Scaling of hydraulic functions in heterogeneous soil using nonlinear least squares minimization method

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Presenting soil heterogeneity precisely in various spatial scales is the main key to simulate water and solute transport through it. The method described by Richards is mostly used to study water flow through vadose zone. It requires spatial representation of hydraulic functions and water retention relationship in the soil. To represent the spatial relationship of soil hydraulic functions, scaling approach is being used since the last few decades. In this study, a simple scaling method using nonlinear least squares minimization technique has been used to scale soil matric potential, hydraulic conductivity as well as simultaneous scaling of soil matric potential and hydraulic conductivity data. Simultaneous scaling is necessary as it reduces the volume of data by producing a single set of scale factors for hydraulic functions in a heterogeneous soil. Van Genuchten's semi-empirical expressions were used in this study to parameterize soil hydraulic functions. Results showed that correlation coefficient from raw and descaled data was superior when soil matric potential and hydraulic conductivity data were scaled separately than simultaneously. Improvement of correlation coefficient in simultaneous scaling can be obtained by adding more weight to the soil matric potential data than unsaturated hydraulic conductivity data, which enhances the overall correlation coefficient in simultaneously scaling. Statistical analysis of the scale factors showed that they are lognormally distributed. Scale factors calculated by solving simple equations obtained using the method described in this study can be used to simulate water movement through heterogeneous soil conditions using HYDRUS model.

Keywords: Effective saturation, lognormal distribution, scaling, soil matric potential, unsaturated hydraulic conductivity.

INFORMATION about hydraulic functions of the soil is important for studying water and solute transport in the vadose zone. Soil scientists have been using deterministic and stochastic modelling to simulate water flow and solute/nutrient transport through the soil in different spatial scales. The hydraulic functions signify the relationship of soil water content with matric potential and hydraulic conductivity. Both these functions vary even in a field plot which apparently seems homogeneous. The differences in soil hydraulic functions are generally attributed to the variability in soil bulk density, texture and organic matter content. In large spatial scale, i.e. watershed, different simulation models are used to study water and solute transport. The efficiency of water flow prediction through the soil depends on the spatial representation of hydraulic properties. Irrespective of the scale, Richards' equation is used to simulate unsaturated water flow in the soil, which requires soil water potential, unsaturated hydraulic conductivity and water content as a function of time and space[1]. Both soil water potential and unsaturated hydraulic conductivity functions are nonlinear and these nonlinearities make the application of Richards' equation inherently problematic across the scales. Until last century, tabular forms of average hydraulic functions derived from different soil groups were used for different purposes such as land evaluation, environment and hydrological studies, etc. However, it was necessary to quantify and represent the variability of hydraulic functions derived from soil groups with minimum number of functions. In this study, van Genuchten's model (VGM)[2] based on statistical pore size distribution model[3] has been used to represent the soil hydraulic functions. It parameterizes the soil hydraulic functions and represents soil water potential and unsaturated conductivity as a function of soil moisture content. The 'scaling approach' is extensively used to scale and represent spatial variability of the soil hydraulic properties.

The concept of scaling approach has been developed from the similar media concept based on the hypothesis of microscopic geometric similitude[4]. It construes that the distribution of the spatial variation of soil hydraulic functions is described by a set of scale factors α_r, relating to the soil hydraulic properties at each location r to a representative mean[5]. Various methods have been evolved to compute scaling factors on using regression analysis[6,7], polynomial functions[6] and logarithmic expressions[8]. Most of these studies reported that the scale factors are lognormally distributed. Water flow through the soil was simulated using the distribution function of scale factors in heterogeneous soil from field to basin scale[9–13]. The soil hydraulic functions proposed by Brooks and Corey[14]

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have also been used by several studies for scaling[15]. Earlier, the soil matric potential and unsaturated hydraulic conductivity were scaled and used separately. The scale factors computed separately are not necessarily identical[6,7]. However, it was necessary to describe soil hydraulic functions by a single set of scale factors for simulating water flow in spatially variable field soils. Recent works on simultaneous scaling are available in the literature[5,16–19].

The present study was carried out with the aim to compute scale factors separately as well as simultaneously for both soil hydraulic functions using nonlinear least squares minimization technique, by minimizing the sum of residuals in a laboratory experiment. The HYDRUS model can be used to feed these three types of scale factors to simulate infiltration through soil using Richards' equation under different situations, such as constant or falling head conditions, etc.[20]. Apart from infiltration, scaling factors are used to study distribution of water in soil profile in irrigation experiments. However, calculation of scale factors using nonlinear least squares minimization method alone is considered in this study. Simple equations are proposed here for computing scale factors in comparison to other studies[5,18]; these may be used effectively to study soil water budget in irrigation experiments. The efficiency of the scaling technique expressed in terms of correlation coefficient (r) by Clausnitzer et al.[5], is comparable with the results obtained in this present study.

Material and methods

Theory

According to Miller and Miller[4], it is possible to obtain detailed similitude of interface shapes and microscopic patterns between two media whose solid geometries differ only by a constant magnifying factor. Two such media will be called 'similar media'. When the interface geometries are also similar, the two media are said to be in 'similar states'. Peck et al.[9] expressed scaling factor as the ratio between microscopic characteristic length λ_r of a soil at location r to the characteristic length $\hat{\lambda}$ of a reference soil

$$\alpha_r = \frac{\lambda_r}{\hat{\lambda}}, \tag{1}$$

scaling of hydraulic functions represents soil matric potential (h_r) and unsaturated hydraulic conductivity (k_r) of any location r with respect to the mean soil matric potential $\hat{h}(\theta)$ and hydraulic conductivity $\hat{k}(\theta)$ of the reference soil as follows

$$h_r = \frac{\hat{h}}{\alpha_r}, \tag{2}$$

$$k_r = \alpha_r^2 \hat{k}, \tag{3}$$

It is to be noted that though h_r is negative in the vadose zone, it is considered to be positive for notational convenience. Soil matric potential and unsaturated hydraulic conductivity functions can be parameterized following VGM[2]

$$h = [(S_e^{-1/m} - 1)^{1/n}]\beta^{-1}, \tag{4}$$

$$k = K_s S_e^{1/2}[1 - (1 - S_e^{1/m})^m]^2, \tag{5}$$

$$S_e = \frac{(\theta - \theta_r)}{(\theta_s - \theta_r)}, \tag{6}$$

$$m = 1 - \frac{1}{n}, \tag{7}$$

where θ (in volume basis) is expressed in terms of the dimensionless form S_e, θ_r, θ_s, β, n and K_s are the effective water content, residual and saturated water content, inverse of the bubbling pressure, pore size distribution index and saturated hydraulic conductivity respectively. These are used as fitting parameters for soil hydraulic functions in VGM expressions. For similar media, eqs (4) and (5) hold for equal water content. Owing to the fact that soils in general are not strictly similar, h and k are written as a function of the degree of saturation S ($S = \theta/\theta_S$) or as a function of S_e rather than the volumetric water content[5].

Experimental measurements

A 26.4 m long, 0.88 m wide and 1.0 m deep experimental tank was constructed using steel profiles combined with Plexiglas at Hubert-Engels Laboratory, Dresden University of Technology, Germany (Figure 1). The tank was carefully filled with soil layer by layer and compacted uniformly with 50 tonnes of silty loam soil to achieve, as far as possible, a homogeneous distribution of soil properties ($\theta_s = 41\%$, $\theta_r = 14\%$, bulk density = 15.1 kN m^{-3}, particle density = 25.7 kN m^{-3}). A parabolic furrow with a top of width of 0.35 m and initial depth of 0.184 m was formed along the central longitudinal axis of the tank. A magnetic-inductive flow meter was used to record inflow into the furrow. Tensiometers and time-domain reflectometry (TTDR) probes were placed at three cross-sections at a distance of 6.3, 12.3 and 18.3 m respectively, in the tank to measure soil matric head and moisture content (Figure 1). Irrigation was applied at a rate and time of 0.0012 m^3 s^{-1} and 2.55 h respectively. Water extraction from the soil was controlled by rye grass (*Lolium multiflorum*). The experimental tank was illuminated for

10 h per day in order to ensure optimal plant growth. Soil hydraulic parameters were calculated by 'inverseparameter estimation' using pressure head measurements. The objective function Z was expressed as follows

$$Z(K_s, \beta, n) = 0.5 \sum_{probe=1}^{n} [h_{mea}(t, x) - h_{sim}(t, x)] \rightarrow \min, \quad (8)$$

where $h_{mea}(t, x)$ denotes the measured pressure head at time and location t and x respectively, whereas $h_{sim}(t, x)$ is the corresponding pressure head simulated by HYDRUS-2D[21]. The VGM parameters K_s, β and n were used as decision variables during optimize run. The values of θ_s and θ_r were determined from soil analysis and kept constant during the optimization process. HYDRUS-2D includes a routine for inverse parameter estimation. Inverse modelling is a form of model calibration which is frequently used in hydrology. It requires a set of observed data such as measured pressure heads. In model calibration, the objective is usually to obtain better predictions. In case of parameter optimization, the objective is to determine the best estimate of the parameters as an alternative to directly measuring them. Minimization of the objective function is accomplished in HYDRUS using the Levenberg–Marquardt nonlinear minimization[22]. Before the start of the experiment, initial soil water content and corresponding matric potential were noted at the TTDR probe locations. Average soil matric potential readings in each depth obtained from tensiometer reading probes were fed into HYDRUS-2D as initial conditions. Initial parameter values were assumed to solve for water flow through the soil in HYDRUS using known soil matric potential at different points of time after the experiment started. In this way, VGM parameter set for the soil hydraulic functions for each TTDR location was optimized and later scaled. Details of the methodology can be found in Schmitz et al.[23].

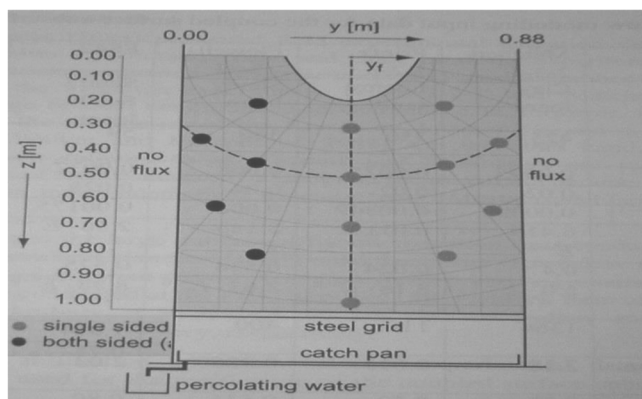Figure 1 shows the locations of the TTDR probes. It can be seen from the figure that nine TTDR probes were installed in the first and second sections, whereas 14 probes were installed in the third section. It was found that one, two and eight probes in the first, second and third sections respectively, were malfunctioning during the experiment and thus data from these probes were not considered for the study. Hence, data recorded from 21 probes in all three sections were used for scaling. Appendix 1 shows the optimized VGM parameters of 21 probes.

Scaling procedure

The scaling procedure minimizes the sum of squared differences (SS) between hydraulic data of the experimental probes and the mean curve. First, hydraulic data corresponding to each probe are parameterized using VGM expressions. The same expressions were used to fit the mean curve using nonlinear least squares minimization method for soil matric potential, hydraulic conductivity as well as simultaneous scaling. This requires the upper, lower and initial values of VGM parameters (n, β and K_s). The range of each VGM parameter was obtained from the experimental TTDR probes. Fitted mean curves take combinations of the VGM parameters during soil matric potential and hydraulic conductivity scaling. However, the value of VGM parameter n was kept the same for both the mean curves fitted during simultaneous scaling of soil matric potential and hydraulic conductivity. The total sum of squared differences in all observed locations R (number of probe locations) is defined by

$$SS = \sum_{r=1}^{R} SS_r, \quad (9)$$

here, three cases of scaling were considered. Soil matric potential (case A) and hydraulic conductivity data (case B) were scaled separately. In the third case (case C), matric potential and hydraulic conductivity data were scaled simultaneously. The method of calculating SS_r (sum of squared differences at the rth location) varies for the three cases as follows

Case A – SS_r is calculated from h versus S_e

$$SS_r = Wh_r \sum_{i=1}^{p} [\hat{h}_i - \alpha_r h_{r,i}]^2, \quad (10)$$

where p and Wh_r refer to the number of $h(S_e)$ data pairs and a weighting factor respectively, for the sampled location r.

Case B: SS_r is expressed as $\ln k$ versus S_e

$$SS_r = Wk_r \sum_{i=1}^{q} [\ln k_{r,i} - \ln \hat{k}_i - 2\ln \alpha_r]^2, \quad (11)$$

where q and Wk_r refer to the number of $k(S_e)$ data pairs and a weighting factor respectively, for sampled location r.

Figure 1. Location of tensiometers and time-domain reflectometry probes in the experimental tank.

Table 1. Optimized values of van Genuchten's parameters for different types of scaling

Soil matric potential scaling		Conductivity scaling		Simultaneous scaling		
β	n	K_s (m/day)	n	β	n	K_s (m/day)
1.086	1.29	0.59	1.17	1.086	1.29	0.40

Case C – SS_r is calculated from h and $\ln(k)$ simultaneously versus S_e

$$SS_r = Wh_r \sum_{i=1}^{p} [\hat{h}_i - \alpha_r h_{r,i}]^2 + Wk_r \sum_{i=1}^{q} [\ln k_{r,i} - \ln \hat{k}_i - 2\ln \alpha_r]^2,$$

(12)

Weighting factor is used to account for and balance the variability between data value ranges and number of data points of the sampled locations, and is defined as follows

$$Wh_r = 1/[(\Delta h)(\Delta h)(p)],$$

(13)

$$Wk_r = 1/[(\Delta \ln k)(\Delta \ln k)(q)],$$

(14)

where Δh and $\Delta \ln k$ are the difference between the maximum and minimum h value and $\Delta \ln k$ respectively. Scaling factors and scaled mean curves are determined by minimizing SS. The scale factor varies spatially. Scale factors are calculated for different cases by minimizing SS

$$\frac{dSS}{d\alpha_r} = 0, \quad r = 1,...,R,$$

(15)

Scale factors obtained for different cases are shown below

Case A

$$\alpha_i = \sum_{j=1}^{p} \frac{\widehat{(h_j h_{i,j})}}{h_{i,j}^2}, \quad i = 1,...,R,$$

(16)

Case B

$$\alpha_i = \sum_{j=1}^{q} \left(\frac{k_i k_{i,j}}{m} \right)^{1/2q}, \quad i = 1,...,R,$$

(17)

where

$$m = \hat{k}_1 \hat{k}_2,...,\hat{k}_n.$$

Case C

$$Wh_i (A.\alpha_i - B.\alpha_i^2) + 2Wk_i (C - 1 - 2q\ln \alpha_i) = 0, \quad i = 1,...,R,$$

18)

where

$$A_i = \sum_{j=1}^{p} \hat{h}_{i,j} h_{i,j}, \quad B_i = \sum_{j=1}^{p} h_{i,j}^2,$$

$$C_i = \sum_{j=1}^{q} \ln k_{i,j} \quad \text{and} \quad D_i = \sum_{j=1}^{q} \ln \hat{k}_{i,j}.$$

Equation (18) is nonlinear equation and can be solved by the Newton–Raphson method using MATLAB software.

Finally, extra weights assumed arbitrarily apart from the weighing functions mentioned in eqs (13) and (14) were used in soil matric potential as well as conductivity data in simultaneous scaling (case C) to observe the effect of weight factors on the results. It was found that more weights applied to the soil matric potential data improved the correlation coefficient of these data, but slightly decreased the correlation coefficient of the conductivity data.

Results and discussion

For each location of TTDR probes (21 numbers) in the soil profile, soil matric potential functions were scaled over eight equally divided matric potential increments (0, 28.57, 57.14, 85.71, 114.29, 142.86, 171.43 and 200.00 cm). Hydraulic conductivity curves were scaled on the same S_e data points corresponding to the same matric potential increments. The lower limit of conductivity for scaling was assumed as 10^{-7} m/day, as below this limit the soil is considered to be impermeable. The measure of degree of success of scaling approach is the 'correlation coefficient' between the original and the descaled soil matric potential and conductivity data.

Soil matric potential scaling

Soil matric potential functions (hereafter raw soil matric potential curves) for different TTDR locations were parameterized using optimized VGM parameters at fixed soil matric potential points as mentioned above and corresponding S_e values were calculated. A mean curve was fitted through the raw soil matric potential curves using nonlinear least squares minimization technique in MATLAB. It was assumed that the mean curve also follows eq. (4) and the range of VGM parameters (β, n and K_s) adopted from the raw soil matric potential functions was used during the least square minimization process. Table 1 provides the optimized VGM parameter values of the mean curves for three types of scaling. Next, the mean curve soil matric potential values corresponding to each S_e were calculated. Scale factors were calculated after
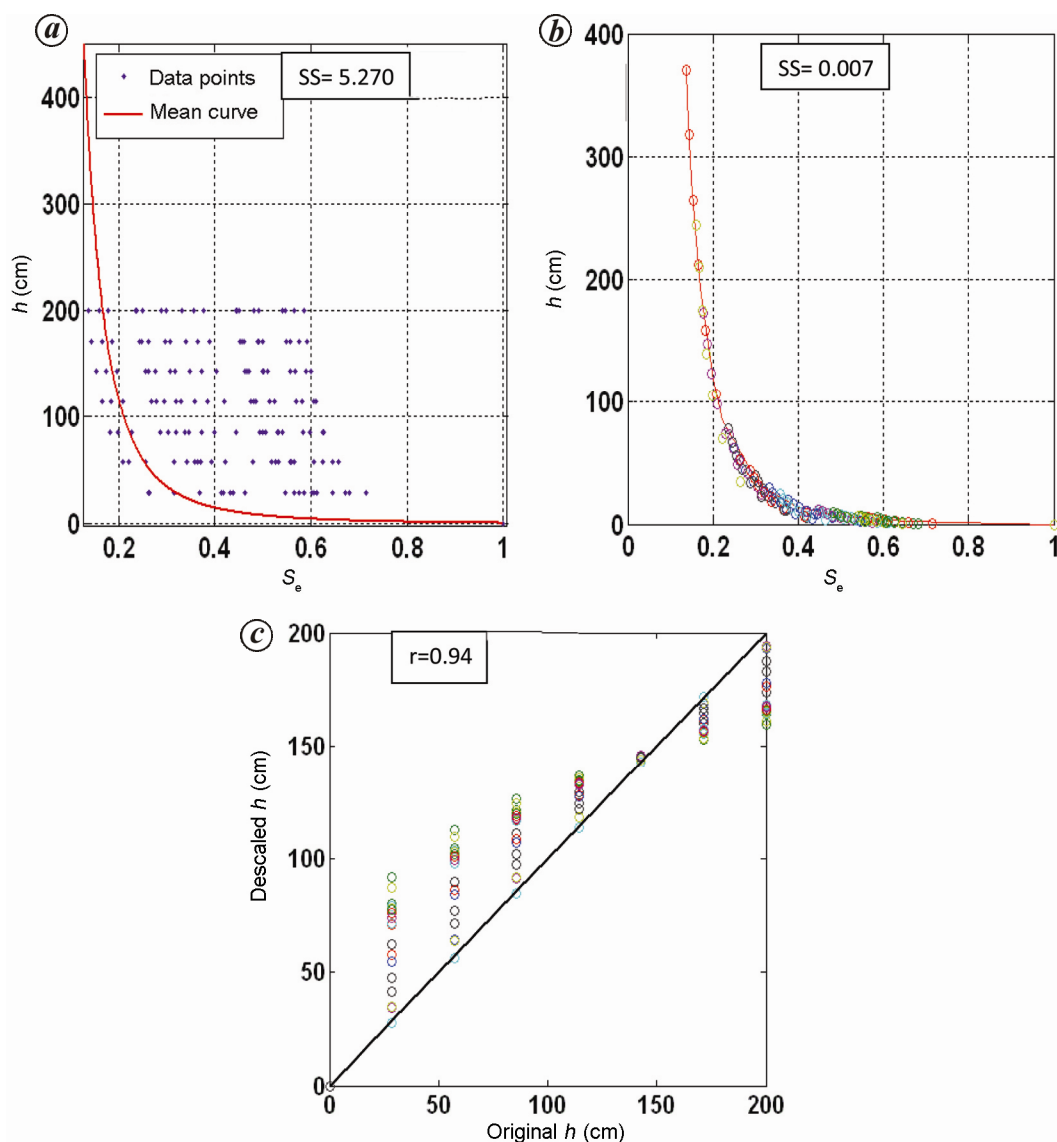
Figure 2. (*a*) Unscaled data with mean curve, (*b*) scaled curve and (*c*) original and descaled data in soil matric potential scaling.

solving eq. (16). Scaled soil matric potential value for each TTDR location was calculated after multiplying scale factor with raw soil matric potential data. Descaled soil matric potential data for each TTDR probe location were obtained by dividing mean curve soil matric potential data with the respective scale factor. Figure 2 *a–c* shows the raw soil matric potential functions with mean curve, scaled curves, and original and descaled data respectively. Equation (10) was used for calculation of SS. The first SS calculated using eq. (10) signifies the sum of square difference between mean and raw curve soil matric potential data, whereas the second SS calculated using eq. (10) signifies the sum of square difference between mean and scaled raw curve soil matric potential data. Sum of squares was reduced from 5.270 to 0.007 after scaling in this case. The correlation coefficient (*r*) between the raw and descaled data was obtained as 0.95.

Hence, the scaling method presented here represents the raw data well after descaling.

Soil hydraulic conductivity scaling

Soil hydraulic conductivity functions were parameterized using VGM parameters corresponding to S_e values that were obtained in soil matric potential scaling. The same procedure was followed as mentioned above for fitting the mean curve through raw conductivity curves. Table 1 shows optimized VGM parameters of the mean curve (K_s, *n*). Figure 3 *a–c* shows the unscaled mean curve, scaled curve, and original and descaled data respectively. In this scaling, SS is reduced from 0.66 to 0.09. The value of the correlation coefficient (*r*) between the original and descaled data was 0.97, which is considered excellent in terms of retrieving the original raw conductivity values.
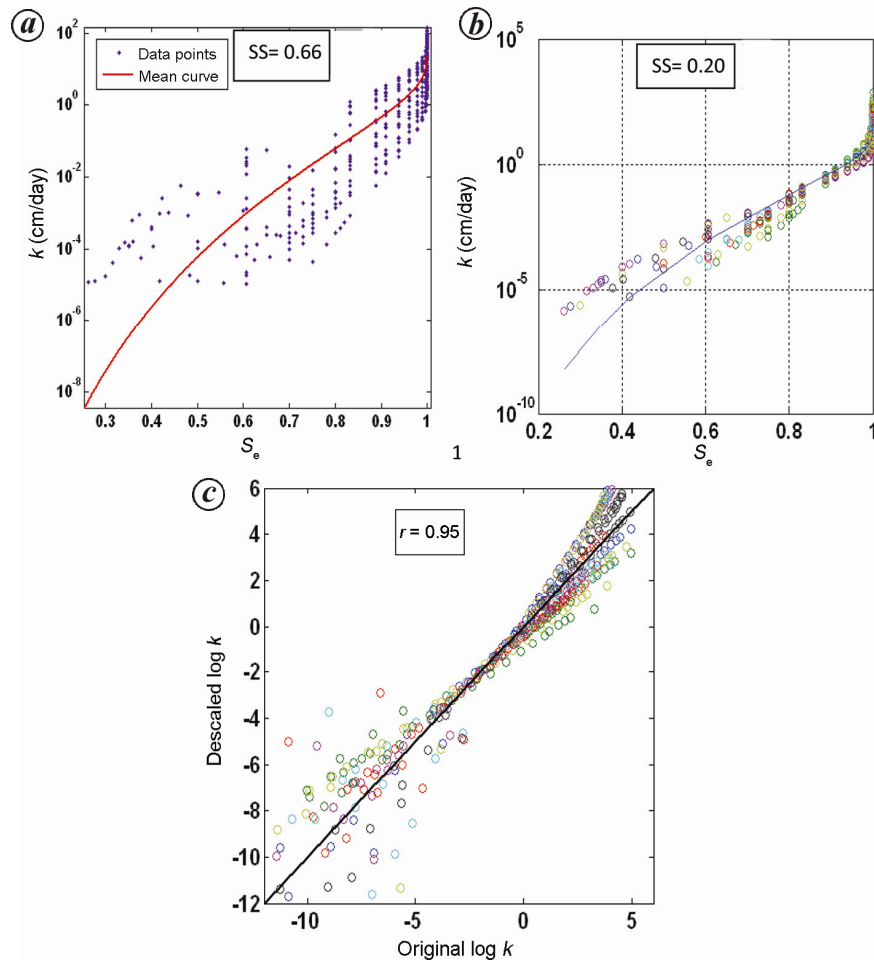
Figure 3. (*a*) Unscaled data with mean curve, (*b*) scaled data and (*c*) original and descaled conductivity data in conductivity scaling.

Simultaneous scaling of soil hydraulic functions

In simultaneous scaling, soil matric potential and hydraulic conductivity functions were scaled together. Table 1 shows the optimized value of soil matric potential mean curve parameters (β, n) and hydraulic conductivity mean curve parameters (K_s, n). The VGM parameter n acts as a bridge between two mean curves, and in this case the value of n was 1.294. Figures 2 *a* and 4 *a*, *b* show the unscaled functions with mean curve, scaled curve, and original and descaled data for soil matric potential scaling respectively, whereas Figure 4 *c*–*e* shows the unscaled functions with mean curve, scaled curve, and original and descaled data for hydraulic conductivity data scaling respectively. Figure 4 *c* shows a poor fit to the conductivity data due to the fact that both soil hydraulic functions are tied together by common parameter n. In simultaneous scaling, SS value was reduced from 5.270 to 0.056 for soil matric potential scaling and from 1.07 to 0.26 for conductivity scaling. The correlation coefficients were 0.78 and 0.92 between original and descaled soil matric potential and hydraulic conductivity

data respectively. Though the results showed that the scale factors obtained from individual scaling of soil hydraulic functions represented the original data more accurately than simultaneous scaling, it is highly desirable to get a single set of scaling factors for generating single random fields for calculating water flow through the soil. Several workers studied the scaling approach for soil water flow and reported similar results[5,16,17]. Nasta *et al.*[16] adopted the root mean square error as the success of scaling whereas Clausnitzer *et al.*[5] considered correlation coefficient and percentage reduction of the sum of squares as the success of scaling. The scaling method presented here is straight forward and simple, and the results obtained are comparable with those from other studies.

Effect of weight factors on simultaneous scaling

Incorporation of weighting factor balances data ranges and the number of data points in individual as well as simultaneous scaling. Results show that correction
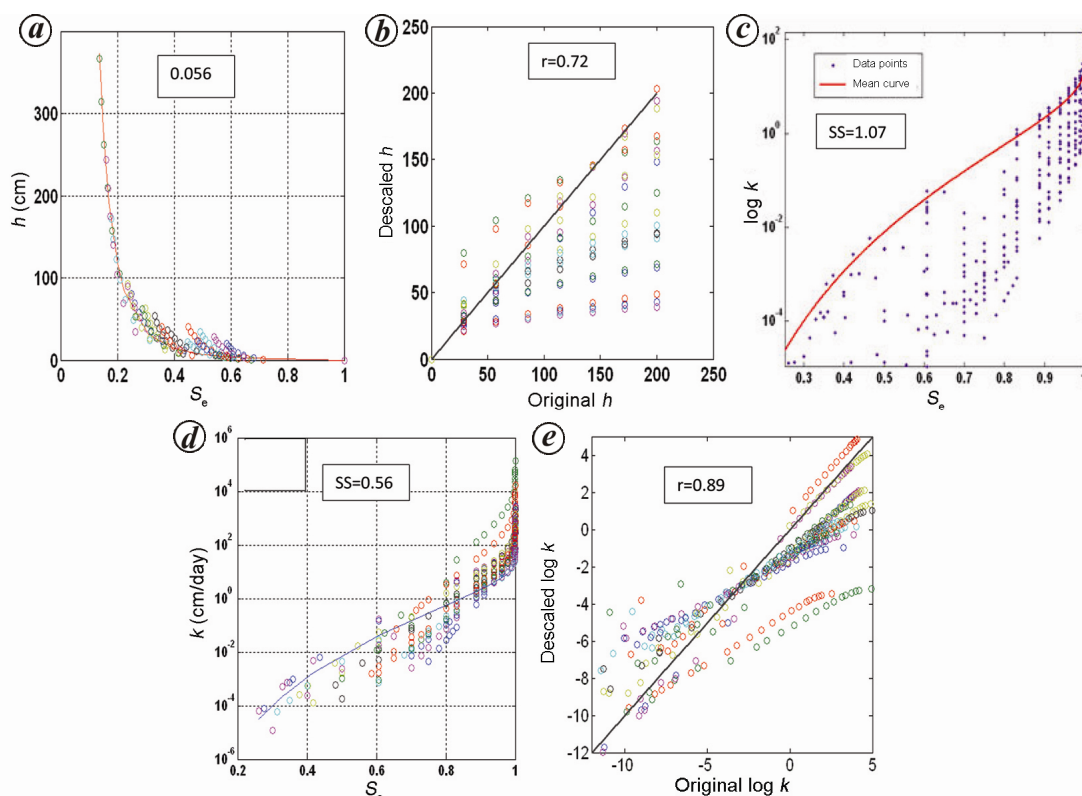
Figure 4. (*a*) Scaled data and (*b*) original and descaleds oil matric potential with mean curve. (*c*) Unscaled data with mean curve, (*d*) scaled data and (*e*) original and descaled conductivity data in simultaneous scaling.

Table 2. Effect of weight factors on simultaneous scaling

Correlation coefficient (without weight)		Correlation coefficient (with weight)			
		0.75		0.9	
Pressure scaling	Conductivity scaling	Pressure scaling	Conductivity scaling	Pressure scaling	Conductivity scaling
0.72	0.89	0.760	0.882	0.818	0.886

coefficient obtained in individual is superior that in simultaneous scaling. However, simultaneous scaling is required as it reduces the volume of data by generating a single set of scaling factors. To enhance the correlation factor in simultaneous scaling, additional arbitrary weights have been incorporated here. Different proportions of weights were applied to both components in simultaneous scaling (Table 2). More weights applied to soil matric potential data result in better correlation coefficient between the original and descaled data. Table 2 shows that 0.9 weight to soil matric potential data and 0.1 weight to conductivity data improve the correlation coefficient both for original and descaled soil matric potential as well as conductivity data compared to 0.75 weight to soil matric potential and 0.25 to conductivity data respectively. When weights of 0.95 and 0.05 were imposed on soil matric potential and conductivity data respectively,

correlation coefficient in case of original and descaled data of soil matric potential increased to 0.83; however, for conductivity data it reduced to 0.85. Therefore, it has been generalized that incorporation of weights up to certain proportions is effective in increasing the efficiency of scaling.

Statistical analysis

As mentioned previously, most of the studies reported that scale factors are lognormally distributed. Therefore, scale factors obtained from the three cases (A–C) were checked for their lognormal distribution. The following steps were performed for the determination of lognormal distribution of the scale factors:

(1) Rank the values of scale factors from low to high in ascending order.

(2) Compute the cumulative probability distribution of logarithm of scale factors ($\ln \alpha$) as approximated by $P = (i - 0.5)/N$, $i = 1,\ldots, N$, where N is the total number of observations.

3. Determine the standard normal deviate (μ_i) for the distributed variable from the standard normal probability table, i.e. $P[\mu < \mu_i]$, for each value computed above.

4. Plot the standard normal deviate (μ_i) determined in step 3 versus the variable $\ln \alpha$.

5. If the plot can be described by a straight line, then the scale factors are considered to be lognormally distributed.

The scale factors obtained from soil matric potential, conductivity and simultaneous scaling were calculated and plotted using the above procedure (Figure 5). Figure 5 shows that the best fit of scale factors can be described by a straight line. Hence, it can be deduced that scale factors calculated from this study are lognormally distributed. Table 3 shows the statistical parameters of the lognormally distributed scale factors as well as coefficient of determination (R^2) when fitted with straight line. The scale factors were also tested for other distributions such as Pearson, Dagum, Burr, log-logistic, gamma, Gumbel, etc. along with lognormal. However, chi-squared test statistics was compared for these distributions and found

minimum in case of lognormal distribution. This confirms that the distribution function of scale factors can be described as lognormal type. Thus, scale factors calculated using the scaling approach presented in this study can be utilized in HYDRUS to simulate and study water flow through heterogeneous soil.

Conclusion

In this study nonlinear least squares method was used to scale soil hydraulic functions in a laboratory experiment. Van Genuchten's semi-empirical expressions were used to parameterize soil hydraulic functions. This requires the following parameters: residual water content (θ_r), saturated water content (θ_s), saturated hydraulic conductivity (K_s), inverse of the air entry value or bubbling pressure (β) and pore size distribution index (n). Soil matric potential and hydraulic conductivity data were scaled separately as well as simultaneously. Correlation coefficient (r) between the original soil hydraulic data and descaled data was adopted as the measure of success of the presented scaling approach. The descaled soil hydraulic data were obtained by incorporating scale factors on the mean curves. More the r value, better would be the degree of success of scaling. When soil matric potential and hydraulic conductivity data were scaled independently, the correlation coefficient obtained between the original and descaled data was more compared to simultaneous scaling. The correlation coefficients obtained from simultaneous scaling can be improved after incorporating additional weight factors. The best weight was found to be 0.9 to soil matric potential and 0.1 to conductivity
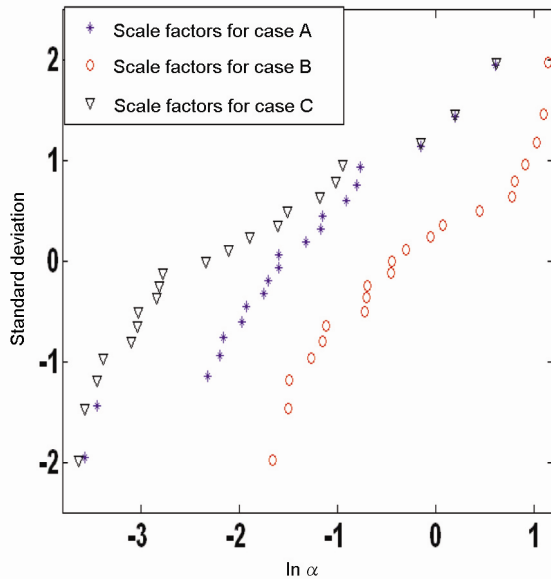

Figure 5. $\ln \alpha$ versus standard deviation plot.

Table 3. Lognormally distributed parameters of the scale factors

Case	Mean	Variance	Mode	Median	R^2
A	0.258	0.33	0.024	0.126	0.93
B	1.219	2.16	0.317	0.778	0.94
C	0.533	1.08	0.051	0.243	0.97

Appendix 1. Van Genuchten parameters for experimental probes

Probe no.	β (m^{-1})	n	K_s (cm day^{-1})	θ_r	θ_s
1	0.600	1.289	47.696	0.140	0.410
2	1.535	1.105	142.108	0.140	0.410
3	0.523	1.122	12.814	0.140	0.410
4	2.761	1.112	56.744	0.140	0.410
5	0.676	1.293	64.071	0.140	0.410
6	2.780	1.116	116.305	0.140	0.410
7	5.580	1.172	135.204	0.140	0.410
8	17.080	1.135	143.711	0.140	0.410
9	4.734	1.078	26.059	0.140	0.410
10	3.945	1.187	38.918	0.140	0.410
11	1.846	1.135	49.694	0.140	0.410
12	2.544	1.129	6.735	0.140	0.410
13	4.515	1.090	47.740	0.140	0.410
14	2.764	1.228	93.048	0.140	0.410
15	0.799	1.192	9.155	0.140	0.410
16	2.436	1.118	8.820	0.140	0.410
17	3.493	1.124	16.880	0.140	0.410
18	1.838	1.337	57.946	0.140	0.410
19	10.528	1.135	29.578	0.140	0.410
20	1.815	1.293	40.694	0.140	0.410
21	6.432	1.256	107.264	0.140	0.410

data. The scale factors obtained from soil matric potential, hydraulic conductivity and simultaneous scaling were lognormally distributed.

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