

Learning to think like a scientist and a mathematician

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This article proposes that the goals of math and science education ought to go beyond understanding and applying a body of established knowledge, to aim at developing the capacity to construct knowledge. This would empower students to think like mathematicians, experimental scientists and theoretical scientists. These mental capacities and the concepts that underlie them serve as the trans-disciplinary foundations for research, allow individuals to function well in their professional, public and personal lives, and integrate knowledge across domains. It also functions as an effective means for the popularization of mathematics and the sciences.

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The challenge

QUESTIONS of serious concern to the science–maths education world include the following:

- How do we attract the young to specialize in, and choose careers in, science and math?
- How do we develop among them the capacity to think like scientists and mathematicians?
- How do we build a component of research into tertiary education?

Responses to these questions from different quarters have been varied:

- Build in a ‘hands-on’ or ‘experiential learning’ component in science classrooms.
- Set-up science clubs in schools and colleges.
- Incorporate ‘project work’ into the curriculum.
- Conduct science competitions.
- Award prizes and scholarships to those who score high marks.
- Conduct workshops for science and maths teachers.
- Hold science camps.
- Provide research training through internships.

None of these solutions confronts the roots of the problem we face, namely poorly designed science and maths curricula. This includes not just what teachers and students do in the classroom, but the entirety of syllabi,

textbooks and exam questions that shape and constrain what students end up learning.

Inquiry-oriented education

It is unrealistic to expect secondary and tertiary teachers to be researchers in all areas of study – in maths, science, philosophy, history and so on. Granting this, how do we design syllabi, textbooks, lesson plans and examination questions to empower teachers to pursue activities in their classrooms to help students develop the capacity to think like mathematicians, scientists, philosophers and historians? We asked ourselves this question more than 30 years ago, and have been working on it ever since, designing appropriate syllabi, teaching–learning materials and assessment tasks, and testing them out in a range of field trials.

If an educational programme seeks to develop the capacity for mathematical inquiry, its syllabus must specify that by the end of the programme, students must have the following abilities:

- Noticing patterns and formulating them as conjectures.
- Proving conjectures.
- Critically evaluating the validity of proofs.
- Coming up with definitions, formulating them precisely and evaluating them.
- Creating abstractions and formulating the axioms governing them.

A textbook aligned to such a syllabus needs to include graded tasks that develop the specifics of each of these strands of abilities, and examination questions need to probe into the attainment of these abilities. Likewise, the

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syllabus of an educational programme that seeks to develop the capacity for scientific inquiry needs to specify that by the end of the programme, students must have the ability to

- Notice regularities in observable phenomena and formulate them as observational generalizations (including correlational hypotheses, distinguishing among them those that are causal).
- Design appropriate ways to test those hypotheses, using experimental or non-experimental observations, with or without instrumentation, within or outside laboratories, with or without quantitative (counting or numerical measurement) data.
- Come up with theoretical explanations for the established observational hypotheses, and critically evaluate them by deducing the predictions of the theory and testing them.
- Choose between competing theoretical explanations, using the criteria of simplicity and generality.

Similar syllabi need to be formulated for conceptual inquiry, ethical inquiry and historical inquiry, but we will not go into them here. Suffice it to say that cutting across these different modes of inquiry are tools like observing, reporting observations, noticing patterns, generalizing, classifying, building classificatory systems, defining, reasoning, justifying, debating and evaluating.

We use the term inquiry-oriented education (IOE) to refer to the strand of education that aims at inquiry abilities of the kind outlined above. IOE would help students become producers and evaluators of knowledge, rather than mere consumers of knowledge.

IOE is not a replacement for knowledge-oriented education (KOE) – the strand of education that aims to help students understand the concepts and propositions of knowledge. We believe that the two strands must combine in a mutually enriching and meaningful fashion. For this, the knowledge component in mainstream syllabi has to be significantly reduced to make room for IOE.

We must note that most approaches to educational reform, as signalled by movements like inquiry-based learning, constructivism, hands-on learning, discovery learning, experiential learning, activity-based learning, peer learning, collaborative learning, problem-based learning, project-based learning and blended learning are knowledge-oriented: they view the goals of education as being restricted to ‘knowledge transfer’, at best to understanding the concepts of knowledge. Their innovativeness and value lie in the pedagogical means to achieve the traditional goals. What we are proposing is a radical rethinking of the goals of secondary and tertiary education.

We must also add that the ideas for incorporating trans-disciplinary inquiry in school syllabi, textbooks and assessments do not replace the current attempts at institu-

tions like IISc and IISERs to provide apprenticeship in discipline-specific research to Bachelor’s and Master’s students. Rather, they provide school students with the trans-disciplinary foundations needed for pursuing research (typically discipline-specific) at the university level.

Syllabus specifications and learning tasks for IOE: examples

When combining KOE and IOE, it is crucial to ensure that the activities to develop inquiry abilities do not require a level of knowledge and mental capacities beyond what can be expected of students of a given age group. Thus, it would be unrealistic to expect eighth grade students to come up with a proof of the Pythagoras theorem or the infinity of prime numbers. However, it is indeed feasible to introduce them to the idea of scientific and mathematical proofs through a lesson plan sequence like the following:

Activity 1: Practice in constructing and evaluating definitions

The teacher raises the question, ‘What is a straight line?’ Students work in groups. Each group comes up with a definition. Chances are that none of the definitions will work, except for: ‘The line from A to B is a straight line if and only if it is the shortest path from A to B ’.

Activity 2: Practice in noticing regularities and formulating them as conjectures/hypotheses

The teacher asks students to draw random triangles and measure the length of each side. They then compare the length of each side with the sum of the lengths of the other two sides. The results are displayed in the form of a table on the blackboard. Students are invited to look for regularities in the data. Chances are that some of them will come up with: ‘In a triangle, no side can be longer than the sum of the other two sides’.

Activity 3: Practice in looking for counterexamples

Is the conjecture in activity 2 true? The first step is to look for counterexamples in the sample of triangles that we already have. Students check each instance and verify that no triangle in the sample violates the conjecture. But can there be other triangles outside the given sample that would be counterexamples? Students draw more triangles to test the conjecture further and discover that they cannot find a counterexample.

Activity 4: Practice in the use of inductive sample-to-population reasoning in scientific proofs

In activity 3, we were not able to find a counterexample to the conjecture/hypothesis from activity 2. The scientific proof for the hypothesis would be along the following lines: We examined a large random sample of triangles, and found no instance of a side that is longer than the sum of the other two sides. Until we find evidence to the contrary, therefore, it is reasonable to conclude that the statement, ‘In a triangle, no side is longer than the sum of the other two sides,’ is true of the entire population of triangles.

Activity 5: Practice in the use of deductive reasoning in mathematical proofs

As far as science is concerned, the degree of certainty of the proof in activity 4 is sufficient. However, it does not establish beyond doubt that there are no triangles in which one of the sides is longer than the sum of the other two sides. Mathematical proofs need certainty beyond such doubt; hence the above proof does not satisfy the criterion of truth in mathematics.

At this point the teacher may ask the students to form groups to come up with a mathematical proof based on axioms and definitions. If necessary, they can be given the following hint: Is it possible to come up with a mathematical proof based on the definitions of (a) polygons, (b) triangles and (c) straight lines?

Given this hint, students in most eighth-grade classrooms come up with a proof. The combination of the definitions of polygons and triangles requires that every side in a triangle be a straight line. In a triangle ABC , then, if AB is longer than $AC + CB$, AB is not the shortest path from A to B . Hence it is not a straight line; and ABC cannot be a triangle. Therefore AB cannot be longer than $AC + CB$ in any triangle.

Activity 6: Practice in generalizing

The teacher can now ask students to extend the conjecture from triangles to quadrilaterals, pentagons, hexagons and octagons, to generalize it to polygons, and prove the generalized conjecture. This should be easy, following the strategy in activity 5.

Many educators and textbook writers seem to think that ‘hands-on’ activities call for the use of one’s physical hands (such as measuring the period of a simple pendulum), ignoring the far more important ‘minds-on’ activities. Notice that none of the activities above, except for activity 2 calls for the use of physical hands. They all crucially require that students engage with the tasks using their minds.

The minds-on experience is especially important in helping students develop thinking abilities required in experimental science. If we distinguish the design of experiments from the execution of experiments, it becomes obvious that a great deal of experimental science can be learnt outside labs, without fancy instruments and other equipment. As an example, consider the following:

Activity 7: Practice in separating variables in experiment design

Miko and her brother Jomo planted the seeds of some very hot chillies in their garden. When the plants grew and produced chillies, they found that the chillies were not hot. Miko said the chillies were not hot because the plants did not get enough sunlight. Jomo said it was because the plants did not get enough water.

Was Miko right and Jomo wrong? Or was Jomo right and Miko wrong? Were they both right? Or were they both wrong? The teacher asks the students to work in groups to design an experiment to examine these options.

Activity 8: Practice in factoring out confounding variables

Miko and Jomo’s grandmother thinks that drinking *tulsi* (holy basil) tea cures common colds. The teacher asks the students to work in groups to design a ‘double blind’ experiment to test this hypothesis, with instructions to browse the internet to find out what ‘placebo effect’ and ‘double blind’ mean.

We believe that the so-called ‘lab’ sessions in secondary and tertiary education that aim at expertise in the competent execution of experimental procedures (measurement, instrumental observation, sensory-motor skills such as in dissection, etc.) nurture neither knowledge nor inquiry. It is important that school education helps all children develop the capacity to think like an experimental scientist. Our mainstream curriculum does not attempt to do this – it only provides the kind of training needed for lab technicians, not experimental scientists. For a science curriculum to be truly meaningful, it may be useful to shift the focus from implementing given procedures in lab classes to designing experiments in regular classes, and reflecting on the implications and consequences of their results.

Finally, although it is possible to help students develop the capacity for theoretical inquiry in science without any more content knowledge than they already have, they are hardly ever exposed to the art and craft of theory construction; this is true even of graduate students. Here is an example that we have successfully tried out with students ranging from grade 8 to Ph D:

Activity 9: Practice in the construction of scientific theories

The teacher invites students' attention to the following periodic correlations between time and temperature on any part of the earth.

1. If we measure the temperature anywhere on earth once every hour for several days, and plot it on a graph, we find that the temperature starts going up in the morning, and coming down in the afternoon, forming a daily cycle.
2. If we plot the average temperature of a day anywhere on earth for a few years, we find that the temperature goes up over a few months, and then comes down, forming a yearly cycle.

Students form groups to come up with an explanation for these two correlations, in terms of both the geocentric and heliocentric theories of the solar system.

They typically come up with two classes of explanation for the correlations. One is the distance theory, which holds that it is hotter when the sun's rays travel a smaller distance. At noon, for instance, the sun's rays travel less distance through the atmosphere than in the morning or in the afternoon. This explains correlation 1. Likewise, if the sun is at the centre of an elliptical orbit (in the heliocentric theory), it would be winter when the earth is far away and summer when it is closer. The other explanation relies on the distribution of energy across a larger or smaller area, depending on the angle of incidence of light.

Getting students to develop these theories with rigour and precision such that they can narrow down the set of admissible theories to a small subset is a fairly complex task. However, it is feasible in secondary or tertiary classrooms, and certainly desirable if nurturing future scientists is one of the goals of science education. It does not matter if they do not zero in on the 'correct' theory of seasons (tilt of the earth's rotation): what matters is the process of theory construction and evaluation.

Each of the activities illustrated above has a well-defined learning outcome that constitutes an important strand of inquiry. Going through a variety of such carefully constructed activities from relatively simple to somewhat challenging ones will give students adequate preparation not only to undertake research in science and mathematics, but also to extend the essentials of mathematical and scientific inquiries to domains beyond these disciplinary boundaries.

To supplement inquiry activities of this kind, we also need activities that nurture critical thinking and critical reading. And to supplement these in turn, it is important that we help students develop a critical understanding of some of the core elements of 'knowledge' that constitutes

the modern worldview. By 'critical understanding', we mean an understanding of the relevant evidence and arguments that support (or refute) the conclusions taken as 'knowledge', allowing students to address questions such as the following:

Our experience tells us that the earth is stationary. Why should we accept the conclusion that the earth revolves around the sun and rotates on an axis tilted to the plane of revolution?

Both ancient Greeks and ancient Indians believed that air is an element; why should we reject this position, and accept the conclusion that air is a mixture?

Aristotle believed that matter can in principle be divided infinitely, without the process ever ending. Democritus, in contrast, believed that the process of dividing matter comes to an end when we get to indivisible particles of matter that he called 'atomos'. Modern science rejects Aristotle's and accepts Democritus' position. Why should we do the same?

Darwin claimed that all existing and extinct life-forms on earth evolved from unicellular ancestor species. What is the evidence for us to accept this claim?

The questions we are raising above are analogous to the following questions in mathematics:

Why should we believe that the sum of angles in a triangle is two right angles?

Why should we believe that the sum of n consecutive integers is divisible by n if and only if n is an odd number?

In mathematics education, the response to such questions comes in the form of mathematical proofs, often provided in math textbooks. Science textbooks do not have a corresponding tradition: they do not provide proofs. In pointing to the need for critical understanding, we are essentially saying that we should minimally extend this tradition of maths education to science education as well.

Raising the quality of science and maths education through IOE

Inspired by these ideas, a number of like-minded educators have come together over the last few years to form a collective called ThinQ (www.schoolofthinq.com), dedicated to the cause of helping secondary and tertiary students develop the capacities of mathematical, scientific, conceptual and ethical inquiry. A comprehensive documentation of IOE, along the lines illustrated above, with extensive samples of syllabi, video and print teaching-learning materials and assessment tasks, is available as open source at the ThinQ website.

ThinQ also conducts an on-line course called Inquiry and Integration in Education (IIE), for anyone who

resonates to the IOE quest, to help them develop the capacity for inquiry across disciplinary boundaries, and the capacity to help students develop inquiry abilities (<http://www.schoolofthing.com/statics/ije2017>). A small subset of participants from the on-line course is invited to an advanced and intensive nine-day face-to-face workshop in IIE. These participants go on to pursue IOE as advocates, parents, teachers, teacher educators, materials producers and educational administrators. As a result of these initiatives, IOE has gradually been spreading to secondary and tertiary educational institutions.

To return to the three questions we raised at the beginning of this article, IOE helps develop:

- The trans-disciplinary foundations for research in a wide range of domains.
- The mental capabilities that allow individuals to function well in their professional, public and personal lives.

- The integration of knowledge across domains, otherwise fragmented as specialized fields, disciplines and discipline groups.

It also functions as an effective means for the popularization of mathematics, physical sciences, biological sciences, human sciences, philosophy and so on. It has been our experience, at secondary school, high school, college and university levels, that once they are exposed to the excitement of inquiry, students who otherwise hate a particular subject, whether maths, biology or history, become excited, realizing that even in those very subjects, they can move from the passenger's to the driver's seat.

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