Improved criteria on robust analysis for linear system using convex combination and geometric sequence methods

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This article addresses the robust analysis on a delayed system with uncertainties. A geometric sequence division (GSD) method is applied for delay partition. Then, a GSD-dependent Lyapunov–Krasovskii functional (LKF) is newly proposed, in which the integral interval relevant with the state variables forms in geometric progression. In addition, by applying the convex combination method, parameter uncertainties and the delay derivative $\dot{d}(t)$ can thus be flexibly overcome. As a result, unnecessary enlargement for estimating the LKF derivative is eliminated. Numerical example shows that this proposed work achieves expected results.

Keywords: Convex combination, delay partition, geometric sequence, parameter uncertainties.

IN the real world, delay is inevitably experienced in dynamic systems, such as chemical reaction processes, biological systems, mechanical systems, etc. The existence of delay can often yield poor performance or even instability[1]. Therefore, how to overcome the negative influence of delays has attracted vast attention in recent academic research. Meanwhile, stability is considered as the priority in many applications, a lot of research has been done on stability analysis for various types of delayed processes in recent decades[2–5]. Generally, stability conditions of delayed systems are categorized as delay-independent and delay-dependent ones. Less conservative results can be obtained using delay-dependent conditions in case of a relatively small delay or interval time-varying delay.

For stability analysis, the essential issue for a system with delay is to obtain maximum delay upper bound that guarantees the studied system to be asymptotically stable. For obtaining a higher delay upper bound, various forms of Lyapunov–Krasovskii functional (LKF) are constructed, such as discretized LKF, augmented LKF and delay-partitioning LKF[6–10]. In fact, expected stability results will be achieved if well-developed inequalities are employed for estimating LKF derivative. Therefore, estimation of LKF derivative alternatively for reducing conservatism is another considerable option. Three techniques are commonly applied: Jensen-inequality[11–13], free weight matrix methods[14,15], convex optimization method including their combinations[16–21]. The well known Jensen-inequality and its modification are proposed in refs 18 and 19. However, as a result of handling LKF derivative, some terms are neglected. So, it leads to conservative conditions because of the estimation. How to avoid unnecessary conservatism in the estimation of LKF derivative is still a challenge.

In addition, uncertain dynamic behaviours commonly exist in practical implementation due to modelling errors, immeasurable issues and perturbations, which could degrade the performance of system or even cause system instability[22,23]. The analysis of uncertainties is another hot topic for studying dynamic systems[24–27]. However, reducing conservatism in such systems is normally accompanied with extra computational complexity[28]. How to compromise conservative reduction and computation burden is also full of challenges.

In this article, stability analysis on an uncertain system is studied. Delay-dependent criteria are derived to ensure that the linear system is globally asymptotically stable under the maximum upper bound. Comparing with existing results, the main outcome would be as follows:

(1) The recently developed geometric sequence division (GSD) method is first applied on time-varying delay in a linear system. Using this method, a new modified LKF is constructed, which contains new GSD-dependent integral forms with unfixed intervals. This approach can dramatically reduce the number of decision variables. This new development can provide expected stability conditions with high efficiency and less computational complexity.

(2) Convex combination method is introduced to represent the parameter uncertainties and solve the delay derivative. This estimation approach can handle the LKF derivative without using extra inequalities or constraint conditions. Thus, unnecessary enlargement can be eliminated.

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(3) Using convex combination and GSD methods, the stability criteria on robust analysis for delay linear system are improved remarkably.

Notations: $\mathbb{R}^n$ is the Euclidean space with n-dimensional. $P > (\geq)0$ indicates positive (semi-positive) definite matrix $P \cdot I_n(0_n)$ is the n-dimension identity (zero) matrix; $\mathrm{He}(A) = A + A^T$.

Preliminaries

A nominal system is given as

$$\dot{x}(t) = A(t)x(t) + A_d(t)x(t - d(t)), \quad t \geq 0,$$

$$x(t) = \phi(t), \quad t \in [-d_N, 0], \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is state vector, $A(t)$, $A_d(t)$ are the real matrices with appropriate dimensions, $d(t)$ is time-varying delay, $\phi(t) \in C([-d_N, 0], \mathbb{R}^n)$ is the initial function.

For any $t \geq 0$, the time-varying delay $d(t)$ is described as two categories:

Case 1: $d(t)$ – a differentiable function satisfying.

$$0 \leq d_0 \leq d(t) \leq d_N, \ \mu_1 \leq \dot{d}(t) \leq \mu_2. \tag{2}$$

Case 2: $d(t)$ – a continuous function satisfying.

$$0 \leq d_0 \leq d(t) \leq d_N, \tag{3}$$

where d_0, d_N, μ_1, μ_2 are constants.

Considering that uncertainties exist in the system, parameters are represented as $\mathcal{J}_v(t)$, $(v = 1, 2)$, and $\mathcal{J}_1(t) = A(t)$, $\mathcal{J}_2(t) = A_d(t)$, which are not exactly known and might be taken from an interval $\mathcal{J}_v(t) \in [\mathcal{J}_{v1}, \mathcal{J}_{v2}]$. Then the parameters with uncertainties satisfy

$$\mathcal{J}_v(t) = \Pi_1(t)\mathcal{J}_{v1} + \Pi_2(t)\mathcal{J}_{v2} = \sum_{o=1}^{2}\Pi_0(t)\mathcal{J}_{v0}, \tag{4}$$

with any constant $\Pi_1(t) \geq 0$, $\Pi_2(t) \geq 0$ satisfying $\Pi_1(t) + \Pi_2(t) = 1$.

Some lemmas are employed as follows:

Lemma 1 (ref. 19). *Considering any matrix $\tilde{\Re} > 0$, and a continuously differentiable function z: $[\alpha^-, \alpha^+] \to \mathbb{R}^n$, the next inequality holds*

$$-(\alpha^+ - \alpha^-)\int_{\alpha^-}^{\alpha^+} \dot{z}^T(s)\tilde{\Re}\dot{z}(s)\mathrm{d}s \leq \varepsilon^T(t)\Theta \in (t), \tag{5}$$

where

$$\varepsilon(t) = \left[z^T(\alpha^+) z^T(\alpha^-) \frac{1}{(\alpha^+ - \alpha^-)} \int_{\alpha^-}^{\alpha^+} z^T(s)\mathrm{d}s \right]^T$$

and

$$\Theta = \begin{bmatrix} -4\tilde{\Re} & -2\tilde{\Re} & 6\tilde{\Re} \\ * & -4\tilde{\Re} & 6\tilde{\Re} \\ * & * & -12\tilde{\Re} \end{bmatrix}.$$

Lemma 2 (ref. 29). *Let z: $[\alpha^-, \alpha^+] \to \mathbb{R}^n$ be a differentiable function, $Z \in \mathbb{R}^{n\times n}$ and $\mathcal{J}_1$, $\mathcal{J}_3 \in \mathbb{R}^{3n\times 3n}$ be symmetric matrices, and $\mathcal{J}_2 \in \mathbb{R}^{3n\times 3n}$, $\mathcal{S}_1$, $\mathcal{S}_2 \in \mathbb{R}^{3n\times 3n}$ satisfying this condition,*

$$\begin{bmatrix} \mathcal{J}_1 & \mathcal{J}_2 & \mathcal{S}_1 \\ * & \mathcal{J}_3 & \mathcal{S}_2 \\ * & * & \mathcal{Z} \end{bmatrix} \geq 0$$

it holds

$$-\int_{\alpha^-}^{\alpha^+} \dot{z}^T(s)\mathcal{Z}\dot{z}(s)\mathrm{d}s \leq \varsigma^T \Pi \varsigma, \tag{6}$$

where

$$\varsigma = \left[z^T(\alpha^+) z^T(\alpha^-) \frac{1}{\alpha^+ - \alpha^-} \int_{\alpha^-}^{\alpha^+} z^T(s)\mathrm{d}s \right]^T,$$

$$\Pi = (\alpha^+ - \alpha^-)\left(\mathcal{J}_1 + \frac{1}{3}\mathcal{J}_3 \right) + \mathrm{He}(\mathcal{S}_1\vartheta_1 + \mathcal{S}_2\vartheta_2),$$

$$\vartheta_1 = \tilde{e}_1 - \tilde{e}_2, \ \vartheta_2 = 2\tilde{e}_3 - \tilde{e}_1 - \tilde{e}_2, \ \tilde{e}_1 = [I \ \ 0 \ \ 0],$$

$$\tilde{e}_2 = [0 \ \ I \ \ 0], \ \tilde{e}_3 = [0 \ \ 0 \ \ I].$$

Lemma 3 (ref. 30). *For any vectors $f_1, \ldots, f_N$, scalar $\gamma_i(t) \in [0, 1]$, $\sum_{i=1}^{N}\gamma_i(t) = 1$, and matrices $\Re_i > 0$, there exists matrix $\mathcal{S}_{ij}(i = 1, \ldots, N-1, j = i+1, \ldots, N)$ satisfies*

$$\begin{bmatrix} \Re_i & \mathcal{S}_{ij} \\ * & \Re_j \end{bmatrix} \geq 0,$$

then the next inequality holds:

$$-\sum_{i=1}^{N}\frac{1}{\gamma_i(t)} f_i^T \Re_i f_i \leq -\begin{bmatrix} f_1 \\ \vdots \\ f_2 \end{bmatrix}^T \begin{bmatrix} \Re_1 & \cdots & \mathcal{S}_{1,N} \\ * & \ddots & \vdots \\ * & * & \Re_N \end{bmatrix} \begin{bmatrix} f_1 \\ \vdots \\ f_N \end{bmatrix}.$$

Lemma 4 (ref. 31). *For a symmetric positive matrix $\tilde{\Re} \in \mathbb{R}^{n\times n}$ and differentiable function* z: $[\alpha^-, \alpha^+] \to \mathbb{R}^{n\times n}$, *then the next double integral inequality holds*

$$-\int_{\alpha^-}^{\alpha^+}\int_{\theta}^{\alpha^+} \dot{z}^T(s)\tilde{\Re}\dot{z}(s)\mathrm{d}s\mathrm{d}\theta \le \eta^T(t)\Omega\eta(t), \tag{7}$$

where

$$\eta(t) = \left[z^T(\alpha^+), \frac{1}{(\alpha^+ - \alpha^-)}\int_{\alpha^-}^{\alpha^+} z^T(s)\mathrm{d}s, \right.$$

$$\left. \frac{1}{(\alpha^+ - \alpha^-)^2}\int_{\alpha^-}^{\alpha^+}\int_{\theta}^{\alpha^+} z^T(s)\mathrm{d}s\mathrm{d}\theta \right]^T$$

and

$$\Omega = \begin{bmatrix} -6\tilde{\Re} & -6\tilde{\Re} & 24\tilde{\Re} \\ * & -18\tilde{\Re} & 48\tilde{\Re} \\ * & * & -144\tilde{\Re} \end{bmatrix}.$$

Lemma 5 (ref. 32). *Let* $\xi \in \mathbb{R}^n$, $\Phi = \Phi^T \in \mathbb{R}^{n\times n}$ *and* $\mathfrak{B} \in \mathbb{R}^{m\times n}$ *with* $\operatorname{rank}(\mathfrak{B}) < n$. *The next statements are equivalent:*

(i) $\xi^T\Phi\xi < 0$, $\forall \mathfrak{B}\xi = 0$, $\xi \ne 0$;

(ii) $\mathfrak{B}^{\perp T}\Phi\mathfrak{B}^{\perp} < 0$;

(iii) $\exists \mathfrak{D} \in \mathbb{R}^{n\times m}$: $\Phi + \mathrm{He}(\mathfrak{D}\mathfrak{B}) < 0$.

where

$\mathfrak{B}^{\mathrm{T}} \in \mathbb{R}^{n\times(n-\operatorname{rank}(\mathfrak{B}))}$ *is the right orthogonal complement of* $\mathfrak{B}$.

Stability analysis

A GSD based delay partition method is employed in Figure 1.

For any integral $N \ge 1$, the interval $[d_0, d_N]$ is separated into N subintervals as

$$\begin{cases} \rho_i = \lambda^i \\ d_i = d_0 + \sum_{a=1}^{i} \lambda^{\alpha}, \ i = 1,\ldots,N, \end{cases} \tag{8}$$

where λ is a real positive number, and ρ_i is the length of the ith subinterval that is equal to λ^i. It is obtained as $[d_0, d_N] = \bigcup_{k=1}^{N} l_k$. There exists an integer $k \in \{1, \ldots, N\}$ $\forall \ge 0$, s.t. $d(t) \in l_k$.

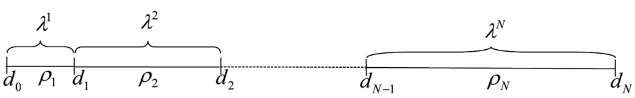

Figure 1. GSD delay partition method.

$$e_j = \Big[\underbrace{0_n,\ldots,0_n}_{j-1}, I_n, \underbrace{0_n,\ldots,0_n}_{4N-j+5}\Big]^T \in R^{(4N+5)n\times n},$$

$$j = 1, \ldots, 4N+5. \tag{9}$$

The augmented vector is defined as

$$\varphi(t) = \begin{bmatrix} x^T(t),\ x^T(t-d_0),\ \varphi_1^T(t),\ \dot{x}^T(t),\ x^T(t-d(t)), \\ \varphi_2^T(t),\ \int_{t-d(t)}^{t} x^T(s)\mathrm{d}s,\ \varphi_3^T(t),\ \varphi_4^T(t) \end{bmatrix}, \tag{10}$$

where

$$\varphi_1(t) = [x^T(t-d_1),\ldots,x^T(t-d_N)]^T,$$

$$\varphi_2(t) = \left[\int_{t-d_1}^{t} x^T(s)\mathrm{d}s,\ldots,\int_{t-d_N}^{t} x^T(s)\mathrm{d}s\right]^T,$$

$$\varphi_3(t) = \left[\frac{1}{\rho_1}\int_{t-d_1}^{t-d_0} x^T(s)\mathrm{d}s,\ldots,\frac{1}{\rho_N}\int_{t-d_N}^{t-d_{N-1}} x^T(s)\mathrm{d}s\right]^T,$$

$$\varphi_4(t) = \left[\frac{1}{(\rho_1)^2}\int_{-d_1}^{-d_0}\int_{t+\theta}^{t-d_0} x^T(s)\mathrm{d}s\mathrm{d}\theta,\ldots,\right.$$

$$\left.\frac{1}{(\rho_N)^2}\int_{-d_N}^{-d_{N-1}}\int_{t+\theta}^{t-d_{N-1}} x^T(s)\mathrm{d}s\mathrm{d}\theta\right]^T.$$

Theorem 1. *Given an integer* $N > 0$, *and* $\rho_i = \lambda^i$. *Consider delay* $d(t)$ *satisfying Case 1. The system (1) is asymptotically stable if there exists symmetric positive definite matrices* $\hat{R}_i, \tilde{R}_i, P_i, \hat{P}, \hat{Z}_i \in \mathbb{R}^{n\times n}$ $(i = 1, \ldots, N)$, $\tilde{P}_i \in \mathbb{R}^{n\times n}$ $(i = 1, \ldots, k)$, $\tilde{U}_i \in \mathbb{R}^{n\times n}$ $(i = k, \ldots, N)$, $\mathfrak{B} \in \mathbb{R}^{(N+2)n\times(N+2)n}$, *symmetric matrices* $\mathcal{J}_1, \mathcal{J}_3 \in R^{3n\times 3n}$, *matrices* $\mathcal{J}_2 \in R^{3n\times 3n}$, $\mathcal{S}_1, \mathcal{S}_2 \in R^{3n\times n}$, $\mathcal{J} \in \mathbb{R}^{n\times n}$, *and* $\mathcal{G} \in \mathbb{R}^{(4N+5)n\times n}$*d, such that the next LMIs hold*

$$\begin{bmatrix} \mathcal{J}_1 & \mathcal{J}_2 & \mathcal{S}_1 \\ * & \mathcal{J}_3 & \mathcal{S}_2 \\ * & * & \tilde{R}_i \end{bmatrix} \ge 0, i = 1,\ldots,N, \tag{11}$$

$$\sum_{o=1}^{2}\Pi_o(t)\sum_{s_{1,2}=1}^{2}\sigma_{s_{1,2}}(t)\sum_{s=1}^{2}\sigma_s(t)(\Xi_{ks_1s_2} + \mathrm{He}(\mathcal{G}\mathfrak{I}_0)) < 0,$$

$$k = 1,\ldots,N, \tag{12}$$

where

$$\Im_o = A(t)e_1^T + A_d(t)e_{N+4}^T - e_{N+3}^T$$

$$\Xi_{ks_1s_2} = \Xi_{1s} + \Xi_{2s} + \Xi_{3ks_1s_2} + \Xi_{4,k} + \Xi_5 + e_{N+3}\mathfrak{Z}\, e_{N+3}^T$$

$$\Xi_{1s} = \mathrm{He}\left\{\begin{bmatrix} e_1^T \\ e_{N+5}^T \\ \vdots \\ e_{2N+4}^T \\ e_{2N+5}^T \end{bmatrix}^T \mathfrak{B} \begin{bmatrix} e_{N+3}^T \\ e_1^T - e_3^T \\ \vdots \\ e_1^T - e_{N+2}^T \\ e_1^T - (1-\mu_s)e_{N+4}^T \end{bmatrix}\right\}$$

$$\Xi_{2s} = \sum_{i=1}^{N}(e_{i+1}P_i e_{i+1}^T - e_{i+2}P_i e_{i+2}^T) + e_1\hat{P}e_1^T$$

$$-(1-\mu_s)e_{N+4}\hat{P}e_{N+4}^T$$

$$\Xi_{3ks_1s_2} = \sum_{i=1}^{k-1}(e_{i+1}\tilde{P}_i e_{i+1}^T - e_{i+2}\tilde{P}_i e_{i+2}^T) + e_{k+1}\tilde{P}_k e_{k+1}^T$$

$$-(1-\mu_{s_1})e_{N+4}\tilde{P}_k e_{N+4}^T + (1-\mu_{s_2})e_{N+4}\tilde{U}_k e_{N+4}^T$$

$$-e_{k+2}\tilde{U}_k e_{k+2}^T + \sum_{i=k+1}^{N}(e_{i+1}\tilde{U}_i e_{i+1}^T - e_{i+2}\tilde{U}_i e_{i+2}^T)$$

$$\Xi_{4,k} = \sum_{i=1}^{N}\begin{bmatrix} e_1^T \\ e_{i+2}^T \\ \frac{1}{d_i}e_{N+4+i}^T \end{bmatrix}^T \Lambda_1 \begin{bmatrix} e_1^T \\ e_{i+2}^T \\ \frac{1}{d_i}e_{N+4+i}^T \end{bmatrix}$$

$$+\sum_{i=1,i\neq k}^{N}\begin{bmatrix} e_{i+1}^T \\ e_{i+2}^T \\ e_{2N+5+i}^T \end{bmatrix}^T \Lambda_2 \begin{bmatrix} e_{i+1}^T \\ e_{i+2}^T \\ e_{2N+5+i}^T \end{bmatrix} - \begin{bmatrix} e_{k+1}^T \\ e_{N+4}^T \\ e_{k+2}^T \end{bmatrix} \Lambda_3 \begin{bmatrix} e_{k+1}^T \\ e_{N+4}^T \\ e_{k+2}^T \end{bmatrix}$$

$$\Xi_5 = \sum_{i=1}^{N}\begin{bmatrix} e_{i+1}^T \\ e_{2N+5+i}^T \\ e_{3N+5+i}^T \end{bmatrix} \Lambda_4 \begin{bmatrix} e_{i+1}^T \\ e_{2N+5+i}^T \\ e_{3N+5+i}^T \end{bmatrix}$$

with

$$\mathfrak{Z} = \sum_{i=1}^{N}\lambda^{2i}\hat{R}_i + \sum_{i=1}^{N}\left(d_0 + \sum_{\alpha=1}^{i}\lambda^{\alpha}\right)^2 \tilde{R}_i$$

$$+\frac{1}{2}\left[(d_1^2 - d_0^2)\hat{Z}_1 + \sum_{i=2}^{N}\left(\left(d_0 + \sum_{\alpha=1}^{i}\lambda^{\alpha}\right)^2\right.\right.$$

$$\left.\left. -\left(d_0 + \sum_{\alpha=1}^{i-1}\lambda^{\alpha}\right)^2\right)\hat{Z}_i\right],$$

$$\Lambda_1 = d_i^2\left(\mathcal{J}_1 + \frac{1}{3}\mathcal{J}_3\right) + d_i\mathrm{He}(\mathcal{S}_1\vartheta_1 + \mathcal{S}_2\vartheta_2),$$

$$\Lambda_2 = \begin{bmatrix} -4\hat{R}_i & -2\hat{R}_i & 6\hat{R}_i \\ * & -4\hat{R}_i & 6\hat{R}_i \\ * & * & -12\hat{R}_i \end{bmatrix},$$

$$\Lambda_3 = \begin{bmatrix} \hat{R}_k & -\hat{R}_k + J & -J \\ * & 2\hat{R}_k - J^T - J & -\hat{R}_k + J \\ * & * & \hat{R}_k \end{bmatrix},$$

$$\Lambda_4 = \begin{bmatrix} -6\hat{Z}_i & -6\hat{Z}_i & 24\hat{Z}_i \\ * & -18\hat{Z}_i & 48\hat{Z}_i \\ * & * & -144\hat{Z}_i \end{bmatrix}.$$

Proof. For $\forall t \geq 0$, there exists an integer $k \in \{1, \ldots, N\}$, such that $d(t) \in l_k$. The LKF is proposed as

$$\mathcal{V}(x_t, k)|_{d(t)\in lk} = \mathcal{V}_1(x_t) + \mathcal{V}_2(x_t)$$

$$+\mathcal{V}_3(x_t,\, k) + \mathcal{V}_4(x_t,\, k) + \mathcal{V}_5(x_t), \qquad (13)$$

where

$$\mathcal{V}_1(x_t) = \begin{bmatrix} x(t) \\ \varphi_2(t) \\ \int_{t-d(t)}^{t} x^T(s)\mathrm{d}s \end{bmatrix}^T \mathfrak{B} \begin{bmatrix} x(t) \\ \varphi_2(t) \\ \int_{t-d(t)}^{t} x^T(s)\mathrm{d}s \end{bmatrix},$$

$$\mathcal{V}_2(x_t) = \sum_{i=1}^{N}\int_{t-\lambda^i}^{t} x^T(s-d_{i-1})P_i x(s-d_{i-1})\mathrm{d}s$$

$$+\int_{t-d(t)}^{t} x^T(s)\hat{P}x(s)\mathrm{d}s,$$

$$\mathcal{V}_3(x_t, k) = \sum_{i=1}^{k-1}\int_{t-\lambda^i}^{t} x^T(s-d_{i-1})\tilde{P}_i x(s-d_{i-1})\mathrm{d}s$$

$$+\int_{t-d(t)}^{t-d_{k-1}} x^T(s)\tilde{P}_k x(s)\mathrm{d}s+\int_{t-d_k}^{t-d(t)} x^T(s)\tilde{U}_k x(s)\mathrm{d}s$$

$$+\sum_{i=k+1}^{N}\int_{t-\lambda^i}^{t} x^T(s-d_{i-1})\tilde{U}_i x(s-d_{i-1})\mathrm{d}s,$$

$$\mathcal{V}_4(x_t,k)=\sum_{i=1}^{N}\left(d_0+\sum_{\alpha=1}^{i}\lambda^\alpha\right)$$

$$\int_{-(d_0+\sum_{\alpha=1}^{i}\lambda^\alpha)}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)\tilde{R}_i\dot{x}(s)\mathrm{d}s\mathrm{d}\theta$$

$$+\sum_{i=1}^{m}\lambda^i\int_{-(d_0+\sum_{\alpha=1}^{i}\lambda^\alpha)}^{-(d_0+\sum_{\alpha=1}^{i-1}\lambda^\alpha)}\int_{t+\theta}^{t}\dot{x}^T(s)\hat{R}_i\dot{x}(s)\mathrm{d}s\mathrm{d}\theta,$$

$$\mathcal{V}_5(x_t)=\sum_{i=1}^{N}\int_{-(d_0+\sum_{\alpha=1}^{i}\lambda^\alpha)}^{-(d_0+\sum_{\alpha=1}^{i-1}\lambda^\alpha)}\int_{\theta}^{0}\int_{t+\beta}^{t}\dot{x}^T(s)\hat{Z}_i\dot{x}(s)\mathrm{d}s\mathrm{d}\beta\mathrm{d}\theta,$$

In the case of $\mu_1 \le \dot{d}(t) \le \mu_2$, for any $\sigma_1(t) \ge 0$, $\sigma_2(t) \ge 0$ satisfying $\sigma_1(t) + \sigma_2(t) = 1$, let

$$\dot{d}(t)=\sigma_1(t)\mu_1+\sigma_2(t)\mu_2=\sum\nolimits_{s=1}^{2}\sigma_s(t)\mu_s,$$

then the LKF derivative along the trajectory of the linear system (1) is indicated as:

$$\dot{\mathcal{V}}(x_t,\ k)\,|_{d(t)\in I_k}=\dot{\mathcal{V}}_1(x_t)+\dot{\mathcal{V}}_2(x_t)$$

$$+\dot{\mathcal{V}}_3(x_t,\ k)+\dot{\mathcal{V}}_4(x_t,\ k)+\dot{\mathcal{V}}_5(x_t),\qquad(14)$$

where

$$\dot{\mathcal{V}}_1(x_t)=2\begin{bmatrix}x(t)\\ \varphi_2(t)\\ \int_{t-d(t)}^{t}x^T(s)\mathrm{d}s\end{bmatrix}^T\mathfrak{B}\begin{bmatrix}\dot{x}(t)\\ \dot{\varphi}_2(t)\\ x(t)-(1-\dot{d}(t))x(t-d(t))\end{bmatrix}$$

$$=2\begin{bmatrix}x(t)\\ \varphi_2(t)\\ \int_{t-d(t)}^{t}x^T(s)\mathrm{d}s\end{bmatrix}^T\mathfrak{B}\begin{bmatrix}\dot{x}(t)\\ \dot{\varphi}_2(t)\\ x(t)-(1-\sum\nolimits_{s=1}^{2}\sigma_s(t)\mu_s)x(t-d(t))\end{bmatrix}$$

$$=\varphi^T(t)\sum_{s=1}^{2}\sigma_s(t)\Xi_{1s}\varphi(t).$$

The derivative of $\mathcal{V}_2(x_t)$ is derived as

$$\dot{\mathcal{V}}_2(x_t)=\sum_{i=1}^{N}(x^T(t-d_{i-1})P_i x(t-d_{i-1})$$

$$-x^T(t-d_i)P_i x(t-d_i))+x^T(t)\hat{P}x(t)$$

$$-\left(1-\sum_{s=1}^{2}\sigma_s(t)\mu_s\right)x^T(t-d(t))\hat{P}x(t-d(t))$$

$$=\varphi^T(t)\sum_{s=1}^{2}\sigma_s(t)\Xi_{2s}\varphi(t).\qquad(15)$$

$\dot{\mathcal{V}}_3(x_t,\ k)$ is derived as

$$\dot{\mathcal{V}}_3(x_t,k)=\sum_{i=1}^{k-1}(x^T(t-d_{i-1})\tilde{P}_i x(t-d_{i-1})$$

$$-x^T(t-d_i)\tilde{P}_i x(t-d_i))$$

$$+x^T(t-d_{k-1})\tilde{P}_k x(t-d_{k-1})$$

$$-\left(1-\sum_{s_1=1}^{2}\sigma_{s_1}\mu_{s_1}\right)x^T(t-d(t))\tilde{P}_k x(t-d(t))$$

$$+\left(1-\sum_{s_2=1}^{2}\sigma_{s_2}\mu_{s_2}\right)x^T(t-d(t))\tilde{U}_k x(t-d(t))$$

$$-x^T(t-d_k)\tilde{U}_k x(t-d_k)$$

$$+\sum_{i=k+1}^{N}(x^T(t-d_{i-1})\tilde{U}_i x(t-d_{i-1})$$

$$-x^T(t-d_i)\tilde{U}_i x(t-d_i))$$

$$=\varphi^T(t)\sum_{s_1=1}^{2}\sum_{s_2=2}^{2}\sigma_{s_1}\sigma_{s_2}\Xi_{3ks_1s_2}\varphi(t).\qquad(16)$$

The derivative of $\mathcal{V}_4(x_t, k)$ is deduced as

$$\dot{\mathcal{V}}_4(x_t,k)=\dot{x}^T(t)\left(\sum_{i=1}^{N}\lambda^{2i}\hat{R}_i+\sum_{i=1}^{N}\left(d_0+\sum_{\alpha=1}^{i}\lambda^\alpha\right)^2\tilde{R}_i\right)\dot{x}(t)$$

$$-\sum_{i=1}^{N}\left(d_0+\sum_{\alpha=1}^{i}\lambda^\alpha\right)\int_{t-(d_0+\sum_{\alpha=1}^{i}\lambda^\alpha)}^{t}\dot{x}^T(s)\tilde{R}_i\dot{x}(s)\mathrm{d}s$$

$$-\sum_{i=1}^{N}\lambda^i\int_{t-(d_0+\sum_{\alpha=1}^{i}\lambda^\alpha)}^{t-(d_0+\sum_{\alpha=1}^{i-1}\lambda^\alpha)}\dot{x}^T(s)\hat{R}_i\dot{x}(s)\mathrm{d}s.\qquad(17)$$

Applying Lemma 2 to deal with the second term of (17), it is given as

$$-\sum_{i=1}^{N}\left(d_0+\sum_{\alpha=1}^{i}\lambda^{\alpha}\right)\int_{t-(d_0+\sum_{\alpha=1}^{i}\lambda^{\alpha})}^{t}\dot{x}^T(s)\tilde{R}_i\dot{x}(s)\mathrm{d}s$$

$$\leq\sum_{i=1}^{N}\varrho_{1i}^T(t)\Lambda_1\varrho_{1i}(t), \tag{18}$$

where

$$\varrho_{1i}(t)=\left[x^T(t)x^T(t-d_i)\frac{1}{d_i}\int_{t-d_i}^{t}x^T(s)\mathrm{d}s\right]^T,$$

Λ_1 is defined in Theorem 1.

Using Lemma 1 and 3 for the third term of (17), it is derived as

$$-\sum_{i=1}^{N}\lambda^i\int_{t-(d_0+\sum_{\alpha=1}^{i}\lambda^{\alpha})}^{t-(d_0+\sum_{\alpha=1}^{i-1}\lambda^{\alpha})}\dot{x}^T(s)\hat{R}_i\dot{x}(s)\mathrm{d}s$$

$$=-\sum_{i=1,i\neq k}^{N}\lambda^i\int_{t-(d_0+\sum_{\alpha=1}^{i}\lambda^{\alpha})}^{t-(d_0+\sum_{\alpha=1}^{i-1}\lambda^{\alpha})}\dot{x}^T(s)\hat{R}_i\dot{x}(s)\mathrm{d}s$$

$$-\rho k\int_{t-d_k}^{t-d_{k-1}}\dot{x}^T(s)\hat{R}_k\dot{x}(s)\mathrm{d}s$$

$$\leq\sum_{i=1,i\neq k}^{N}\varrho_{2i}^T(t)\Lambda_2\varrho_{2i}(t)-\varrho_3^T(t)\Lambda_3\varrho_3(t), \tag{19}$$

where

$$\varrho_{2i}(t)=\left[x^T(t-d_{i-1})x^T(t-d_i)\frac{1}{\rho_i}\int_{t-d_i}^{t-d_{i-1}}x^T(s)\mathrm{d}s\right]^T,$$

$$\varrho_{3i}(t)=[x^T(t-d_{k-1})x^T(t-d(t))x^T(t-d_k)]^T,$$

Λ_2 and Λ_3 are defined in Theorem 1.

Then, it follows from eqs (17)–(19) that

$$\dot{\mathcal{V}}_4(x_t)\leq\dot{x}^T(t)\left(\sum_{i=1}^{N}\lambda^{2i}\hat{R}_i+\sum_{i=1}^{N}\left(d_0+\sum_{\alpha=1}^{i}\lambda^{\alpha}\right)^2\tilde{R}_i\right)\dot{x}(t)$$

$$+\varphi^T(t)\Xi_{4,k}\varphi(t). \tag{20}$$

The derivative of $\mathcal{V}_5(x_t)$ is deduced as

$$\dot{\mathcal{V}}_5(x_t)\leq\dot{x}^T(t)\frac{1}{2}\left[(d_1^2-d_0^2)\hat{Z}_1+\sum_{i=2}^{N}\left(\left(d_0+\sum_{\alpha=1}^{i}\lambda^{\alpha}\right)^2\right.\right.$$

$$\left.\left.-\left(d_0+\sum_{\alpha=1}^{i-1}\lambda^{\alpha}\right)^2\right)\hat{Z}_i\right]\dot{x}^T(t)$$

$$-\int_{-d_1}^{-d_0}\int_{t+\theta}^{t}\dot{x}^T(s)\hat{Z}_1\dot{x}(s)\mathrm{d}s\mathrm{d}\theta$$

$$-\sum_{i=2}^{N}\int_{-(d_0+\sum_{\alpha=1}^{i}\lambda^{\alpha})}^{-(d_0+\sum_{\alpha=1}^{i-1}\lambda^{\alpha})}\int_{t+\theta}^{t}\dot{x}^T(s)\hat{Z}_i\dot{x}(s)\mathrm{d}s\mathrm{d}\theta. \tag{21}$$

Applying Lemma 4，the above last two terms are derived as

$$-\int_{-d_1}^{-d_0}\int_{t+\theta}^{t}\dot{x}^T(s)\hat{Z}_1\dot{x}(s)\mathrm{d}s\mathrm{d}\theta$$

$$-\sum_{i=2}^{N}\int_{-(d_0+\sum_{\alpha=1}^{i}\lambda^{\alpha})}^{-(d_0+\sum_{\alpha=1}^{i-1}\lambda^{\alpha})}\int_{t+\theta}^{t}\dot{x}^T(s)\hat{Z}_i\dot{x}(s)\mathrm{d}s\mathrm{d}\theta$$

$$\leq\sum_{i=2}^{N}\varrho_{4i}^T(t)\Lambda_4\varrho_{4i}(t), \tag{22}$$

where

$$\varrho_{4i}(t)=\left[x^T(t-d_{i-1}),\frac{1}{\rho_i}\int_{t-d_i}^{t-d_{i-1}}x^T(s)\mathrm{d}s,\right.$$

$$\left.\frac{1}{(\rho_i)^2}\int_{-d_i}^{-d_{i-1}}\int_{t+\theta}^{t-d_{i-1}}x^T(s)\mathrm{d}s\mathrm{d}\theta,\right]^T.$$

Λ_4 is defined in Theorem 1. Then it is derived as

$$\dot{\mathcal{V}}_5(x_t)\leq\dot{x}^T(t)\frac{1}{2}\left[(d_1^2-d_0^2)\hat{Z}_1+\sum_{i=2}^{N}\left(\left(d_0+\sum_{\alpha=1}^{i}\lambda^{\alpha}\right)^2\right.\right.$$

$$\left.\left.-\left(d_0+\sum_{\alpha=1}^{i-1}\lambda^{\alpha}\right)^2\right)\hat{Z}_i\right]\dot{x}^T(t)+\varphi^T(t)\Xi_5\varphi(t). \tag{23}$$

Hence, the next inequality holds

$$\dot{\mathcal{V}}(x_t,k)|_{d(t)\in l_k} \le \varphi^T(t)\sum_{s_{1,2}=1}^{2}\sigma_{s_{1,2}}(t)\sum_{s=1}^{2}\sigma_s(t)\Xi_{ks_{1,2}}\varphi(t). \quad (24)$$

Applying the vector (eq. 10) with the simplified expression (eq. 9), the linear system (eq. 1) is represented as

$$0 = \mathfrak{F}_0\varphi(t), \quad (25)$$

where $\mathfrak{F}_0$ is described in Theorem 1.

Hence, the asymptotic stability result of the system (eq. 1) is presented as

$$\varphi^T(t)\sum_{s_{1,2}=1}^{2}\sigma_{1,2}(t)\sum_{s=1}^{2}\sigma_s(t)\Xi_{ks_1,s_2}\varphi(t) < 0 \quad (26)$$

subject to: $0 = \mathfrak{F}_0\varphi(t)$.

So, using Lemma 5, there will exist a matrix $\mathcal{G}$ such that eq. (26) is equivalent to

$$\varphi^T(t)\sum_{o=1}^{2}\Pi_0(t)\sum_{s_{1,2}=1}^{2}\sigma_{s_{1,2}}(t)\sum_{s=1}^{2}\sigma_s(t)(\Xi_{ks_1s_2}+\mathrm{He}(\mathcal{G}\mathfrak{F}_0))\varphi(t) < 0. \quad (27)$$

Thus the newly proposed LKF derivative is obtained as $\dot{\mathcal{V}}(x_t,\,k)|_{d(t)\in lk} < 0$. It means $\dot{\mathcal{V}}(x_t,\,k)|_{d(t)\in l_k} \le \xi\,\|\,x(t)\,\|^2$ for a sufficiently small $\xi > 0$. So the system (eq. 1) is asymptotically stable. This completes the proof.

Theorem 2. *Given an integer* $N > 0$, *and* $\rho_i = \lambda^i$. *Consider delay* $d(t)$ *satisfying Case 2. The system* (*eq.* 1) *is asymptotically stable if there exists symmetric positive definite matrices* $\hat{R}_i, \tilde{R}_i, P_i, \hat{Z}_i \in \mathbb{R}^{n\times n}(i = 1,\ldots,N)$, $\hat{\mathfrak{B}} \in \mathbb{R}^{(N+1)n\times(N+1)n}$, *symmetric matrices* $\mathcal{J}_1, \mathcal{J}_3 \in \mathbb{R}^{3n\times3n}$, *matrices* $\mathcal{J}_2 \in \mathbb{R}^{3n\times3n}$, $\mathcal{S}_1, \mathcal{S}_2 \in R^{3n\times n}$, $\mathcal{J} \in \mathbb{R}^{n\times n}$, *and* $\mathcal{G} \in \mathbb{R}^{(4N+5)n\times n}$, *such that the next LMIs hold*

$$\begin{bmatrix} \mathcal{J}_1 & \mathcal{J}_2 & \mathcal{S}_1 \\ * & \mathcal{J}_3 & \mathcal{S}_2 \\ * & * & \tilde{R}_i \end{bmatrix} \ge 0, i = 1,\ldots,N. \quad (28)$$

$$\sum_{o=1}^{2}\Pi_0(t)(\hat{\Xi}_k + \mathrm{He}(\mathcal{G}\mathfrak{F}_0)) < 0. \quad (29)$$

where $\Xi_{ks_1s_2}$ *is modified to* $\hat{\Xi}_k$ *by removing* $\Xi_{3ks_1s_2}$ *and replacing* Ξ_{1s}, Ξ_{2s} *to* $\hat{\Xi}_1, \hat{\Xi}_2$ *as follows*

$$\hat{\Xi}_k = \hat{\Xi}_1 + \hat{\Xi}_2 + \hat{\Xi}_{4,k} + \Xi_5 + e_{N+3}\mathfrak{Z}e_{N+3}^T,$$

$$\hat{\Xi}_1 = \mathrm{He}\left\{\begin{bmatrix} e_1^T \\ e_{N+5}^T \\ \vdots \\ e_{2N+4}^T \end{bmatrix}^T \mathfrak{B}\begin{bmatrix} e_{N+3}^T \\ e_1^T - e_3^T \\ \vdots \\ e_1^T - e_{N+2}^T \end{bmatrix}\right\},$$

$$\hat{\Xi}_2 = \sum_{i=1}^{N}(e_{i+1}P_ie_{i+1}^T - e_{i+2}P_ie_{i+2}^T),$$

$\mathfrak{F}_o$ *are defined in Theorem 1.*

Proof. Modify the Lyapunov functionals (eq. (13)) by changing $\mathcal{V}_1(x_t)$, $\mathcal{V}_2(x_t)$ and removing $\mathcal{V}_3(x_t, k)$ as

$$\mathcal{V}(x_t,\,k)|_{d(t)\in l_k} = \hat{\mathcal{V}}_1(x_t) + \hat{\mathcal{V}}_2(x_t) + \mathcal{V}_4(x_t,\,k) + \mathcal{V}_5(x_t), \quad (30)$$

where

$$\hat{\mathcal{V}}_1(x_t) = \begin{bmatrix} x(t) \\ \varphi_2(t) \end{bmatrix}^T \hat{\mathfrak{B}}\begin{bmatrix} x(t) \\ \varphi_2(t) \end{bmatrix},$$

$$\hat{\mathcal{V}}_2(x_t) = \sum_{i=1}^{N}\int_{t-\lambda^i}^{t} x^T(s-d_{i-1})P_ix(s-d_{i-1})\mathrm{d}s.$$

Then following the same procedures of proof of Theorem 1, the stability criteria will be equivalent to

$$\varphi^T(t)\sum_{o=1}^{2}\Pi_0(t)(\hat{\Xi}_k + \mathrm{He}(\mathcal{G}\mathfrak{F}_0))\varphi(t) < 0. \quad (31)$$

This completes the proof.

Illustrative examples

Example 1. Consider a nominal system (eq. (1)) with the parameters discussed in refs 28 and 34 are given as

$$A = \begin{bmatrix} -0.5 & -2 \\ 1 & -1 \end{bmatrix},\ A_d = \begin{bmatrix} -0.5 & -1 \\ 0 & 0.6 \end{bmatrix}.$$

Considering ±20% parameters uncertainties regarding eq. (4), i.e. $\mathcal{J}_v(t) \in [\mathcal{J}_v(t) \times 80\%, \mathcal{J}_v(t) \times 120\%]$. By choosing $d_0 = 0$ and the maximum value of d_N is obtained with different μ.

In the case of $\mu = 0.9$ and $d_N = 0.5412$, the state response of eq. (1) is shown in Figure 2.

Table 1 compares maximum upper bounds d_N with different values of μ and $d_0 = 0$. It clearly shows that for $\mu = 0.9$ and $\mu \ge 1$ this proposed approach presents a bigger d_N than the results in refs 28 and 34, with $N = 3$. Figure 2 indicates that the state converges to zero with $d_N = 0.5412$, which means the system (eq. (1)) is globally asymptotically stable under the obtained maximum value of $d(t)$.

Example 2. Consider a nominal system (eq. (1)) with the parameters discussed in refs 31 and 33, are given as

$$A = \begin{bmatrix} 0 & 1 \\ -10 & -1 \end{bmatrix},\ A_d = \begin{bmatrix} 0 & 0.1 \\ 0.1 & 0.2 \end{bmatrix}.$$

Considering time-varying delay satisfying case 2, the maximum value of d_N is obtained based on Theorem 2.

In Table 2, considering unknown μ and different d_0, the maximum upper bounds d_N are obtained. It shows that the proposed method improves the stability results compared to the previous works[31,33].

Remark 1. In this article, parameter uncertainties are taken into account for stability analysis of delayed linear system. Different from the existing results[28,31,33,34], a new LKF is constructed, which contains GSD-dependent integral forms with unfixed intervals. This novel design can reduce the number of decision variables and provide expected stability conditions with high efficiency and less computational complexity. A new expression of uncertainties is formulated in (eq. (4)) using convex combination method. Extra inequalities and constraint conditions can be eliminated compared to earlier research[14,21,31]. The stability criteria on robust analysis for linear system with time-varying delay is improved.

Table 1. Maximum value of d_N with $d_0 = 0$ and different vaules μ

Method	Peng and Tian[34]	Liu and Li[28]	Theorem 1 ($N = 3$)
$\mu = 0.9$	0.4760	0.5390	0.5412
$\mu \geq 1$	0.4760	0.5390	0.5407

Table 2. Upper bounds of d_N for unknown μ and different d_0

Method	Lee *et al.*[33]	Park *et al.*[31]	Theorem 2 ($N = 3$)
$d_0 = 0$	1.35	1.64	1.71
$d_0 = 1$	2.31	2.91	3.13

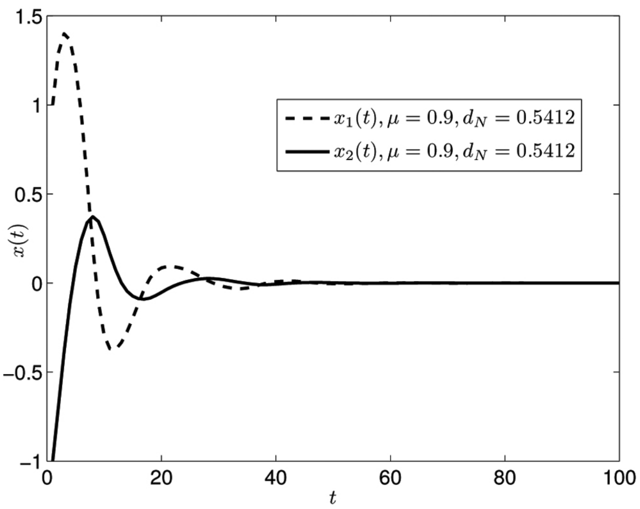

Figure 2. State of systems (eq. (1)).

Remark 2. This employed GSD method considerably improves the efficiency for obtaining the maximum value of $d(t)$. Sum formulation of the GSD method is $a_1(1 - \lambda^N)/(1 - \lambda)$ where a_1 is the first term and $\lambda \neq 1$. For example, let the first term $a_1 = 1$, $\lambda = 2$ and the partitioning number $N = 4$, that is $a_1(1 - \lambda^N)/(1 - \lambda) = 15$. This means that the partition number $N = 15$, if the common equivalent division method is selected. However, by using our GSD approach, the partition number $N = 4$, which is 30% less than the equivalently partitioning method. Hence, the decision variables are reduced considerably. In addition, if the common ratio $\lambda = 1$, then the sum will be $N \times a_1$ that coverts the length of subinterval to equal. Thus the previously produced works[14,35] using equivalent partition approach are the special cases of this proposed method.

Remark 3. When a system has high dimension, the computation burden is increased. It becomes more difficult to work out feasible solution. Lower dimension systems are commonly used for stability study. Additionally, big partitioning number requires much more computing time. In future, we will try to discover an improved method to reduce the computation cost.

Conclusion

In this study, stability conditions of nominal system with parameter uncertainties and interval time-varying delay are investigated by utilizing the GSD delay-partitioning method and convex combination approach. New GSD-dependent LKF is developed, which includes integral forms with geometric progression interval. Additionally, the convex combination method is proposed to flexibly estimate LKF derivative instead of using extra inequalities. Therefore, unexpected enlargement can be appropriately reduced. Meanwhile, less decision variables are used, because the proposed GSD approach reduces the number of partitioning subintervals. As a result, the computational burden is lessened. Numerical results demonstrate a good stability criteria. Due to the complex dynamics of nonlinear system, control of such systems is full of challenges. Recently, this research area has attracted a lot of attention. Thus, future studies should carry on the robust control of nonlinear systems with stochastic disturbances and uncertainties.

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1. Richard, J. P., Time-delay systems: an overview of some recent advances and open problems. *Automatica*, 2003, **39**, 1667–1694.
2. Ekramian, M., Ataei, M. and Talebi, S., Stability of nonlinear time-delay systems satisfying a quadratic constraint. *Trans. Inst. Meas. Control*, 2016; doi:10.1177/0142331216668003.

3. Fridman, E. and Shaikhet, L., Delay-induced stability of vector second-order systems via simple Lyapunov functionals. *Automatica*, 2016, **74**, 288–296.
4. Gouaisbaut, F. and Peaucelle, D., Delay-dependent stability analysis of linear time delay systems, IFAC Workshop on Time Delay System (TDS'06), L'Aquila, Italy, 2006.
5. Wu, Z.-G., Shi, P., Su, H. and Chu, J., Local synchronization of chaotic neural networks with sampled- data and saturating actuators. *IEEE Trans. Cybernetics*, 2014, **44**, 2635–2645.
6. Shi, K., Liu, X., Zhu, H., Zhong, S., Liu, Y. and Yin, C., Novel integral inequality approach on master-slave synchronization of chaotic delayed Lur's systems with sampled-data feedback control. *Nonlinear Dynam.*, 2016, **83**, 1259–1274.
7. Hua, C. C., Yang, X., Yan, J. and Guan, X. P., New exponential stability criteria for neural networks with time-varying delay. *IEEE Trans. Circuits Systems II: Exp. Briefs*, 2011, **58**, 931–935.
8. Zheng, C. D., Zhang, H. and Wang, Z., Novel exponential stability criteria of high-order neural networks with time-varying delays. *IEEE Trans. Syst. Man Cyb.*, 2011, **41**, 486–496.
9. Shi, K., Liu, X., Tang, Y., Zhu, H. and Zhong, S., Some novel approaches on state estimation of delayed neural networks. *Inform. Sci.*, 2016, **372**, 313–331.
10. Lakshmanan, S., Park, J. H., Jung, H., Kwon, O. and Rakkiyappan, R., A delay partitioning approach to delay-dependent stability analysis for neutral type neural networks with discrete and distributed delays. *Neurocomputing*, 2013, **111**, 81–89.
11. Gyurkovics, É., A note on Wirtinger-type integral inequalities for time-delay systems. *Automatica*, 2015, **61**, 44–46.
12. Niamsup, P., Ratchagit, K. and Phat, V., Novel criteria for finite-time stabilization and guaranteed cost control of delayed neural networks. *Neurocomputing*, 2015, **160**, 281–286.
13. Zhang, H., Wang, J., Wang, Z. and Liang, H., Mode-dependent stochastic synchronization for Markovian coupled neural networks with time-varying mode-delays. *IEEE Trans. Neural Net. Learn. Syst.*, 2015, **26**, 2621–2634.
14. Wang, C. and Shen, Y., Delay partitioning approach to robust stability analysis for uncertain stochastic systems with interval time-varying delay. *IET Control Theory Appl.*, 2012, **6**, 875–883.
15. Sun, J., Liu, G., Chen, J. and Rees, D., Improved delay-range-dependent stability criteria for linear systems with time-varying delays. *Automatica*, 2010, **46**, 466–470.
16. Gao, H. and Chen, T., Stabilization of nonlinear systems under variable sampling: a fuzzy control approach. *IEEE Trans. Fuzzy Syst.*, 2007, **15**, 972–983.
17. Fridman, E., Shaked, U. and Liu, K., New conditions for delay-derivative-dependent stability. *Automatica*, 2009, **45**, 2723–2727.
18. Park, P., Ko, J. W. and Jeong, C., Reciprocally convex approach to stability of systems with time-varying delays. *Automatica*, 2011, **47**, 235–238.
19. Seuret, A. and Gouaisbaut, F., Wirtinger-based integral inequality: application to time-delay systems. *Automatica*, 2013, **49**, 2860–2866.
20. Chen, W.-H., Jiang,Z., Lu, X. and Luo, S., H_∞ synchronization for complex dynamical networks with coupling delays using distributed impulsive control. *Nonlinear Anal: Hybrid Syst.*, 2015, **17**, 111–127.
21. Kchaou, M., Hajjaji, A. E. and Toumi, A., Non-fragile H_∞ output feedback control design for continuous-time fuzzy systems. *ISA Trans.*, 2015, **54**, 3–14.
22. Tang, Z., Park, J. H. and Lee, T. H., Dynamic output-feedback-based design for networked control systems with multipath packet dropouts. *Appl. Math. Comput.*, 2016, **275**, 121–133.
23. Tang, Z., Park, J. H. and Feng, J., Impulsive effects on quasi-synchronization of neural networks with parameter mismatches and time-varying delay. *IEEE Trans. Neural Net. Learn. Syst.*, 2017; doi:10.1109/TNNLS.2017.265102412.
24. Ali, M. S. and Saravanan, S., Robust finite-time H_∞ control for a class of uncertain switched neural networks of neutral-type with distributed time varying delays. Neurocomputing, 2016, **177**, 454–468.
25. Shi, K., Tang, Y., Liu, X. and Zhong, S., Non-fragile sampled-data robust synchronization of uncertain delayed chaotic Lurie systems with randomly occurring controller gain fluctuation. *ISA Trans.*, 2017, **66**, 185–199.
26. Bouarar, T., Guelton, K. and Manamanni, N., Robust fuzzy Lyapunov stabilization for uncertain and disturbed Takagi-Sugeno descriptors. *ISA Trans.*, 2010, **49**, 447–461.
27. Khazaee, M., Markazi, A. H. and Omidi, E., Adaptive fuzzy predictive sliding control of uncertain nonlinear systems with bound-known input delay. *ISA Trans.*, 2015, **59**, 314–324.
28. Liu, Y. and Li, M., Improved robust stabilization method for linear systems with interval time-varying input delays by using Wirtinger inequality. *ISA Trans.*, 2015, **56**, 111–122.
29. Zeng, H., He, Y., Wu, M. and She, J., Free-matrix-based integral inequality for stability analysis of systems with time-varying delay. *IEEE Trans. Automat. Control*, 2015, **60**, 2768–2772.
30. Wang, Z., Liu, L., Shan, Q.-H. and Zhang, H., Stability criteria for recurrent neural networks with time-varying delay based on secondary delay partitioning method. *IEEE Trans. Neural Net. Learn. Syst.*, 2015, **26**, 2589–2595.
31. Park, P., Lee, W. I. and Lee, S. Y., Auxiliary function-based integral inequalities for quadratic functions and their applications to time-delay systems. *J. Frank. Inst.*, 2015, **352**, 1378–1396.
32. de Oliveira, M. C. and Skelton, R. E., Stability tests for constrained linear systems. *Perspectives in Robust Control* (ed. Moheimani, S. R.), Springer-Verlag, Berlin, 2001, pp. 241–257.
33. Lee, W. I., Lee, S. Y. and Park, P., Improved criteria on robust stability and performance for linear systems with interval time-varying delays via new triple integral functionals. *Appl. Math. Comput.*, 2014, **243**, 570–577.
34. Peng, C. and Tian, Y.-C., Improved delay-dependent robust stability criteria for uncertain systems with interval time-varying delay. *IET Control Theory Appl.*, 2008, **2**, 752–761.
35. Zhao, Y., Gao, H., Lam, J. and Du, B., Stability and stabilization of delayed T–S fuzzy systems: a delay partitioning approach. *IEEE Trans. Fuzzy Syst.*, 2009, **17**, 750–762.

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