehicle (AUV). In many military and scientific applications including sonar-based robotic navigation, underwater weapon systems and infrared seeker based tracking, bearing only tracking (BOT) is used. For underwater weapon guidance system, passive tracking sensors are used in BOT applications. For aerospace and naval applications, target tracking is generally performed using seekers or sonars. The sensor provides either only bearing angle information or both range and bearing information or bearing and elevation information. Passive tracking of maneouvering objects using line of sight (LOS) angle measurements only is an important field of research in the application areas of submarine tracking, aircraft surveillance, autonomous robotics and mobile systems[1–5].

Due to immanent property of environment, the sensor data such as range, bearing and elevation are often noisy, which also result in nonlinear relation between the states and measurements. These inaccuracies of the measurements have a direct impact on the performance of the tracking algorithm.

In the ocean environment, two approaches are commonly used for target tracking. The first approach is a linear Kalman filter (KF)[6], designed by R. E. Kalman in 1960, wherein the measurements are linear functions of the states and designed for prediction/estimation problem. KF can be defined as an optimal recursive data processing algorithm and is characterized by accurate estimation of state variables under noisy condition. It is suitable for drives, robotic manipulators and other industrial applications. The algorithm is formulated in two steps which involve prediction and updating. The second approach is an extended Kalman Filter (EKF)[7], wherein the measurements are nonlinear functions of the states. It is well-established fact that in EKF the initial covariance is based on the initial converted measurement and the gain is based on the accuracies of the subsequent linearization; and therefore the overall performance depends on these accuracies.

In earlier research, the bearing-only filtering problem is considered as discrete-time EKF with relative Cartesian coordinates[8] in two-dimension (2-D). The filter was implemented on basis of the nearly constant velocity model (NCVM)[9] and nonlinear measurement model for bearing-only tracking. The EKF, unscented Kalman filter (UKF), Gauss–Hermite Kalman filter (GHKF) and cubature Kalman filter (CKF) are implemented only for 2-D tracking problems proposed by Scala *et al.*[10] and Jouni *et al.*[11].

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The EKF is implemented for both the predicted state estimate and covariance using a discretized linear approximation[12]. All the approaches mentioned use a 2-D state estimation. The performance of the EKF, UKF, and particle filter (PF) for the angle-only filtering problem in three dimension (3-D) using bearing and elevation measurements from a single manoeuvring sensor are compared[13]. Estimation of the kinematics such as position and velocity of a target, using noisy measurements of the target from a single moving observation platform is a nonlinear function.

Earlier research algorithms, based on EKF which linearizes the measurement model, often result in unstable performances, including poor track accuracy and divergences[7,8]. A new approach of EKF tracking algorithm has been proposed here to solve the problems of underwater environment. Examining the case of single manoeuvring sensor/observer, bearing and elevation only tracking (BEOT) problem authenticates good accuracy and efficiency as the inaccuracies can be handled effectively in this method.

The target tracking basics are covered by Bar–Shalom *et al.*[14] and most aspects of tracking are covered by Blackman *et al.*[15]. Comparison of different tracking methods derives a tracking filter that is well-suited for angle-only target tracking[16,17]. The approach followed in this study is that the non-linearities are modified before being subjected to the tracking algorithm with angle-only measurements. Modified gains for the bearings and elevation problem have also been derived in an elegant manner. The results have been promising and have shown improved performance over the standard EKF.

A modified gain extended Kalman filter (MGEKF) for nonlinear estimation problems was derived by Song *et al.*[18]. This MGEKF algorithm was further developed by Galkowiski *et al.*[19] based on pseudo measurements. Further, the improved modified gain functions for 3-D angles-only tracking was presented by Longbin *et al.*[20,21]. The non-linearities are modified and then applied to a tracker with bearing and elevation only measurements. It shows an improved performance over the standard EKF. Modified gains in a simpler manner for the bearings and elevations problem have been derived.

Tracking algorithm

The foremost problem in bearing and elevation only tracking is in estimating the target trajectory from noisey-corrupted sensor data[22]. In this scenario, the observer tracks a moving target with sensor, which measures only the bearings and elevations of the target, with respect to positions of the sensor. There is one moving target in the scene and one sensor for tracking it. The state of the target motion model (TMM) is described by a nearly constant velocity model (NCVM) and at time step (k) consists of the position in 3-D Cartesian coordinates x, y and z and the velocity towards those coordinate axes v_x, v_y and v_z. Thus, the dynamics of the target is modelled as a state space model. The state of the target is defined in the tracker coordinate frame (T frame) for which the X, Y, and Z axes are along the local east, north, and upward directions, respectively as shown in Figure 1. The target and ownship/observer states in Cartesian coordinates are defined.

$$X_t = [v_{xt} v_{yt} v_{zt} x_t y_t z_t]', \tag{1}$$

and

$$X_o = [v_{xo} v_{yo} v_{zo} x_o y_o z_o]'. \tag{2}$$

The relative state vector in the T frame is defined by

$$X = X_t - X_o. \tag{3}$$

Let the relative state vector in the T frame be

$$X = [v_x v_y v_z x\ y\ z]'. \tag{4}$$

Then, $x = x_t - x_o$, $v_x = v_{xt} - v_{xo}$, etc.

The range vector of the target from the observer (or sensor) in the T frame is

$$r_t = [x\ y\ z]' = [x_t - x_o\ \ y_t - y_o\ \ z_t - z_o]'. \tag{5}$$

Then, the range is defined as

$$r = \sqrt{x^2 + y^2 + z^2}. \tag{6}$$

The range vector can be expressed in terms of range, bearing (φ) and elevation (θ), as defined in Figure 1 by

$$r_t = r\begin{bmatrix} \cos\varphi * \sin\theta \\ \sin\varphi * \sin\theta \\ \cos\theta \end{bmatrix}. \tag{7}$$

TMM is described in the Cartesian coordinate system by linear discrete–time difference equation with some additive noise as

$$X_t(k) = F(k)X_t(k-1) + w(k-1), \tag{8}$$

where the state vector (X_t) consists of the position and velocity components of the target moving in the 3-D space, i.e.

$$X_t(k) = [v_x(k) v_y(k) v_z(k)\ x(k)\ y(k)\ z(k)], \tag{9}$$

where $F(x)$ and $w(k)$ are the state transition matrix and integrated process noise respectively, for the time interval

$[t(k), t(k-1)]$ and process noise is assumed to be zero mean white Gaussian noise.

$$dt = t(k) - t(k-1), \quad (10)$$

$$F(k) = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ dt & 0 & 0 & 1 & 0 & 0 \\ 0 & dt & 0 & 0 & 1 & 0 \\ 0 & 0 & dt & 0 & 0 & 1 \end{bmatrix}. \quad (11)$$

The target is tracked by a sonar under water which provides measurements of bearing (φ_m) and elevation (θ_m).

The measurement model is given as

$$Z_m(k) = hX_t(k) + v(k-1). \quad (12)$$

$$hX_t(k) = \begin{bmatrix} \varphi_m(k) \\ \theta_m(k) \end{bmatrix} = \begin{bmatrix} \tan^{-1} \frac{y(k)}{x(k)} \\ \tan^{-1} \frac{\sqrt{x^2(k)+y^2(k)}}{z(k)} \end{bmatrix}, \quad (13)$$

where $v(k)$ is uncorrelated, zero-mean white Gaussian noise with variances σ_φ^2, σ_θ^2 in the bearing (φ) and elevation (θ) measurements respectively.

The measured range, bearing and elevation from sonar are converted to target positions in Cartesian coordinates with respect to ownship as origin using the following relations.

$$x(k) = r_m * \cos\varphi_m * \sin\theta_m,$$

$$y(k) = r_m * \sin\varphi_m * \sin\theta_m,$$

$$z(k) = r_m * \cos\theta_m,$$

$$R_m = [x(k)y(k)z(k)]. \quad (14)$$

As the measurement model is non-linear, we replace KFA with EKF. The dynamic model using NCVM in 3-D is linear and the measurement model for bearing and elevation is nonlinear for this problem. In general, EKF is based on linearized approximations to non-linear dynamic and/or measurement models[9,23] and is widely used. For this problem, the linearized approximation is performed in the measurement update step[9,23].

The basic idea of this algorithm is to estimate the state of the object X_n at nth instant based on the measured data up to and including $(n-1)$th state estimate iteratively.

$$X_k^- = F_k X_{k-1}, \quad (15)$$

$$P_k^- = F_k P_{k-1} F_k^T + Q_k, \quad (16)$$

$$X_k = X_k^- + K_k(Z_k - H_k X_k^-), \quad (17)$$

$$K_k = P_k^- H_k^T (H_k P_k^- H_k^T + R)^{-1}, \quad (18)$$

$$P_k = (I - K_k g(Z_k, X_k^-))\ P_k^- (I - K_k g(Z_k, X_k^-))^T + K_k R K_k^T, \quad (19)$$

where X_{k-1} = State estimate at time $k-1$; F_k = State transition matrix at time k; X_k^- = Predicted state estimate at time k; P_{k-1}^- = State covariance matrix estimate at time $k-1$; P_k^- = Predicted state covariance matrix at time k; Q_k = Process noise covariance matrix at kth time; X_k = Updated state estimate at time k; K_k = Filtergain at time k; Z_k = Measurement data at time k; H_k = Measurement matrix at time k; P_k = Updated state covariance matrix at time k; R = Measurement noise covariance matrix; $g(Z_k, X_x^-)$ = Modified gain function and I is the identity matrix.

The main difference between the EKF and MGEKF is the function in the covariance update.

To determine the modified gain function g, we write

$$\begin{bmatrix} (\varphi - \hat{\varphi}) \\ (\theta - \hat{\theta}) \end{bmatrix} = g \begin{bmatrix} (x - \hat{x}) \\ (y - \hat{y}) \\ (z - \hat{z}) \end{bmatrix}, \quad (20)$$

where $\hat{\varphi}$ is estimated bearing from the states $\hat{x}$, $\hat{y}$ and $\hat{z}$. $\hat{\theta}$ is estimated elevation from the states $\hat{x}$, $\hat{y}$ and $\hat{z}$.

Since g is not a function of target velocity, we removed those states for the derivation of g.

The measurement matrix H is given by

$$H = \begin{bmatrix} 0 & 0 & 0 & \frac{-\sin\hat{\varphi}}{\hat{r}_{xy}} & \frac{\cos\hat{\varphi}}{\hat{r}_{xy}} & 0 \\ 0 & 0 & 0 & \frac{\cos\hat{\varphi}\cos\hat{\theta}}{\hat{r}_{xyz}} & \frac{\sin\hat{\varphi}\cos\hat{\theta}}{\hat{r}_{xyz}} & \frac{-\sin\hat{\theta}}{\hat{r}_{xyz}} \end{bmatrix}. \quad (21)$$

Updated measurement of bearing data

If the range in horizontal plane is

$$r_{xy} = \sqrt{x^2 + y^2},$$

then, the estimated range will be

$$\hat{r}_{xy} = \sqrt{\hat{x}^2 + \hat{y}^2}.$$

By adding r_{xy} with $\hat{r}_{xy}$ we can get

$$r_{xy} + \hat{r}_{xy} = x\cos\varphi + y\sin\varphi + \hat{x}\cos\hat{\varphi} + \hat{y}\sin\hat{\varphi}. \quad (22)$$

Adding $-x\cos\hat{\varphi}-y\sin\hat{\varphi}-\hat{x}\cos\varphi-\hat{y}\sin\varphi$ to both sides

$$r_{xy}+\hat{r}_{xy}=\frac{(x-\hat{x})(\cos\varphi-\cos\hat{\varphi})+(y-\hat{y})(\sin\varphi-\sin\hat{\varphi})}{1-\cos(\varphi-\hat{\varphi})}. \quad (23)$$

Similarly,

$$r_{xy}-\hat{r}_{xy}=\frac{(x-\hat{x})(\cos\varphi+\cos\hat{\varphi})+(y-\hat{y})(\sin\varphi+\sin\hat{\varphi})}{1+\cos(\varphi-\hat{\varphi})}. \quad (24)$$

$$2\hat{r}_{xy}=(x-\hat{x})\left[\frac{\cos\varphi-\cos\hat{\varphi}}{1-\cos(\varphi-\hat{\varphi})}-\frac{\cos\varphi+\cos\hat{\varphi}}{1+\cos(\varphi-\hat{\varphi})}\right]$$

$$+(y-\hat{y})\left[\frac{\sin\varphi-\sin\hat{\varphi}}{1-\cos(\varphi-\hat{\varphi})}-\frac{\sin\varphi+\sin\hat{\varphi}}{1+\cos(\varphi-\hat{\varphi})}\right]. \quad (25)$$

Taking eq. (25) and simplifying, we get

$$\left[\frac{\cos\varphi-\cos\hat{\varphi}}{1-\cos(\varphi-\hat{\varphi})}-\frac{\cos\varphi+\cos\hat{\varphi}}{1+\cos(\varphi-\hat{\varphi})}\right]$$

$$=-2\frac{\cos\varphi\cos(\varphi-\hat{\varphi})-\cos\hat{\varphi}}{1-\cos^2(\varphi-\hat{\varphi})}=-2\frac{\sin\varphi}{\sin(\varphi-\hat{\varphi})}. \quad (26)$$

$$\left[\frac{\sin\varphi-\sin\hat{\varphi}}{1-\cos(\varphi-\hat{\varphi})}-\frac{\sin\varphi+\sin\hat{\varphi}}{1+\cos(\varphi-\hat{\varphi})}\right]$$

$$=2\frac{\sin\varphi\cos(\varphi-\hat{\varphi})-\sin\hat{\varphi}}{1-\cos^2(\varphi-\hat{\varphi})}=2\frac{\cos\varphi}{\sin(\varphi-\hat{\varphi})}. \quad (27)$$

Now the coefficients of $(x-\hat{x})$ and $(y-\hat{y})$ are

$$\hat{r}_{xy}=\frac{-\sin\varphi}{\sin(\varphi-\hat{\varphi})}(x-\hat{x})+\frac{\cos\varphi}{\sin(\varphi-\hat{\varphi})}(y-\hat{y}). \quad (28)$$

Equation (28) rewritten as

$$\sin(\varphi-\hat{\varphi})=\frac{-\sin\varphi(x-\hat{x})+\cos\varphi(y-\hat{y})}{\hat{r}_{xy}} \quad (29)$$

Updated measurement of elevation data

As can be seen from the above analysis

$\tan^{-1}\frac{y}{x}=\varphi$ generates

$$\sin(\varphi-\hat{\varphi})=\frac{-\sin\varphi(x-\hat{x})+\cos\varphi(y-\hat{y})}{\hat{r}_{xy}}. \quad (30)$$

In a similar way

$\tan^{-1}\frac{\sqrt{x^2+y^2}}{z}=\tan^{-1}\frac{r_{xy}}{z}=\theta$ generates

$$\sin(\theta-\hat{\theta})=\frac{-\sin\theta(z-\hat{z})+\cos\theta(r_{xy}-\hat{r}_{xy})}{\hat{r}_{xyz}}. \quad (31)$$

From eq. (24), we get

$$r_{xy}-\hat{r}_{xy}=\frac{(x-\hat{x})\cos\left(\frac{\varphi+\hat{\varphi}}{2}\right)+(y-\hat{y})\sin\left(\frac{\varphi+\hat{\varphi}}{2}\right)}{\cos\left(\frac{\varphi-\hat{\varphi}}{2}\right)}. \quad (32)$$

Now replacing eq. (32) in (31)

$$\sin(\theta-\hat{\theta})=\frac{-\sin\theta(z-\hat{z})}{\hat{r}_{xyz}}$$

$$+\frac{\cos\theta}{\hat{r}_{xyz}}\left[\frac{(x-\hat{x})\cos\left(\frac{\varphi+\hat{\varphi}}{2}\right)+(y-\hat{y})\sin\left(\frac{\varphi+\hat{\varphi}}{2}\right)}{\cos\left(\frac{\varphi-\hat{\varphi}}{2}\right)}\right]. \quad (33)$$

By rearranging, eq. (33) becomes

$$\sin(\theta-\hat{\theta})=\frac{\cos\theta\cos\left(\frac{\varphi+\hat{\varphi}}{2}\right)}{\hat{r}_{xyz}\cos\left(\frac{\varphi-\hat{\varphi}}{2}\right)}(x-\hat{x})$$

$$+\frac{\cos\theta\sin\left(\frac{\varphi+\hat{\varphi}}{2}\right)}{\hat{r}_{xyz}\cos\left(\frac{\varphi-\hat{\varphi}}{2}\right)}(y-\hat{y})+\frac{-\sin\theta(z-\hat{z})}{\hat{r}_{xyz}}. \quad (34)$$

$$\begin{bmatrix}(\varphi-\hat{\varphi})\\(\theta-\hat{\theta})\end{bmatrix}$$

$$=\begin{bmatrix}\frac{-\sin\varphi}{\hat{r}_{xy}} & \frac{\cos\varphi}{\hat{r}_{xy}} & 0\\ \frac{\cos\theta\cos\left(\frac{\varphi+\hat{\varphi}}{2}\right)}{\hat{r}_{xyz}\cos\left(\frac{\varphi-\hat{\varphi}}{2}\right)} & \frac{\cos\theta\sin\left(\frac{\varphi+\hat{\varphi}}{2}\right)}{\hat{r}_{xyz}\cos\left(\frac{\varphi-\hat{\varphi}}{2}\right)} & \frac{-\sin\theta}{\hat{r}_{xyz}}\end{bmatrix}$$

$$\times\begin{bmatrix}(x-\hat{x})\\(y-\hat{y})\\(z-\hat{z})\end{bmatrix}. \quad (35)$$

Since the true bearing and elevation angles are not available in practice, the measured bearing and elevations are used to compute the modified gain. Hence eq. (35) can be rewritten as

$$\begin{bmatrix}(\varphi-\hat{\varphi})\\(\theta-\hat{\theta})\end{bmatrix}$$

$$=\begin{bmatrix}\dfrac{-\sin\varphi_m}{\hat{r}_{xyz}\sin\theta_m} & \dfrac{\cos\varphi_m}{\hat{r}_{xyz}\sin\theta_m} & 0\\[2ex] \dfrac{\cos\theta_m\cos\left(\dfrac{\varphi_m+\hat{\varphi}}{2}\right)}{\hat{r}_{xyz}\cos\left(\dfrac{\varphi_m-\hat{\varphi}}{2}\right)} & \dfrac{\cos\theta_m\sin\left(\dfrac{\varphi_m+\hat{\varphi}}{2}\right)}{\hat{r}_{xyz}\cos\left(\dfrac{\varphi_m-\hat{\varphi}}{2}\right)} & \dfrac{-\sin\theta_m}{\hat{r}_{xyz}}\end{bmatrix}$$

$$\times\begin{bmatrix}(x-\hat{x})\\(y-\hat{y})\\(z-\hat{z})\end{bmatrix}. \quad (36)$$

By considering the velocity components v_x, v_y and v_z, g is given by

$$g=$$

$$\begin{bmatrix}0\ 0\ 0 & \dfrac{-\sin\varphi_m}{\hat{r}_{xyz}\sin\theta_m} & \dfrac{\cos\varphi_m}{\hat{r}_{xyz}\sin\theta_m} & 0\\[2ex] 0\ 0\ 0 & \dfrac{\cos\theta_m\cos\left(\dfrac{\varphi_m+\hat{\varphi}}{2}\right)}{\hat{r}_{xyz}\cos\left(\dfrac{\varphi_m-\hat{\varphi}}{2}\right)} & \dfrac{\cos\theta_m\sin\left(\dfrac{\varphi_m+\hat{\varphi}}{2}\right)}{\hat{r}_{xyz}\cos\left(\dfrac{\varphi_m-\hat{\varphi}}{2}\right)} & \dfrac{-\sin\theta_m}{\hat{r}_{xyz}}\end{bmatrix}. \quad (37)$$

Results and discussions

The simulated data with different measurement inaccuracies have been generated for validating the algorithm. At the instant of the first angle measurements, ownship is considered at the origin. It has also been assumed that the target movement has a constant velocity and travels in a straight path. Figure 1 shows the tracking angles of the observer and target and Figure 2 shows the trajectories of ownship, true target and predicted target. Figure 3 shows the time history in x, y and z positions.

The performance of the algorithm is evaluated in terms of:

- The percentage fit error (PFE) in x and y

$$\text{PFE}_x = 100 * \frac{\text{norm}(x-\hat{x})}{\text{norm}(x)}, \quad (38)$$

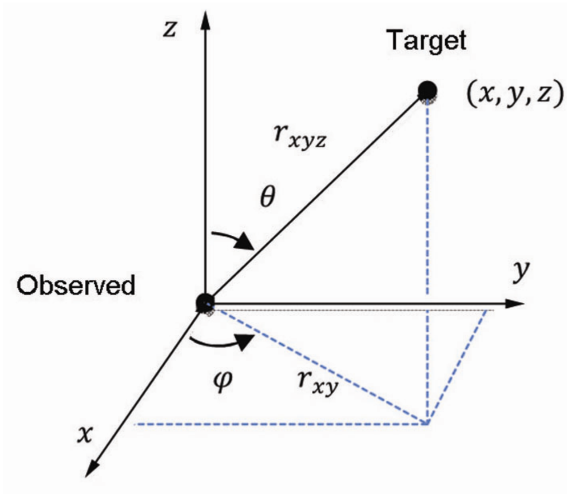

Figure 1. Tracking angles (bearing and elevation).

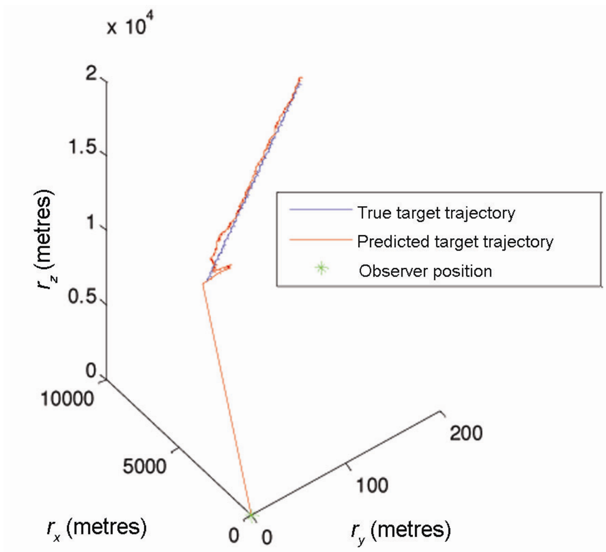

Figure 2. Observer, true (simulated) target and predicted target trajectories.

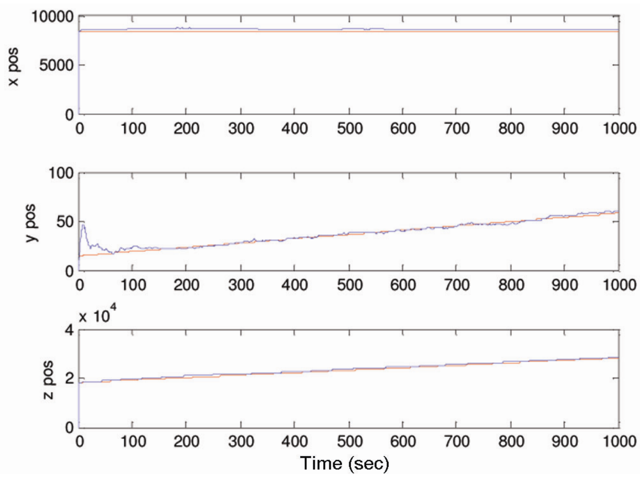

Figure 3. Time history of x, y and z positions (true and predicted).

$$\text{PFE}_y = 100 * \frac{\text{norm}(y - \hat{y})}{\text{norm}(y)}, \quad (39)$$

$$\text{PFE}_z = 100 * \frac{\text{norm}(z - \hat{z})}{\text{norm}(z)}. \quad (40)$$

- The root mean square position error

$$\text{RMSPE} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\frac{(x_i - \hat{x}_i)^2 + (y_i - \hat{y}_i)^2 + (z_i - \hat{z}_i)^2}{3}}, \quad (41)$$

where, $N = 1, 2, 3\ldots 1000$ Monte Carlo runs.

- The root mean square velocity error

$$\text{RMSVE} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\frac{(v_{x_i} - \hat{v}_{x_i})^2 + (v_{y_i} - \hat{v}_{y_i})^2 + (v_{z_i} - \hat{v}_{z_i})^2}{3}}, \quad (42)$$

where $N = 1, 2, 3\ldots 1000$ Monte Carlo runs.

- Root sum square position error

$$\text{RSSPE} = \sqrt{(x - \hat{x})^2 + (y - \hat{y})^2 + (z - \hat{z})^2}. \quad (43)$$

- Root sum square velocity error

$$\text{RSSVE} = \sqrt{(v_x - \hat{v}_x)^2 + (v_y - \hat{v}_y)^2 + (v_z - \hat{v}_z)^2}, \quad (44)$$

where x, y and z are the measurements, $\hat{x}$, $\hat{y}$ and $\hat{z}$ are the estimated target positions, v_x, v_y and v_z are the measurements, $\hat{v}_x$, $\hat{v}_y$ and $\hat{v}_z$ are the estimated target velocities in x, y and z coordinates respectively. Here, RMSPE and RMSVE are calculated at each time step as given in the eqs (41) and (42).

The simulated data has been generated as follows:

Simulation 1:
Initial target velocity = 10 m/sec
Initial target course = 135°
Initial range between ownship/observer and target = 10 km
Initial bearing between ownship/observer and target = 0.5°.
Initial elevation between ownship/observer and target = 45°.
$\sigma_\varphi = 0.0015$
$\sigma_r = 30$
$\sigma_\theta = 0.0015$.

The estimated errors in simulations 1 are plotted in Figure 4. It has been observed that the convergence duration is 200 sec in case of range, 50 sec in case of bearing, 200 sec in case of velocity, 600 sec in case of course and 800 sec in case of elevation, which indicate the suitability of this method in aiding the AUV for its safe navigation. The root mean square errors in position (RMSPE) and velocity (RMSVE), root sum square errors in position (RSSPE) and velocity (RSSVE) are shown in Figures 5 and 6.

Simulation 2:
Initial target velocity = 10 m/sec
Initial target course = 145°
Initial range between ownship/observer and target = 20 km
Initial bearing between ownship/observer and target = 0.1°
Initial elevation between ownship/observer and target = 25°
$\sigma_\varphi = 0.0015$
$\sigma_r = 30$
$\sigma_\theta = 0.0015$.

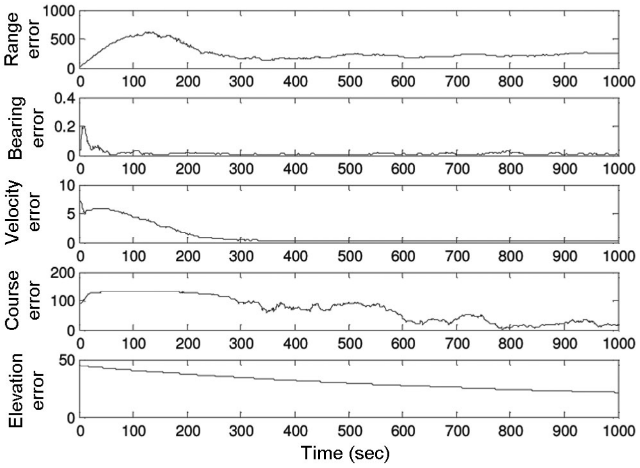

Figure 4. Range, bearing, velocity, course and elevation errors for simulation 1.

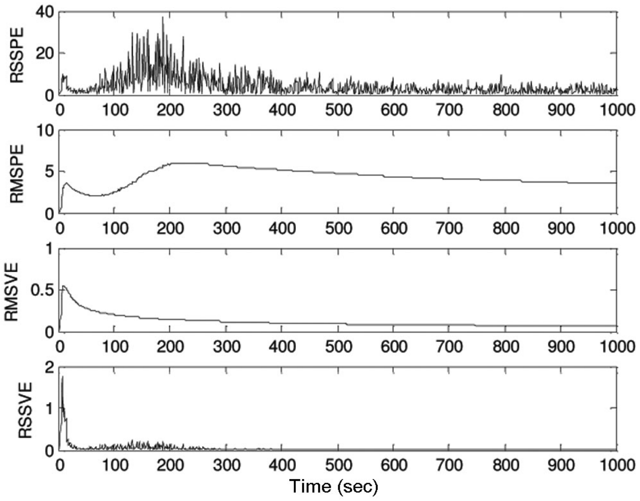

Figure 5. RMSPE, RMSVE, RSSPE and RSSVE in predicted position for simulation 1.

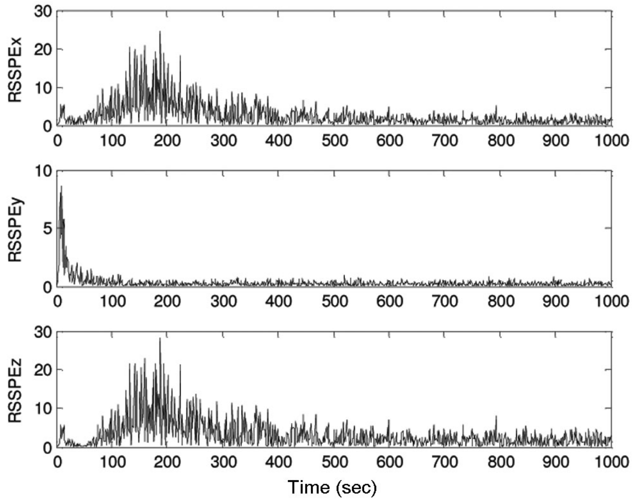

Figure 6. RSSPE in x, y and z positions for simulation 1.

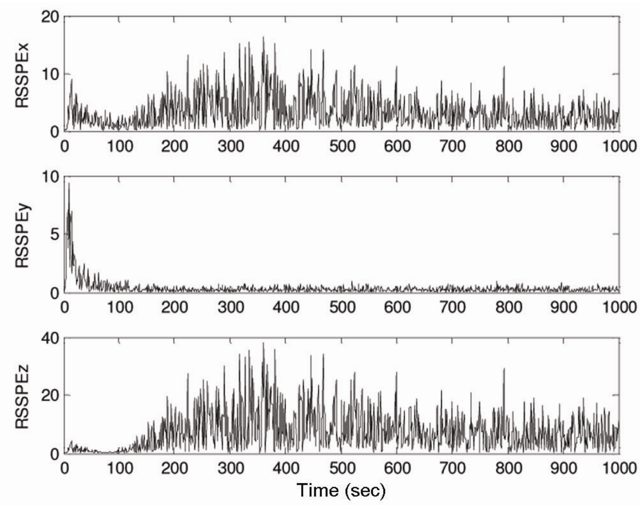

Figure 9. RSSPE in x, y and z positions for simulation 2.

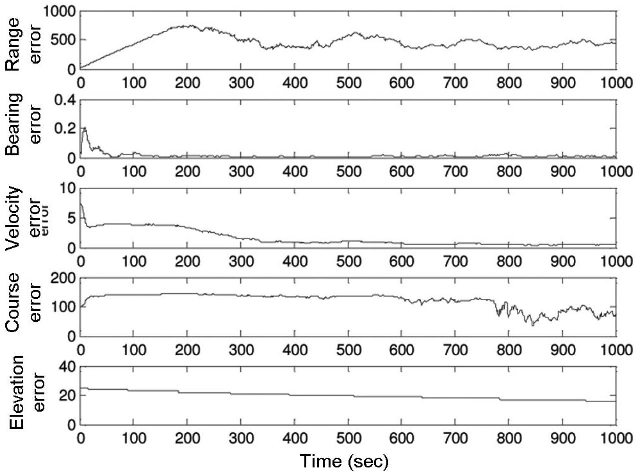

Figure 7. Range, bearing, velocity, course and elevation errors for simulation 2.

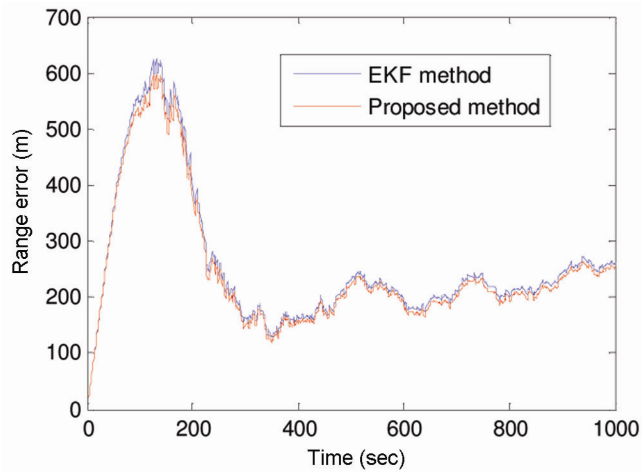

Figure 10. Range error of proposed BEOT algorithm with EKF.

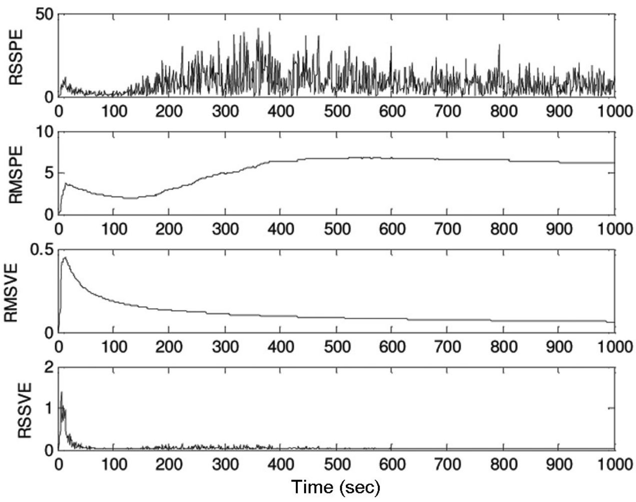

Figure 8. RMSPE, RMSVE, RSSPE and RSSVE in predicted position for simulation 2.

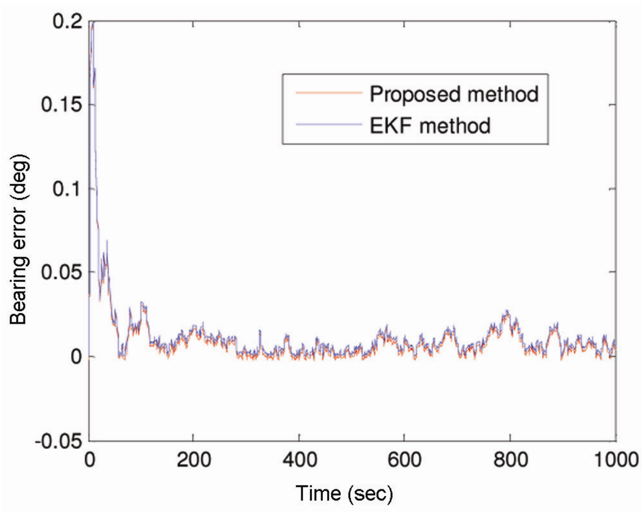

Figure 11. Bearing error of proposed BEOT algorithm with EKF.

The estimated errors in simulations 2 are plotted in Figure 7. It has been observed that the convergence duration is 350 sec in case of range, 70 sec in case of bearing, 300 sec in case of velocity, 850 sec in case of course and 900 sec in case of elevation. The RMSPE and RMSVE, RSSPE and RSSVE are shown in Figures 8 and 9 respectively.

Observer motion is assumed to be stationary and at origin (0, 0, 0). Bearing and elevation measurements are taken at every 1 sec for 1000 Monte Carlo updates.

Filter initializations are as follows:

The initial state has been defined as

$$X_0 = [10\,10\,10\; r_m \cos\varphi_m \sin\theta_m \; r_m \sin\varphi_m \sin\theta_m \; r_m \cos\theta_m], \quad (45)$$

where r_m, θ_m and φ_m are initial bearing and elevation measurements.

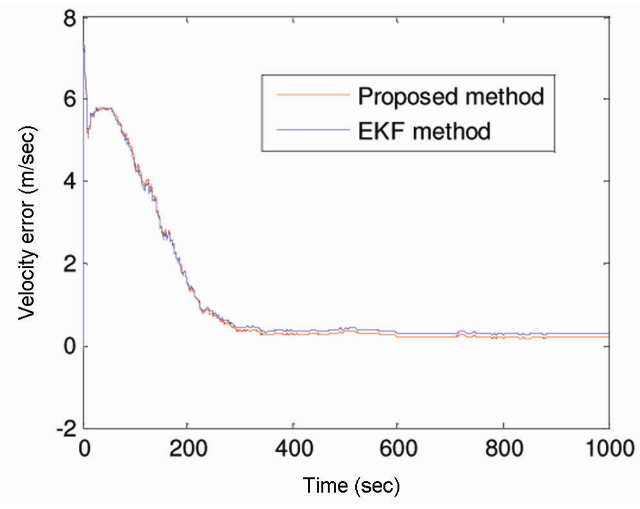

Figure 12. Velocity error of proposed BEOT algorithm with EKF.

$$P_0 = 10^{-5}\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}. \quad (46)$$

Initial covariance matrix is assumed as per the standard procedure[24]. The initial covariance matrix of process and measurement noises are taken as follows.

$$Q = 10^{-5}\begin{bmatrix} dt^2 & 0 & 0 & \frac{dt^3}{2} & 0 & 0 \\ 0 & dt^2 & 0 & 0 & \frac{dt^3}{2} & 0 \\ 0 & 0 & dt^2 & 0 & 0 & \frac{dt^3}{2} \\ \frac{dt^2}{2} & 0 & 0 & \frac{dt^3}{4} & 0 & 0 \\ 0 & \frac{dt^2}{2} & 0 & 0 & \frac{dt^3}{4} & 0 \\ 0 & 0 & \frac{dt^2}{2} & 0 & 0 & \frac{dt^3}{4} \end{bmatrix}, \quad (47)$$

$$R = 10^{-5}\begin{bmatrix} \sigma_\varphi^2 & 0 \\ 0 & \sigma_\theta^2 \end{bmatrix}, \quad (48)$$

where dt is the time step. σ_φ^2 is the variance of noise in bearing measurement. σ_θ^2 is the variance of noise in elevation measurement.

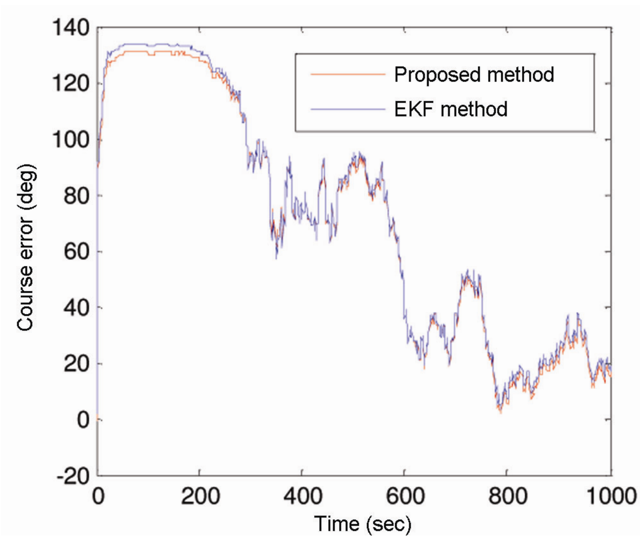

Figure 13. Course error of proposed BEOT algorithm with EKF.

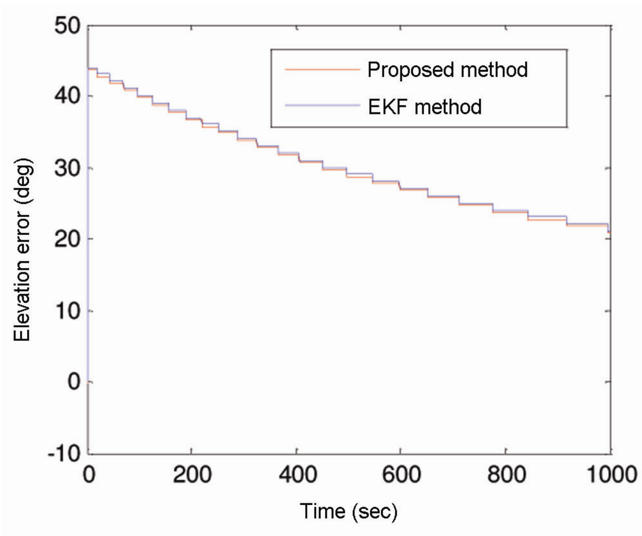

Figure 14. Elevation error of proposed BEOT algorithm with EKF.

Comparison of proposed BEOT algorithm with conventional EKF

In Figures 10–14, the range, bearing, velocity, course and elevation errors for methods proposed, BEOT and conventional EKF are compared for every time step of 1000 Monte Carlo runs and found to be minimum.

It is observed that the range, bearing, velocity, course and elevation errors are optimum for the proposed algorithm.

Conclusions

In this study, EKF has been extended to estimation with bearing and elevation measurements for underwater moving object tracking. Convergence issues associated with the newly designed algorithm have been analysed. It is observed that the errors are small and settle down after a filter learns the dynamics. The performance of this algorithm has been evaluated using 1000 Monte Carlo simulations. The results of the proposed method are compared with the existing EKF method and it was observed that range, course, velocity, bearing, and elevation errors of the target are minimized. Due to the reduced measurement residual covariance, the accuracy has been improved when compared to the conventional EKF method. Also, the convergence of the covariance matrix into steady state is improved with the proposed method.

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