Modelling of diurnal profile of surface air temperature and surface dissolved oxygen of lake water

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Fourier analysis method can be used as a mathematical tool for modelling the diurnal profile of surface air temperature and surface dissolved oxygen (DO) of lake water. For developing the model, initially Lake Rudrasagar, Tripura, India has been chosen as the study site. Within the scope of this communication, due to availability of collected data the developed mathematical model has been verified for Subhas Sarobar, Kolkata. It is observed that the diurnal profile of DO and surface air temperature of both the lakes studied almost follows similar trends of sinusoidal variation.

Keywords: Diurnal profile, dissolved oxygen, sinusoidal variation, surface air temperature.

ONE-dimensional harmonic analysis with regard to time-series data has potential applications in oceanography, hydrology[1–4] as well as in geosciences and geology. The two-dimensional extension of harmonic analysis finds significant applications in meteorology and climatology. Fourier analysis may also be termed as spectral analysis which decomposes a new time-dependent regular sensation directly into several sinusoidal features, each of which simply characterized by special amplitude and period valuations. The actual portion involving variance inside initial time-series can be established and every single term of the harmonic analysis can also be worked out. Model designing during the last few years has become a challenging task among researchers and engineers, for scientifically prediction of the future scenario[5]. In hydrodynamics, mathematical simulation has become a vital feature of researches in this regard. In order to generate a mathematical model, experimental data analysis is the fundamental criterion for following the process over the time–space domain. Fourier analysis method has been chosen as the mathematical tool for generating the required model to study the sinusoidal variation of surface air temperature and surface dissolved oxygen (DO) of lakes. Several major physico-chemical characteristics of lake water are controlled by temperature for the Lake Rudrasagar[6] which is in turn responsible for maintaining the lake water quality suitable for human consumption. The progressive variation in air temperature also shows a strong seasonal relationship with lake water temperature[7].

DO concentration regulates the respiration of these organisms. If strong turbulence is not observed in a lake, then photosynthesis by aquatic plants is the only vital source of DO. There exist strong correlations between surface air temperature and surface DO for the lakes Rudrasagar and Subhas Sarobar. Incoming solar radiation may also put a signature on diurnal profile of surface air temperature and surface DO of lake water[8].

The study site, i.e. Lake Rudrasagar (23°29′N and 90°01′E) falls in Melaghar Block under Sonamura Sub-Division of Tripura, the third smallest state of India. This natural lake is 50 km away from the state capital, Agartala. It has three sources of inflow and one outflow. Inflows are Noacherra, Durlavnarayan cherra and Kemtali cherra, whereas Kachigang, the only connective channel which discharges water into the River Gomati acts as an outflow. Figure 1 represents the satellite image of Rudrasagar wetland.

Sampling sites were chosen along the mid reach of lake water for getting maximum water depth. Multipurpose water quality analyser devices were used for continuous recording of the two parameters – surface air temperature and surface DO. These measuring devices store data in their memory cells and at the end of each sampling day these are transmitted to a computer. The recorded temperature and DO data have resolution of 0.01°C and 0.01 mg/l respectively.

The model equation for diurnal profile of surface air temperature of the lake is

$$T(t) = A_0 + A_i \cos\{(2\pi it/T) - \varphi_i\},$$

where i is the running variable taken from $i = 1$ to 10 in 24 h time domain and A_0 represents the mean value of the

Figure 1. Satellite image of Rudrasagar wetland, Tripura, India.

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corresponding hourly mean measurements considering each day of entire three-year period from 2011 to 2013. The time-series equation for the diurnal profile of lake surface water DO is

$$\mathrm{DO}\ (t) = A_0 + A_i \cos\ \{(2\pi it/T) - \varphi_i\}.$$

Cloud cover, wind speed, solar radiation, etc. were not considered in this study due to lack of availability of such meteorological data for Lake Rudrasagar. Only the daily recorded lake surface air temperature has been considered as the significant meteorological parameter influencing DO. This is supported by the strong correlation between lake DO and surface air temperature for both lake Rudrasagar[9] and Subhas Sarobar. As surface air temperature depends on precipitation, cloud cover, wind speed,

Table 1. Fourier analysis outcome from recorded surface air temperature over Lake Rudrasagar during the period 2011–2013

Harmonic	Amplitude (A_i)	Phase angle φ_i (deg)
First	5.709071	61.28831621
Second	1.041727	–2.722895046
Third	0.727675	–75.27999287
Fourth	0.430342	–61.25398895
Fifth	0.221411	45.19953675
Sixth	0.124454	–79.80587513
Seventh	0.126243	–47.32268532
Eighth	0.308735	–3.710364987
Ninth	0.117135	56.02622208
Tenth	0.384484	–55.21472691

Table 2. Surface air temperature variance above Lake Rudrasagar during the period 2011–2013

Harmonic	Variance	Percentage of total variance
First	16.29674645	93.84632098
Second	0.542598033	3.124600931
Third	0.264755739	1.5246204
Fourth	0.092596928	0.53322797
Fifth	0.024511325	0.141150729
Sixth	0.007744415	0.044596932
Seventh	0.007968646	0.045888186
Eighth	0.047658726	0.274447177
Ninth	0.006860344	0.039505923
Tenth	0.07391403	0.425640773
Total	17.36535464	

Table 3. Results of Fourier analysis of dissolved oxygen on the surface of Lake Rudrasagar during the period 2011–2013

Harmonic	Amplitude (A_i)	Phase angle φ_i (deg)
First	3.296978	60.36798147
Second	0.485723	–63.70923695
Third	0.231801	–41.25890414
Fourth	0.224018	–60.92221408
Fifth	0.244302	–65.6292055
Sixth	0.10163	–38.09560988
Seventh	0.054482	–58.66383443
Eighth	0.061749	19.98640837
Ninth	0.120935	–0.621246931
Tenth	0.112459	71.75385406

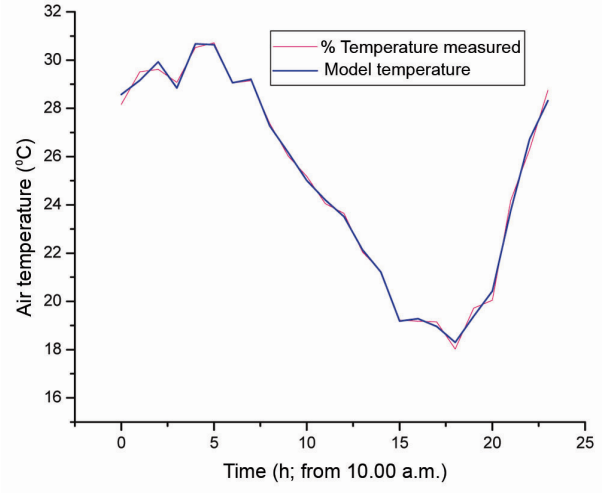

Figure 2. Observed and predicted diurnal profile of surface air temperature over Lake Rudrasagar during 2011–2013.

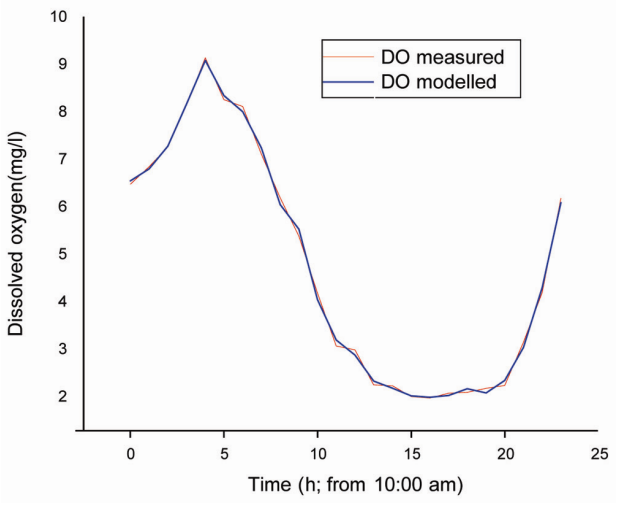

Figure 3. Observed and expected diurnal profile of dissolved oxygen (DO) in Lake Rudrasagar, during 2011–2013.

Table 4. Variance of dissolved oxygen on the surface of Lake Rudrasagar during the period 2011–2013

Harmonic	Variance	Percentage of total variance
First	5.435032298	96.07645917
Second	0.117963548	2.085271873
Third	0.026865816	0.474913924
Fourth	0.025091972	0.443557225
Fifth	0.02984175	0.527520259
Sixth	0.00516432	0.091291016
Seventh	0.001484122	0.026235208
Eighth	0.001906463	0.033701043
Ninth	0.007312676	0.129268047
Tenth	0.006323506	0.111782233
Total	5.656986472	

Table 5. Statistical comparison of observed and predicted temperature of Lake Rudrasagar

	Observed diurnal variation (temperature, °C)			Predicted diurnal variation (temperature, °C)		
Year	Mean	Variance	Standard variation	Mean	Variance	Standard variation
2011–2013	24.99958333	18.10851721	4.255410346	24.9995	18.25529655	4.272621741
2014	26.10208333	8.912608514	2.985399222	26.10208333	8.890302601	2.981661047

Table 6. Statistical comparison of observed and predicted surface DO of Lake Rudrasagar

	Observed diurnal variation (DO, mg/l)			Predicted diurnal variation (DO, mg/l)		
Year	Mean	Variance	Standard variation	Mean	Variance	Standard variation
2011–2013	4.822083333	5.905347645	2.430092106	4.822125	5.89713081	2.428400875
2014	4.012083333	3.682686775	1.919032771	4.012083333	3.680318428	1.918415603

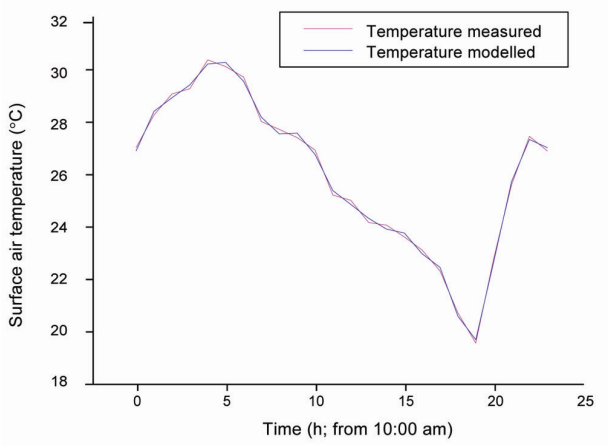

Figure 4. Predicted and observed diurnal profile of surface air temperature of Lake Rudrasagar in 2014.

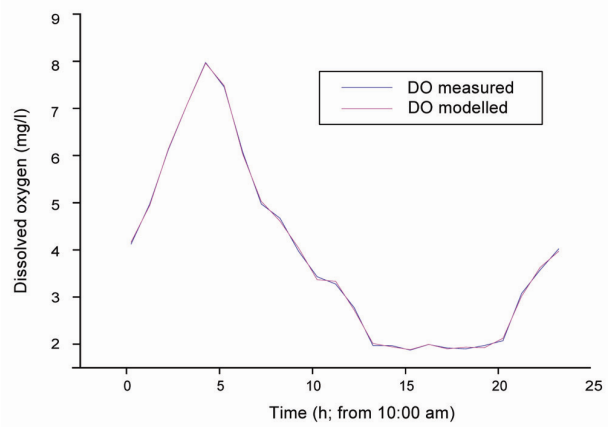

Figure 5. Expected and observed surface DO profile of Lake Rudrasagar in 2014.

solar radiation, etc., strong correlation between surface air temperature and surface DO for both the lakes suggests that the former alone can be taken as driving parameter of the model; this makes the model more simpler. For other lakes where temperature may not show such a strong correlation with DO, parameters like precipitation, cloud cover, wind speed and solar radiation must be taken into consideration.

Model equations have been generated for following the diurnal profile of surface air temperature and DO of Lake Rudrasagar as well as Subhas Sarobar using Fourier analysis method. Both graphical and statistical comparisons have been adopted for calibration and validation of the mathematical model.

A simple model equation has been generated following the observed diurnal profile of air temperature over the surface of Lake Rudrasagar. Data were collected for the entire year and a mean dataset prepared for the diurnal profile. Tables 1 and 2 show the outcome of Fourier analysis applied on the estimated dataset of surface air temperature and variance for different harmonics respectively.

Figure 2 shows the observed diurnal profile of surface air temperature based on estimated mean datasets for the entire three years' domain along with the simulated pattern.

Mathematical modelling approach was also used for surface DO of lake water. Tables 3 and 4 show the results of Fourier analysis applied to the prepared dataset of surface DO of the lake surface and the variance for different harmonics respectively.

Figure 3 shows the observed diurnal profile of surface DO based on prepared average datasets during the period 2011–2013 along with the simulated graph, where the simulated graph almost coincides with the observed variation.

Figures 4 and 5 represent the diurnal profile of observed and predicted surface air temperature over Lake Rudrasagar and those for surface DO of lake water respectively, in the year 2014. Both the figures show a negligible mismatch between observed and predicted variation for the lake.

Tables 5 and 6 show a statistical comparison of the observed and predicted values of surface air temperature

Table 7. Statistical comparison of observed and modelled temperature and DO of Subhas Sarobar during the period 2011–2013

Parameter	Observed diurnal variation			Modelled diurnal variation		
	Mean	Variance	Standard variation	Mean	Variance	Standard variation
Surface air temperature (°C)	26.94958333	9.158397403	3.026284422	26.949625	8.483208071	2.912594732
DO (mg/l)	5.030416667	6.670160688	2.582665423	5.030375	6.642256505	2.577257555

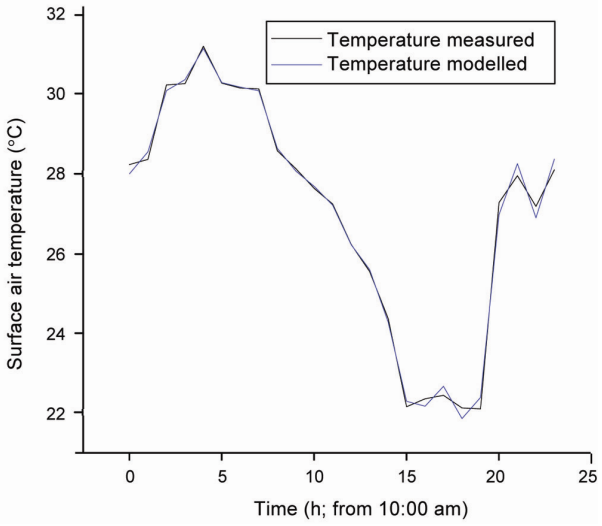

Figure 6. Modelled and observed diurnal profile of surface air temperature of Subhas Sarobar during 2011–2013.

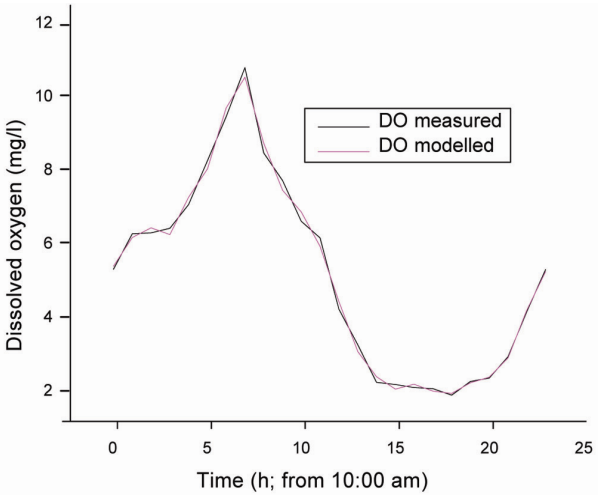

Figure 7. Predicted and observed diurnal variation of surface DO of Subhas Sarobar during 2011–2013.

and surface DO of water for Lake Rudrasagar respectively. Table 7 displays the statistical comparative study of observed and modelled temperature and DO of Subhas Sarobar during the period 2011–2013. In all cases the results are satisfactory.

In order to test the sinusoidal nature of the diurnal curve for surface air temperature and surface DO of water for other lakes, we chose Subhas Sarobar (22°34′7″N, 88°24′2″E), in Bidhan Nagar city, Kolkata. Figures 6 and 7 show the diurnal profile of observed and modelled surface air temperature and those for surface DO of water from Subhas Sarobar for the period 2011–2013.

Thus at the preliminary level, the model has been tested on Lake Rudrasagar and Subhas Sarobar. It can be applied on other Indian lakes as well for its validity. The results have significant importance for the prediction of diurnal profile of surface air temperature and DO at the lake surface. However, change in observed diurnal profile of these parameters, can indicate the change in regional health condition of the lake. For validation of such numerical models more research is needed in this field.

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ACKNOWLEDGEMENTS. We thank the authorities of Rudrasagar Lake for their well cooperation during fieldwork; Civil Engineering Department, NIT, Agartala for help with field measurement programmes; the School of Water Resource Engineering, Jadavpur University, Kolkata for providing experimental data on Subhas Sarobar, Kolkata for the year 2013 and the Director for encouragement. Financial support from the University Grants Commission is acknowledged.

Received 20 April 2015; revised accepted 15 February 2016

doi: 10.18520/cs/v111/i5/882-885