cytoskeletal dynamics indicate that the field of mechanochemistry and mechanobiology are bound to provide tough challenges for theoreticians. Mehmood *et al.* provide a glimpse of the challenges involved in mass spectrometric analysis of protein complexes. In this context it is worth pointing out a recent text (Ghosh, P. K., *Introduction to Protein Mass Spectrometry*, Academic Press, 2016) that provides comprehensive information on the state-of-the-art in the field. The method of collision-induced unfolding is interesting, and it is clear that analysing the unfolding plots is a problem in dynamics that perhaps involves a central role for energy flow pathways in the complex.

In summary, this volume of *ARPC* shows the amazing breadth of physical chemistry. Indeed, it would take a certain amount of patience to go through it all and discover the connections between seemingly unrelated topics. Hopefully, this review has managed to group them in some 'coarse-grained' fashion and hence act as a guide for those who want to read up on the frontiers of a certain corner of physical chemistry. If you are short on time, then read the summary boxes provided at the end of most chapters. However, there is no substitute for reading the review in full. For active researchers, the reviews in this volume are perfect as a reference to the current level of understanding and the grand challenges that remain. Perhaps *ARPC* can make the summary box and an additional open questions box mandatory for every article.

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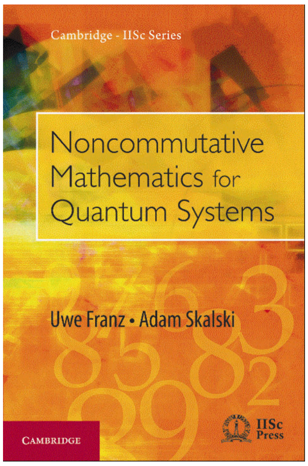

Noncommutative Mathematics for Quantum Systems. Uwe Franz and Adam Skalski. Cambridge–IISc Series, Cambridge University Press, 4843/24, 2nd Floor, Ansari Road, Daryaganj, Delhi 110 002, 2016. 125 pages. Price: Rs 950.

This monograph is based on lectures given by these two young mathematicians at an international workshop-cum-conference held at the Indian Statistical Institute, Bengaluru during 31 December 2012–11 January 2013. The authors are two of the topmost researchers and finest expositors of the subject of non-commutative mathematics. They give a friendly introduction to some recent developments in the field. Currently, there are no books covering some of the topics explained here.

This book is on quantum probability and to explain it we need to begin with classical probability. Consider one of the simplest of random experiments, namely throwing of a die. The outcome could be any number from 1 to 6. Actually if we know all the related physical parameters such as shape and weight distribution of the die, the force applied and the way it is applied and so on, at least in principle, it should be possible to predict the outcome accurately. However, in practice it is a rather difficult thing to do, as the number of parameters involved is too many and the mathematical equations coming from physical laws are too complex. In practice, we may believe that the die is a fair one and model the experiment using a triple $(\Omega, \mathcal{F}, P)$ as follows. The set of all possible outcomes, or sample space, is a set $\Omega = \{1, 2, 3, \ldots, 6\}$. Similarly $\mathcal{F}$ consists of events which are subsets of Ω, for instance, the set of odd numbers here, $A = \{1, 3, 5\}$ is an event. Then we assign probabilities to events, following certain natural rules. If we believe that the die is a fair one, it is natural to assign probability of A as three out of six, that is 1/2. Mathematically we write it as

$$P(A) = 1/2.$$

Starting with this elementary idea, and some combinatorics, we can answer questions such as what is the chance that we get eight successive 'sixes' if we throw the die say 1000 times. Here we are making use of probability theory. This model was originally proposed by A. N. Kolmogorov. It uses Boolean logic of a family of subsets of the set of all possible outcomes. These subsets are called events and probabilities are assigned to events. Here the machinery of measure theory comes in handy. The whole of probability theory is built with this model and it is the accepted model used in modelling all the 'randomness' we see in our daily life. We call this as 'classical probability'.

One of the major surprises of quantum theory is that it is non-deterministic. It has 'randomness' in-built in it. Even more surprisingly, Kolmogorov's model is not applicable here. One uses the logic of subspaces of Hilbert spaces and this probability theory has come to be known as 'quantum probability' or 'noncommutative probability'. It is the model applicable to quantum mechanics, and consequently indispensable for quantum computation, quantum information and other related fields. The phenomenon of non-commutativity appears here in a crucial way with serious mathematical and physical consequences.

The idea of having non-commutative or 'quantum' versions has spread to other areas of mathematics and has become a major trend of modern mathematics. At a basic level we have the theory of C*-algebras and von Neumann algebras as non-commutative topology and non-commutative measure theory respectively. At a much more sophisticated level we have non-commutative geometry and quantum groups. I quote the following paragraph from this book to explain the basic mathematical scheme followed for building these theories:

'The general pattern of the noncommutative mathematics is the following: take a "classical" mathematical theory, say topology, measure theory, differential geometry, or group theory and reformulate it in terms of algebras. These algebras are algebras of functions on the spaces appearing in the "classical" theory, that is, continuous functions (with values in *C*) on a topological space, measurable functions on a measure space, smooth functions on a manifold, and so on. They inherit some additional structure, for example, an involution, a norm or topology, a coproduct, from the space on which they are defined. Finally, these algebras are always commutative. Axiomatizing the additional structure and dropping the commutativity condition one arrives at a noncommutative generalization of the original theory.'

The monograph is divided into two parts. In the first part there are lectures by Uwe Franz on quantum versions of Levy processes (independent increment processes). It begins with a well-written, brief introduction to quantum probability and its need in quantum mechanics. Anybody who is familiar with classical probability and wishes to know the mathematical framework of quantum probability, is welcome to read this section. The probability distributions appearing in Levy processes are infinitely divisible with respect to convolution and hence there is a section detailing the notion of infinite divisibility and associated characterization theorems. Then one moves on to Levy processes on involutive bialgebras and compact quantum groups. Everyone knows that the notion of independence plays a crucial role in classical probability. When we come to the quantum world, there are several possible notions of independence. It is shown that if we make some natural choice of axioms that a good notion of independence must satisfy, then there are precisely five universal independences: tensor, free, Boolean, montone and anti-monotone. This part ends with explaining Levy processes on dual groups and presenting a short list of open problems.

The second part, by Adam Skalski, is about quantum dynamical systems. It describes them mostly in C*-algebraic (topological) and von Neumann algebraic (measure theoretic) framework. Voiculescu's non-commutative topological entropy has been explained at great length with explicit computations of entropy for several examples. Non-commutative ergodic theorem is another major theme covered here.

On the whole, the book presents a timely set of lecture notes on recent developments in non-commutative mathematics. As mentioned before, the initial sections would be useful for anyone interested in knowing basic ideas behind quantum probability. The whole book is aimed more at research students and serious researchers of the field and not at casual readers. This is definitely a welcome addition to the Cambridge University Press–IISc Lecture notes Series.

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