Numerical and experimental studies in prediction of bed levels of aggrading channels

B. R. Andharia, P. L. Patel*, V. L. Manekar and P. D. Porey

Department of Civil Engineering, S.V. National Institute of Technology, Surat 395 007, India

A semi-coupled 1D numerical model is presented to compute transient bed and water levels of aggrading channels due to the overloading of sediments. The numerical model solves mass and momentum equations (i.e. de Saint–Venant equations) for water and continuity equations for sediments simultaneously, using explicit finite difference scheme while considering upstream and downstream boundary conditions in the channel. Series of experimental studies are reported for measurements of bed and water levels in an aggrading channel due to the overloading of uniform sediments, in a flume installed at the Advanced Hydraulics Laboratory of SVNIT. The performance of bed level variation models, with different sediment transport functions, has been validated using the laboratory measurements. The performance of the numerical model is dependent on sediment transport functions. In addition, the performance of the proposed numerical model has been verified with existing numerical models on prediction of bed level variations. The proposed numerical model with recommended sediment transport function has been found to perform better than the existing numerical models on bed level variations of uniform sediment beds.

Keywords: Numerical model, aggradation, alluvial channel, uniform sediments, transport functions.

PREDICTION of riverbed level variation in natural channels is required for efficient designing of hydraulic structures, implementation and execution of river valley projects, study on migration of natural channels and precise prediction of floods and their control along the rivers[1]. A delicate balance between water and sediment discharge, river slope and sediment size is often disturbed by human and natural interferences, which may result in aggradation or degradation of natural streams. In general, aggradation in a stream takes place when the stream carrying capacity decreases in the direction of flow; or sediment inflow is more than its carrying capacity. Aggradation occurs in alluvial streams under various circumstances. The excessive sediment supply from eroding catchments during heavy rainfall or landslides, withdrawal of water for irrigation and domestic usage, formation of delta and alluvial fans in upstream of lakes and reservoirs due to reduction of bed shear stresses and dumping of large quantity of mining wastes in the natural streams are the major factors responsible for the aggradation. Due to rapid industrialization and human interferences along the rivers, the overloading of riverbeds due to landslides and dumping of solid wastes have become more frequent. Under such circumstances, prediction of river morphology and maintenance of their ecological balance have become challenging problems for the fraternity of hydraulic and environmental engineering.

Starting from the pioneering work of Adachi and Nakato[2] on evolution of river bed due to silting of reservoir, numerous analytical aggradation models were proposed and supported with overloading experiments in the laboratory flumes[3–9]. The analytical models developed in previous studies were based on gross assumptions and their applicability is limited in actual field conditions. Further, the development of exact analytical models, while solving continuity and momentum equations for fluids and sediments with appropriate boundary conditions, is extremely difficult due to their nonlinear characteristics. In the modern era, due to the advent of high-ended computational facilities, numerical modelling is the most preferred approach in solving governing equations for prediction of bed and water levels in alluvial streams. The numerical models are classified as uncoupled models[10–15], semi-coupled models[15–23] and fully-coupled models[24–28]. The first step in uncoupled models is the estimation of hydraulic characteristics at a particular time step[14,15,18]. Subsequently, at the same time step, these estimated characteristics are used for predicting the fluvial channel bed levels. Consideration of ‘fixed bed’ channel boundary and exclusion of sediment parameters while solving flow equations, are major limitations of uncoupled models. The uncoupled models fail to simulate the mobile boundary channel flows under peak flow conditions as rapid changes in the bed levels are expected due to strong interaction between the water and sediment phase motions. Park and Jain[29] implemented weighted implicit finite difference method for development of a

---

*For correspondence. (e-mail: premlalpatel1966@gmail.com)

1D uncoupled model for prediction of bed levels in aggrading channels, and concluded that numerical scheme, based on uncoupled approach, becomes unstable when the bed slope becomes too large due to non-implementation of iterative procedure. The applications of coupled models become inevitable due to strong coupling between water and sediment continuity equations[28], as the physical interaction and interchange between water and sediment phase are possible in a coupled approach. The coupled models are not suitable for incompressible flow due to absence of coupling parameters between continuity and momentum equations for fluid flow. Such models are also not cost-effective as the time steps of flow and sediment transport processes are different[30]. Under such circumstances, the semi-coupled models are preferred wherein flow calculations are decoupled from sediment calculations; however, the three components of sediment transport model, viz. sediment transport, bed levels and grain sorting, are solved in a coupled manner. Saiedi[28] developed a 1D coupled numerical model called COUPFLEX using implicit Pressmann scheme and compared the results with uncoupled model using experimental flume data of Saiedi[31]. The study concluded that accuracy gained due to coupling of water routing and sediment continuity equations is often less effective due to inappropriate use of empirical equilibrium sediment transport equations.

Tayfur and Singh[21] used 1D unsteady non-equilibrium sediment transport bed level variation model based on kinematic wave theory proposed by Tayfur and Singh[20] using sediment transport models on different approaches, and compared the results with experimental data of Yen *et al.*[32] and Seal *et al.*[8]. The parameters of sediment transport in overland flow (sheet sediment transport as suggested by Foster[33]) were used in the simulation of laboratory data of Yen *et al.*[32]. Tayfur and Singh[34] developed a mathematical model based on double decomposition method developed by Adomian[35,36] for simulation of aggrading bed and water surface profiles, and solved diffusive partial differential equations proposed by de Vries[3]. The method is simple and straightforward; however, its solution does not consider the transport of suspended and bed loads under non-equilibrium transport processes. Rahman and Matin[37] performed 16 test runs in a laboratory channel with uniform sediment bed for the aggradation processes. They used the Colby[38] type power-law velocity function in numerical modelling with coefficients derived from their own experimental data. Schippa and Pavan[23] presented a finite difference based 1D semi-coupled mobile bed numerical model for aggradation due to overloading, with extremely irregular and complex geometry. The model was tested using the laboratory data of Soni *et al.*[5] and Begin *et al.*[39]. Goutiere *et al.*[22] proposed a semi-coupled model for the solution of St. Venant and Exner equations under transient and trans-critical flow conditions in alluvial channel using the experimental data, generated in a laboratory flume at the Civil and Environmental Engineering Laboratory of the University Catholique de Louvain, Belgium. The propagation of aggradation profile was simulated satisfactorily from the numerical model with slight overestimation of celerity due to excessive sediment input into the model while using Meyer-Peter and Müller formula[40] as sediment transport function. Tayfur and Singh[41] presented a 1D kinematic wave routing model to predict river and bed level variations using experimental data of Guy *et al.*[42] and Soni *et al.*[5]. The study demonstrated that kinematic wave routing model performed better than diffusive wave analytical model; and shear velocity and sediment concentration significantly affect the transient bed levels. Bhallamudi and Chaudhry[17] introduced a 1D numerical model for unsteady gradually varied open channel flow to determine the aggradation due to sediment overloading using explicit MacCormack scheme. The Manning's roughness coefficient and Colby[38] type power functions were used as flow resistance and sediment transport functions respectively, in the modelling. The simulated and experimental results did not agree, when the bed levels were affected by armouring.

Previous studies[14,16,37,41,43,44] indicated that 1D numerical models were developed using Colby[38] type sediment transport functions wherein the coefficient and exponent of velocity are calibrated, and depends on sediment transport data of the corresponding channels. Earlier studies also did not systematically demonstrate the comparative performance of existing uniform sediment transport functions, being used in 1D numerical model, in the prediction of bed and water levels of the aggrading channels.

In the present study, a 1D semi-coupled numerical model was developed using Mac-Cormack Explicit finite difference scheme for the prediction of bed and water level variations in aggrading alluvial channels. The estimated bed level variations from the proposed numerical scheme with different sediment transport functions are compared with actual observed data collected in the present study and reported from Soni *et al.*[5]. The numerical model developed in this study, with best sediment transport function, for aggrading bed channels is also compared with numerical models developed in the previous studies[34,37,41] on prediction of bed and water levels of aggrading channels with uniform sediments.

Model descriptions

Basic equations

The basic 1D partial differential equations describing unsteady open channel flow in a wide rectangular alluvial channel with no lateral discharge, can be expressed in vector matrix form as

$$\frac{\partial U}{\partial t}+\frac{\partial F}{\partial x}=S, \tag{1}$$

where ***U*** is the variable vector, ***F*** the flux vector and ***S*** the source term vector. These vectors can be expressed as

$$\boldsymbol{U}=\begin{bmatrix} h \\ q \\ z+\dfrac{q_T h}{q(1-p)} \end{bmatrix},\quad \boldsymbol{F}=\begin{bmatrix} q \\ \dfrac{q^2}{h}+\dfrac{1}{2}gh^2+ghz \\ \dfrac{q_T}{(1-p)} \end{bmatrix},$$

$$\boldsymbol{S}=\begin{bmatrix} 0 \\ (-ghS_f) \\ 0 \end{bmatrix} \tag{2}$$

where q is the flow discharge of the channel per unit width, h the depth of flow, g the acceleration due to gravity, S_f the friction slope, x the distance along the channel, t the time, z the riverbed elevation, q_T the volumetric rate of sediment transport per unit width of the channel and p is the porosity of riverbed material. The friction slope, S_f, in eq. (2), is determined using Manning's equation for wide rectangular channel as

$$S_f=\frac{q^2n^2}{h^{3.333}}, \tag{3}$$

where n is Manning's roughness coefficient of the channel bed, which depends on the characteristics of the channel bed surface.

Different sediment transport functions can be used to estimate sediment discharge, q_T, for using the same in sediment continuity equations. Brief descriptions of sediment transport functions being used in the present study for estimation of sediment transport rates of uniform sediments are included in Table 1.

Numerical scheme

Equation (1) links unknown dependent variables, viz. h, q and z with independent variables x and t, have been solved using explicit finite difference numerical scheme (MacCormack scheme) with appropriate boundary conditions described in the following section. The McCormack's predictor–corrector scheme is second order accurate in space and time, and is able to capture the 'shock' (i.e. it is able to describe discontinuities due to steep moving slopes, and thus, is very stable and suitable for aggradation processes[45,46]).

The MacCormack's scheme also allows strong coupling between the flow hydraulics and sediment variables. The two-step predictor–corrector approach enables simultaneous solution of the de Saint Venant–Exner equations (eq. 1). The semi-coupled approach being used in the present study provides a nearly identical performance vis-à-vis coupled approach[47]. The finite difference grid used in the foregoing numerical scheme is shown in Figure 1.

The steps in the implementation of numerical scheme are shown in a flowchart for the prediction of transient bed and water levels in an aggrading channel comprising of uniform sediments (Figure 2). In Figure 2, superscripts $*$ and $**$ refer to the values of dependent variables after the predictor and corrector steps at unknown time level $k+1$ (i.e. $t+\Delta t$) respectively; suffix 'o' denotes the initial values; i denotes the space node while k denotes the time node; B_0 is the width of the channel; L the length of channel; Δq_T the change in sediment transport-rate; t_{last} denotes the last computational step; Δt the temporal step(s); Δx the spatial step (m)/distance increment, and C_n is the Courant number. The vector ***U*** must be discretized in eq. (1) to calculate the predictor of dependent variables h, q and z at the time level $t+\Delta t$ at ith computational node.

Boundary conditions

The values of the dependent variables h, q and z at the boundary nodes $i = 1$ and $N + 1$ (last node) are computed using the upstream and downstream boundary conditions respectively. Two conditions were imposed at upstream boundary while one condition was imposed at the downstream boundary for subcritical flow conditions. The initial conditions can be specified for initial known values of depth of flow (h), flow discharge (q) and bed level z at each computational node point along the initial ($t = 0$) time line (i.e. $k = 0$).

In this study, the upstream boundary condition has been defined with constant flow discharge as

$$q_1^{k+1}=q_0 \text{ (known constant value)}, t\geq 0. \tag{4}$$

The flow depth, h, at node 1, for $k + 1$ time line, is determined by explicit forward difference scheme using continuity equation (eq. (1) for water discharge) for

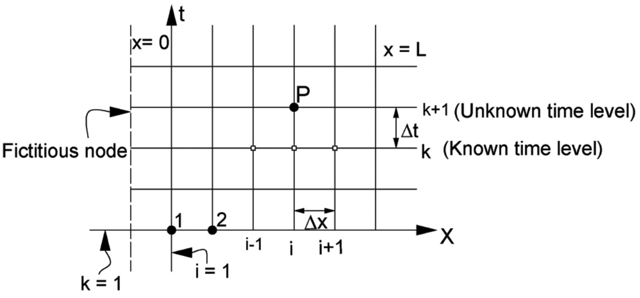

Figure 1. Finite difference grid for MacCormack scheme (P represents the unknown variables, h_i^{k+1}, q_i^{k+1} and z_i^{k+1}).

Table 1. Uniform sediment transport functions used in numerical scheme

Method	Range of applicability	Approach
Meyer-Peter and Muller[40]	0.4 mm < d < 30 mm	Empirical–excess shear stress
Samaga *et al.*[58,59]	0.5 < τ_0/τ_{0c} < 9.8	Empirical
Van Rijn[57]	0.2 mm < d < 2 mm	Stream power
Engelund and Hansen[53]	d > 0.15 mm	Potential energy–stream power
Wong and Parker[62]	0.4 mm < d < 28 mm	Modified MPM–Excess shear stress
Einstein–Brown[49,50]	0.3 mm < d < 28.6 mm	Probabilistic
Karim–Kennedy[55]	d > 0.10 mm	Regression analysis
Hanes[60]	0.05 mm < d < 30 mm	Semi-theoretical–critical shear stress

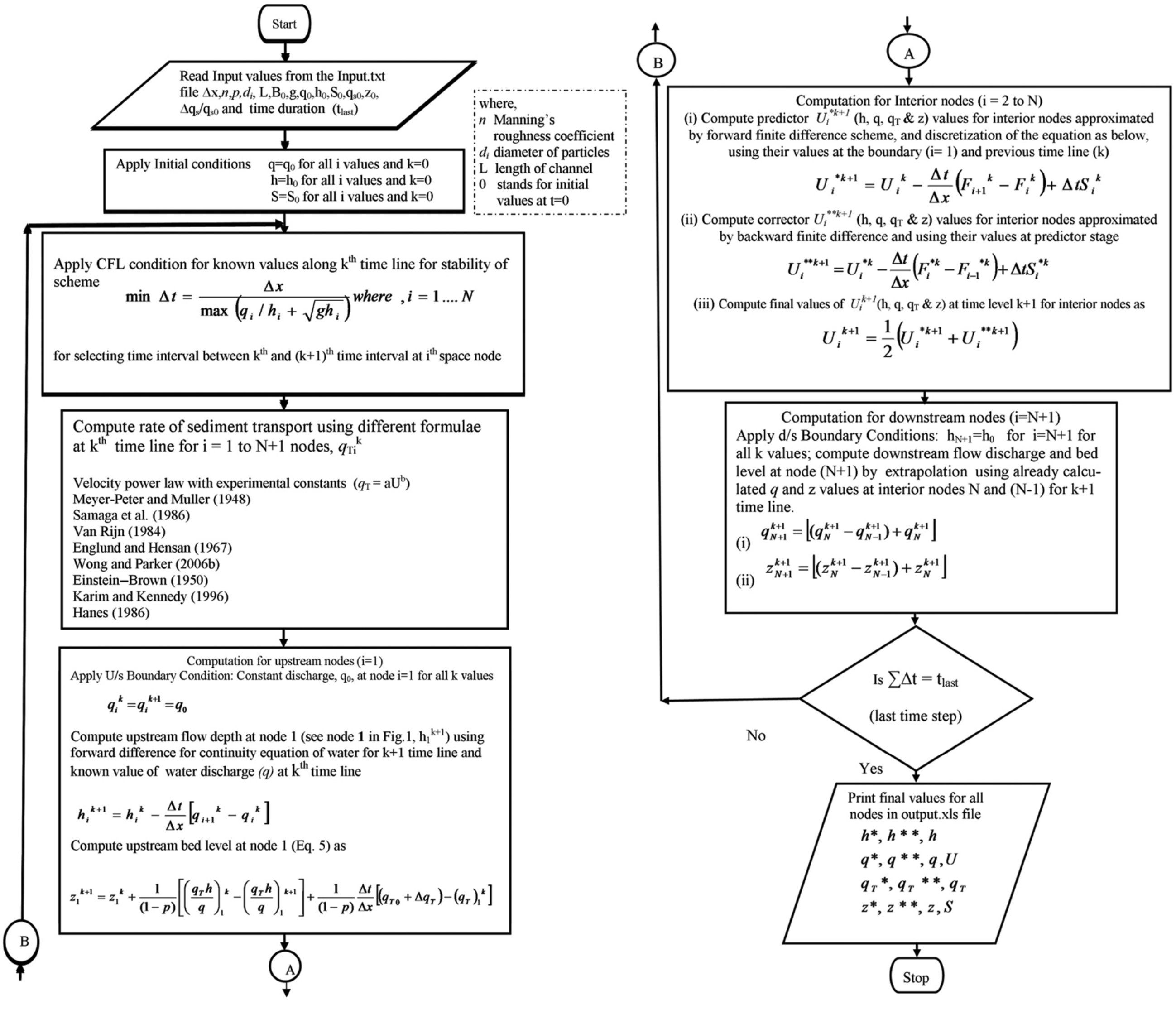

Figure 2. Methodology for development of bed level variation model of aggrading channel (uniform sediment bed).

known discharge and depth along *k*th time line. The equilibrium sediment discharge at upstream boundary (q_T^{k+1}) is computed for known flow parameters (h_1^{k+1} and q_1^{k+1}) at upstream node using appropriate sediment transport function.

However, the second boundary condition at upstream node, $q_T(0,t) = q_{To} + \Delta q_T$; $t \geq 0$ is not straightforward, and it is required to be translated into an equation by which the bed elevation (z_1^{k+1}) at the upstream end is defined. This is achieved by assuming a fictitious node upstream from node one (dotted line in Figure 1) and specifying the sediment discharge at that node equal to $q_{To} + \Delta q_T$ (ref. 17). Applying the backward finite difference approximation on spatial differential term of sediment continuity equations, the bed level (z_1^{k+1}) at node 1 for unknown time level $k+1$ for each time step can be computed as

$$z_1^{k+1} = z_1^k + \frac{1}{(1-p)}\left[\left(\frac{q_T h}{q}\right)_1^k - \left(\frac{q_T h}{q}\right)_1^{k+1}\right]$$

$$+\frac{1}{(1-p)}\frac{\Delta t}{\Delta x}\left[(q_{T0}+\Delta q_T)-(q_T)_1^k\right]. \quad (5)$$

The values of z_i^{k+1} at different time intervals at upstream node can be computed by knowing the values of the previous node (i.e. kth time line and flow parameters h_1^{k+1}, q_1^{k+1} and q_{T1}^{k+1}) at upstream boundary.

The downstream boundary conditions have been specified by considering constant flow depth during the transient water profiles along the channel, such as

$$h(N+1,t) = h_o(\text{known value}),\ t \geq 0. \quad (6)$$

In this study, flow depth has been taken as constant (by considering sufficient long channel) such that the bed transients would not reach the downstream end within the computational time duration. The discharge (q_{N+1}^{k+1}) and bed levels (z_{N+1}^{k+1}) were estimated by linearly extrapolating the values from the interior nodes (i.e. N and $N-1$ for ($k+1$) time line) as

$$q_{N+1}^{k+1} = \left[(q_N^{k+1} - q_{N-1}^{k+1}) + q_N^{k+1}\right], \quad (7)$$

$$z_{N+1}^{k+1} = \left[(z_N^{k+1} - z_{N-1}^{k+1}) + z_N^{k+1}\right]. \quad (8)$$

Stability of numerical scheme

For stability of the numerical scheme, minimum Δt were computed dynamically after every time step using Courant–Friedrichs–Lewy (CFL) condition[48], which can be expressed as

$$C_n = \frac{(q/h + \sqrt{gh})\Delta t}{\Delta x} \leq 1. \quad (9)$$

The minimum values of Δt from eq. (9) were arrived using computed flow parameters along the previous time line (kth time line), for using the same for computation along current time, ($k+1$) time line.

Sediment transport functions for uniform sediments bed

In the first step, the existing sediment transport functions for uniform sediments[38,40,49–62] were assessed for their performance in computation of sediment transport rates using experimental data of Soni *et al.*[5]. Based on their performance, eight sediment transport functions (Table 1) have been used for their inclusion in sediment continuity equations for computation of bed level variation in aggrading alluvial channels.

Data

The description of the experimental set-up, detailed procedure of data collection and salient features of data being used in the present study are described in following sections.

Experimental set-up

The series of flume test runs were undertaken in a straight, recirculating tilting flume of 15 m length, 0.89 m width and 0.6 m height (Figure 3) at the Advanced Hydraulics Laboratory of SVNIT. The measuring test section starts at 5.0 m distance from the inlet into the flume, and consists of steel frame with side glass wall of 6 m length. The experimental set-up is presented in Figure 3. The discharge into the channel was supplied from a downstream reservoir of 45,000 litres capacity, using two 7.5 HP pumps connected to a 25 cm diameter recirculating pipe. Flow rates were recorded by a digital flow meter, having capacity of 24–240 LPS, with an accuracy of ±1%. The discharge rate can also be measured volumetrically with the help of a volumetric tank provided at the downstream end of the channel. The discharges into the flume were controlled with a valve provided in the inlet pipe joining upstream of flume with downstream reservoir. The required slopes of the channel for experimental work were maintained by a screw jack

Figure 3. Sediment transport flume at Advanced Hydraulic Laboratory, SVNIT, Surat, India.

Table 2. Hydraulic characteristics of experimental runs used in the present study

Experimental. run no.	d_a (mm)	Flow discharge (m^3/s)	Flow depth (h_o) (m)	Mean flow velocity (m/s)	Water surface slope (%)	Bed load transport rate × 10^{-6} (m^2/s)	$\Delta q_s/q_{s0}$	Time interval of transient profiles recorded during runs (min)
A_1	0.707	0.0263	0.0785	0.377	0.25	0.36	2	15, 30, 45, 80, 105, 120
A_2	0.707	0.0263	0.0785	0.377	0.25	0.36	4	15, 30, 45, 80, 105, 120
A_3	1.414	0.0496	0.10	0.428	0.25	1.50	6	15, 30, 45, 60, 75, 90
A_4	0.707	0.0532	0.11	0.421	0.125	2.83	4	15, 30, 45, 60, 75
Soni *et al.*[5]*								
U_1	0.32	0.004	0.050	0.400	0.356	1.2	4	15, 30, 40
U_4	0.32	0.0071	0.086	0.413	0.225	1.610	3.5	50
U_6	0.32	0.0071	0.085	0.417	0.263	1.660	1.35	75
Rahman and Matin[37] Run 1	0.285	0.02	0.07	0.286	0.022	–	4	60

*Empirical constants $a = 1.45 \times 10^{-3}$ and $b = 5.0$ derived from uniform flow experiments[5].

drive with the motor arranged at the upstream end of the channel. The water and bed levels were measured at every 1 m interval of the test section using pointer gauge and a gauge having circular flat bottom respectively. Five water level sensors were also used to monitor the water levels throughout the working section of the flume. Such observations were recorded on a digital panel board, installed at the upstream end of the channel.

To develop full turbulent flow conditions on the upstream of the test section, a coarse gravel bed (particle size ranging 16–32 mm) of 3 m length was prepared. At the upstream end of the flume, a sediment feeder was installed to continuously feed the entire channel width. The sediment trap section was used for continuous sampling of bed load without disturbing the flow. It contains two box-type sediment samplers with sliding and lifting arrangements. An electronic weighing machine and an automatic sieve shaker (with sieve set ranging from 0.0625 to 45.0 mm size) were used for measuring the grain size distribution of bed and bed load transport material in the channel.

Experimental procedure

The steps followed in the development of aggrading channel bed and measurement of water and bed surface profiles due to sediment overloading are described in the following sections.

Preparation of fluvial channel bed

The sediments used for the experiments were collected from the natural bed of Tapi River in India, with bed material sizes up to 64 mm at the sampling site. The procured sand was sieved into six prominent sizes.

The characteristics of uniform sediment size fractions used in the present experimental investigations are included in Table 2, where d_a represents the geometric mean size of sediments corresponding to the two sieve sizes between which the sediment fraction is retained. The bed of 0.07 m thick sediment layer was levelled carefully to achieve uniform longitudinal slope throughout the test section.

Establishment of uniform flow conditions and equilibrium sediment transport in channels

The downstream end of the flume is provided with a gate for maintaining uniform depth of flow in the channel. The discharge was increased in small increments by operating the inlet valve. The increase in discharges was continued until adequate movement of sediments were observed in the channel bed. The transported bed material was sampled at regular intervals of 15–30 min using sediment sampler. The collected material was put back into the sediment feeder for re-circulation. When the weight of three consecutive samples from the sediment sampler becomes the same, it can be considered that the equilibrium condition has been achieved.

Development and measurement of aggrading profiles

The aggradation experiments were conducted at different rates of sediment overloading varying from 1.5 to 6.5 q_{s0}, where q_{s0} is observed equilibrium sediment discharge in m^3/sec. Due to sediment overloading, the sediments started settling in the upstream end of the channel, and aggrading profiles were formed (Figure 4). The bed and water surface profiles were measured after 15 min from the commencement of aggrading profile. At the end of the test run, the bed was allowed to drain and become dry. The sediments in the channel bed were mixed thoroughly to maintain initial conditions in the channel and keep the set-up ready for the next experiment. Ranges of flow parameters used for performing experiments are included in Table 2.

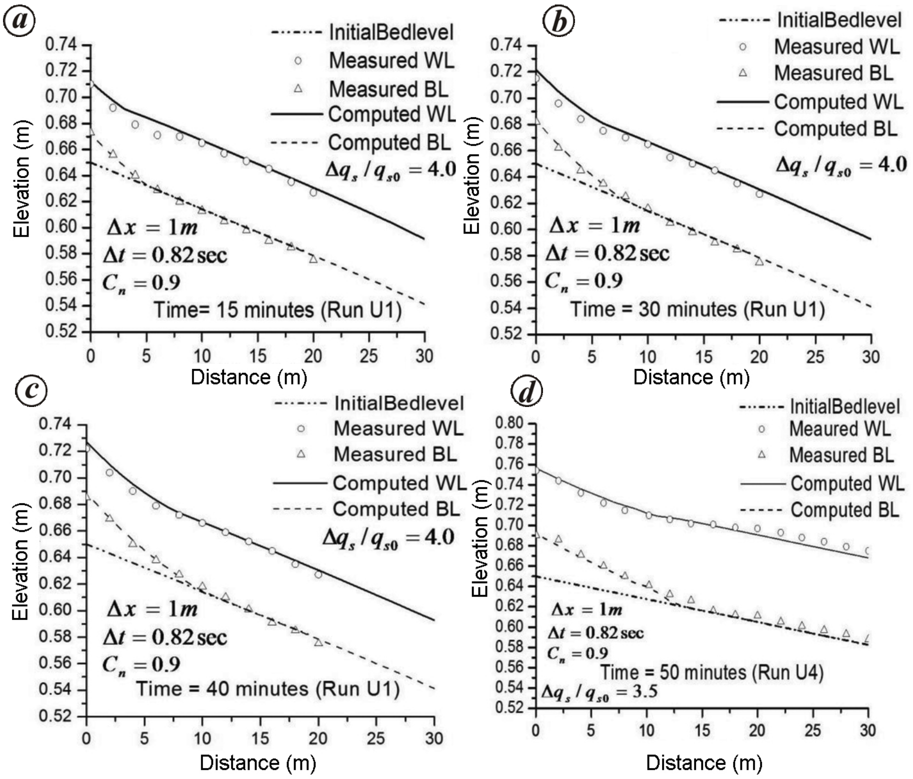

Figure 4. Computed and observed bed profiles for U_1 run (Soni *et al.*[5]). ***a***, Time = 15 min (RMSE = 0.19 cm for bed level); ***b***, 30 min (RMSE = 0.49 cm); ***c***, 40 min (RMSE = 0.27 cm); ***d***, run U_4 (50 min) (RMSE = 0.41 cm).

Data from other sources

The experimental data collected by Soni *et al.*[5] were procured for the analyses. The experimental runs are described in Table 2.

Results and discussion

The performance of numerical scheme being used in the present study has been validated using the Colby[48] type sediment transport function for laboratory flume data of Soni *et al.*[5]. The performance of numerical scheme is also assessed by using different sediment transport functions of uniform sediments. The numerical model with best-suited sediment transport function has been validated with both the experimental data of the present study and procured data from Soni *et al.*[5].

Computation of bed and water surface profiles

The bed and water levels for the flow conditions as described in Table 2 were computed as per the methodology described in Figure 2. The values of Δx and Δt were chosen such that the numerical scheme remained stable for chosen flow conditions. Figure 4 shows the computed bed and water transient profiles for the flow and sediment conditions described in experimental runs of U_1 and U_4 of Soni *et al.*[5]. The values of mean velocity, coefficient *a* and exponent *b* were obtained from Soni *et al.*[5].

From Figure 4 it is apparent that the numerical model now used computes the bed and water levels satisfactorily for the flow parameters used by Soni *et al.*[5]. Detailed descriptions of cited statistical performance indices are available elsewhere[63]. The sediment transport function, $q_s = aU^b$, used by Soni *et al.*[5], has the uncertainty in selection of coefficient *a* and exponent *b* values for different flow and sediment characteristics as they depend on specific flow and sediment conditions. Thus, it is necessary to assess the suitability of other existing sediment transport functions for their inclusion in sediment continuity equations in computation of bed level variations using the numerical scheme used in the present study.

Performance of sediment transport functions

The numerical results using eight selected sediment transport functions (Table 1) for the present datasets were compared with corresponding observed bed and water profiles, to assess their performance in computation of bed and water levels. The numerical computations were

Table 3. Statistical performance of sediment transport functions for computation of bed and water surface profiles

Statistical performance indices	Sediment transport functions	MPM[40]	Samaga *et al.*[58,59]	Englund and Hansen[53]	Van Rijn[57]	Einstein[49,50]	Karim and Kennedy[55]	Hanes[60]	Wong and Parker[62]
Run U_1 40 min (Soni *et al.*[5])									
RMSE (cm)	BL*	0.43	0.3	0.29	0.33	0.64	0.15	0.26	0.39
	WL	0.37	0.37	0.29	0.22	0.83	0.26	0.33	0.39
Run A_1 60 min (present study)									
RMSE (cm)	BL	0.61	0.48	0.61	0.8	0.33	0.13	0.22	0.54
	WL	0.09	0.08	0.22	0.13	0.26	0.065	0.16	0.35
Run A_2 60 min (present study)									
RMSE (cm)	BL	1.9	0.93	0.66	1.97	1.15	0.13	0.26	0.86
	WL	0.58	0.35	0.71	0.65	1.39	0.11	0.36	0.91

*BL: Bed level; WL, Water level; RMSE: Root mean square error.

performed at $\Delta x = 1$ m, and the time step taken corresponding to stability index C_n equals to 0.6. The values of Δt were calculated for each time step, ranging from 0.175 to 0.45 sec.

The computed bed and water levels are compared with the corresponding observed values in run U1 (transient time 40 min) of Soni *et al.*[5], for different sediment transport functions (Table 3 and Figure 5). Also, the data collected in the present study (experimental runs A_1 and A_2) have been used to assess the performance of selected sediment transport functions in the prediction of bed and water levels while using them in the numerical model developed in the present study. The observed bed and water level profiles for a transient period of 60 min (runs A_1 and A_2) are compared with corresponding computed profiles in Table 3. From Figure 5 and Table 3, it is evident that the computed bed and water surface profiles are close to respective observed profiles while using Karim and Kennedy[55] sediment transport function in the computation algorithm for the present datasets.

The best performance of Karim and Kennedy[55] method in prediction of bed and water surface profiles can be attributed to the following points. (i) Its development, for which large sets of experimental and field data were used. (ii) Consideration of wide range of flow and sediment conditions. (iii) Its derivation, which was based on stochastic and non-linear multiple regression with the use of seven basic fluid, flow and sediment characteristics (i.e. flow depth h, mean velocity U, energy slope S_m, median size of bed material d_{50}, bed material gradation σ_g, specific gravity s and kinematic viscosity v).

Further, the sediment transport functions, in the order of their performance for the present datasets, in prediction of bed and water surface profiles can be written using data from previous studies[40,49,50,53,57–60,62]. Thus, depending upon the availability of input data for computation of sediment transport, suitable sediment transport functions can be used for prediction of bed level variations in alluvial streams.

Performance of numerical model for recommended sediment transport function

Figure 6 presents the performance of numerical model with the use of sediment transport function by Karim and Kennedy[55], for the datasets of Soni *et al.*[5] (i.e. experimental run U_6) and data collected in the present study (i.e. experimental runs A_3 and A_4). From Figure 6, it is evident that numerical model performs well for complete datasets in prediction of bed and water level variations in alluvial streams.

Performance of proposed numerical model vis-à-vis existing numerical models

Numerical model by Rahman and Matin[37]*:* Rahman and Matin[37] developed a 1D unsteady gradually varied flow numerical model using finite difference scheme and Colby[38] sediment transport function in prediction of bed level changes in laboratory alluvial channel (having 12 m length, 1 m wide and 0.61 m deep) consisted of uniform sand having median diameter of 0.285 mm (Table 2). The numerical model now developed has been used in the prediction of bed level of alluvial channel using experimental data of Rahman and Matin[37] (experimental run 1 at $t = 1$ h). The computed bed levels from the numerical model now developed and those obtained from the numerical model of Rahman and Matin[37] are plotted in Figure 7 *a* along with the corresponding observed bed levels in the channel. From Figure 7 *a*, it is evident that our numerical model performs better than the numerical model of Rahman and Matin[37], particularly in upstream reach of the channel. The improved performance of the model proposed in the present study is due to the inclusion of robust sediment transport function (function of flow velocity only) in the numerical model instead of using Colby[38] sediment transport function being used by Rahman and Matin[37]. Inadequate performance of the numerical model of Rahman and Matin[37] in upstream reaches of

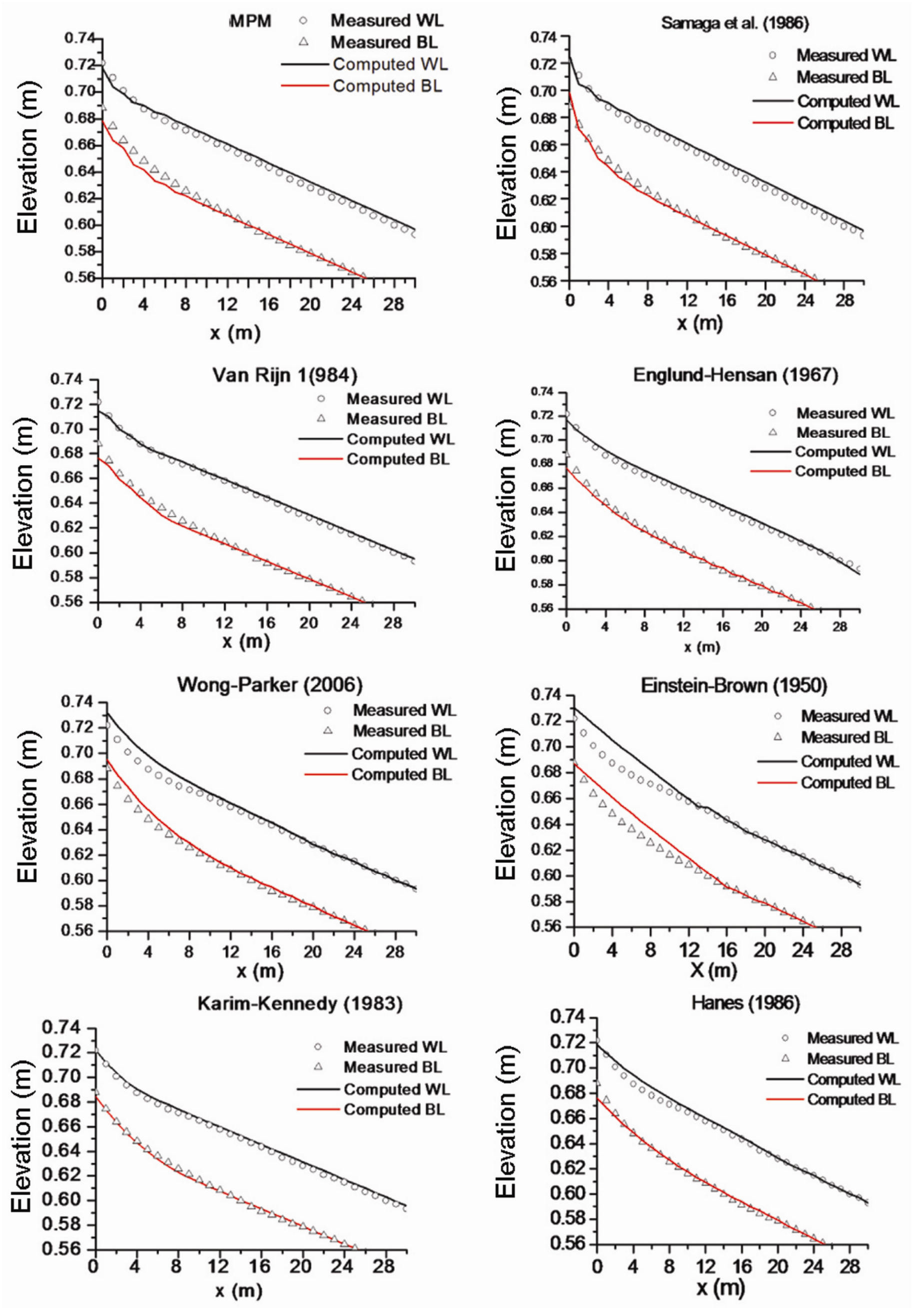

Figure 5. Comparison of predicted and measured bed (BL) and water surface (WL) profiles for run U_1 ($t = 40$ min).

the channel is due to excessive variations in bed slopes of the channel and hence, variations in the coefficients *a* and *b* due to the development of aggrading profile.

Mathematical model developed by Tayfur and Singh^{34,41}

Tayfur and Singh[41] developed a 1D numerical model for prediction of bed levels due to aggradation process using Lax explicit finite difference scheme that use kinematic wave routing. The friction slope, suspended sediment concentration, particle velocity and particle fall (terminal) velocity were estimated using Chezy's equation, Velikanov[64], Bridge and Dominic[65], and Dietrich[66] respectively. Tayfur and Singh[34] also developed a diffusion wave based model using double decomposition algorithm for describing the same process. Tayfur and Singh[34,41] verified the performance of the model using the data collected (experimental run U_6) by Soni *et al.*[5]. The performance of the numerical model developed in the present study has been compared with Tayfur and Singh[34,41] models by estimating the aggradation bed profile (for $t = 75$ min) of the same experimental run

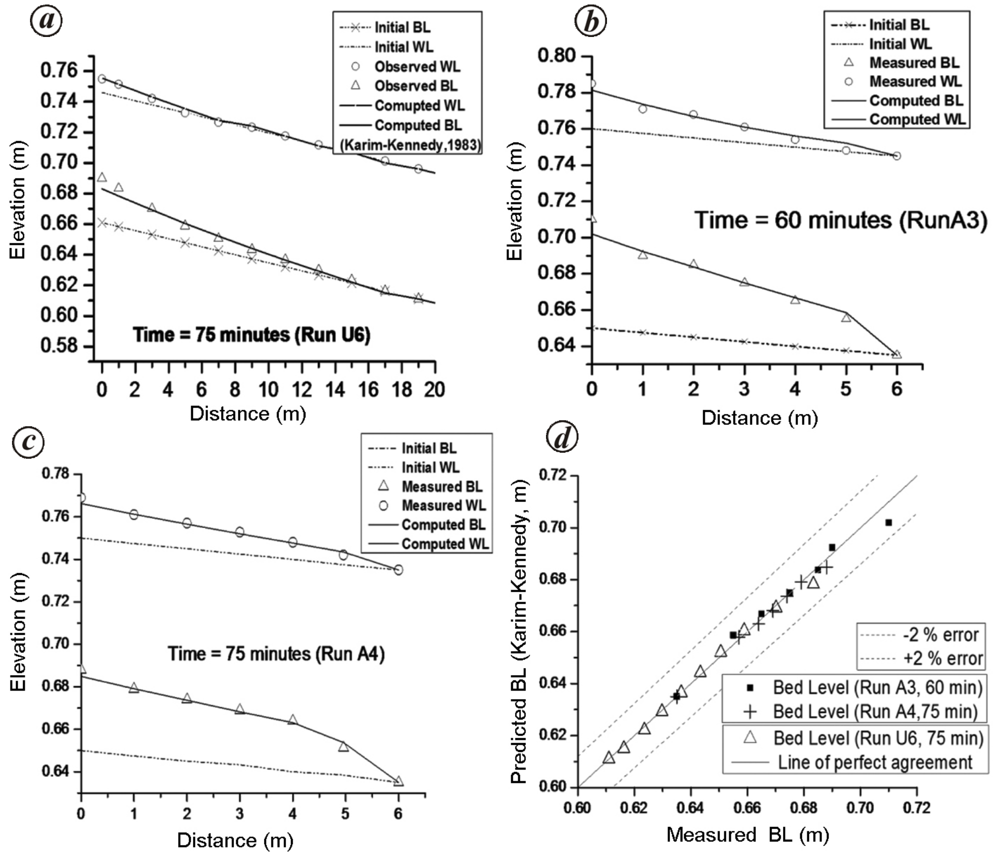

Figure 6. Performance of numerical model using ‘Karim and Kennedy[55]’ sediment transport function for (***a***) run U_6 (Soni *et al.*[5]), (***b***) run A_3 (present study), (***c***) run A_4 (present study), and (***d***) computed and observed levels for runs U_6, A_3 and A_4.

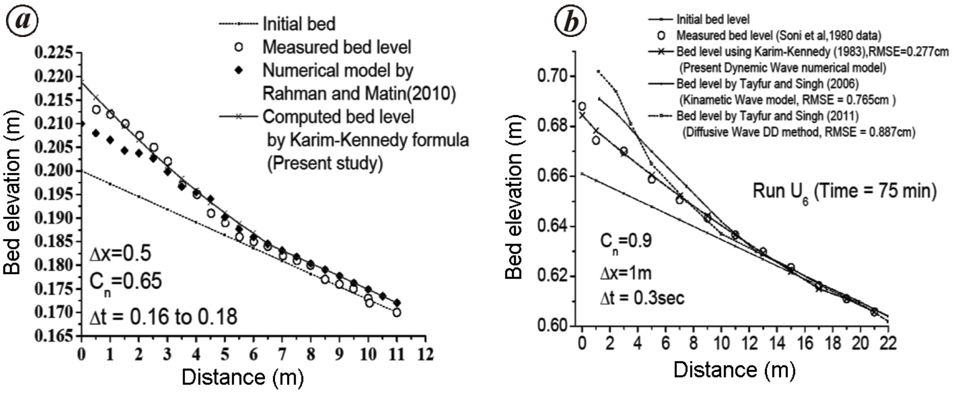

Figure 7. Comparative performance of developed numerical model vis-à-vis (***a***) Rahman and Matin[37], (***b***) Tayfur and Singh[34,41].

U_6 (Figure 7*b*). From Figure 7*b*, it is evident that the numerical model developed in the present study performs better (with RMSE = 0.27 cm) particularly for initial reaches of the channel length. The better performance of the model is mainly due to the inclusion of inertial terms in the water momentum equation of fluid flow, and use of better sediment transport functions in the equations of sediment continuity.

Conclusions

A numerical model, based on 1D de Saint–Venant equations along with sediment continuity equations is developed for prediction of bed levels of aggrading alluvial channels. The performance of the model has been assessed using eight sediment transport functions for their suitability for present datasets in the prediction of bed level profiles. The performance of the developed model has been validated with data recorded in the present experimental study, and those available from previous studies[5]. The model is also compared with recent models, which were developed for predicting bed levels of alluvial channels. The conclusions of the present study are summarized below:

(1) Sensitivity of the numerical model developed in the present study with reference to selected sediment transport functions has been undertaken using the data collected in the present study and that available in the literature. The best performance of the numerical model is obtained while using sediment transport function proposed by Karim and Kennedy[55].

(2) The performance of various sediment transport functions (used in the developed numerical model) while predicting bed and water surface profiles in descending order is Karim and Kennedy[55], Hanes[60], Englund–Hensan[53], Samaga *et al.*[58,59], van Rijn[57], Wong–Parker[62], Meyer–Peter Müller[40] and Einstein–Brown[49,50]. The choice of sediment transport function depends on the availability of input data.

(3) The numerical model developed in the present study performed better than the models developed by Rahman and Matin[37] and Tayfur and Singh[34,41].

---

1. Garde, R. J. and Ranga Raju, K. G., *Mechanics of Sediment Transportation and Alluvial Stream Problems*, New Age Publishers, New Delhi, 2000.
2. Adachi, S. and Nakato, T., Changes of top-set bed in a silted reservoir. 13th Congress, IAHR, 5-1, 1969, pp. 269–272.
3. deVries, M., River bed variation-aggradation and degradation. In International Seminar on Hydraulics of Alluvial Streams, IAHR, Delft, the Netherlands, 1973.
4. Bhamidipaty, S. and Shen, H. W., Laboratory study of degradation and aggradation. *J. Waterways, Harbours Coastal Eng. Div.*, 1971, **97**(4), 615–630.
5. Soni, J. P., Garde, R. J. and Ranga Raju, K. G., Aggradation in streams due to overloading. *J. Hydraul. Eng.*, 1980, **106**(1), 117–131.
6. Mehta, P. J., Garde, R. J. and Ranga Raju, K. G., Transient bed profiles in aggrading streams. In Proceeding of 2nd International Conference on River Sedimentation, Nanjing, China, 1983.
7. Jain, S. C., River bed aggradation due to over loading. *J. Hydraul. Eng.*, 1981, **107**(1), 120–124.
8. Seal, R., Paola, C., Parker, G., Southard, J. B. and Wilcock, P. R., Experiments on downstream fining of gravel. I: narrow-channel runs. *J. Hydraul. Eng.*, 1997, **123**(10), 874–884.
9. Carlos, M., Escobar, T., Paola, C., Parker, G., Wilcock, P. and Southard, J., Experiments on downstream fining of gravel. II: wide and sandy runs. *J. Hydraul. Eng.*, 2000, **126**(3), 198–208.
10. Thomas, W. A. and Prasuhn, A. L., Mathematical modeling of scour and deposition. *J. Hydr. Div.*, 1977, **103**(8), 851–863.
11. Cunge, J. A., Holly, F. M. and Verwey, A., *Practical Aspects of Computational River Hydraulics*, Pitman Advanced Publishing Program, London, 1980.
12. Karim, M. F., Kennedy, J. F., IALLUVIAL: a computer-based flow and sediment routing for alluvial streams and its application to the Missouri River, Iowa. *Inst. Hydraul. Res.*, Rep. No. 250, The Univ. of Iowa, Iowa City, Iowa, 1982.
13. Chang, H. H., Modeling of river channel changes. *J. Hydraul. Eng.*, 1984, **110**(2), 157–172.
14. Lyn, D. A., Unsteady sediment transport modelling. *J. Hydraul. Eng.*, 1987, **113**(1), 1–15.
15. Cui, Y., Parker, G. and Paola, C., Numerical simulation of aggradation and downstream fining. *J. Hydraul. Res.*, 1996, **34**(2), 195–204.
16. Lyn, D. A. and Goodwin, S. M., Stability of a general Preissmann scheme. *J. Hydraul. Eng.*, 1987, **113**(1), 16–28.
17. Bhallamudi, S. M. and Chaudhry, H. M., Numerical modeling of aggradation and degradation in alluvial channel. *J. Hydraul. Eng.*, 1991, **117**(9), 1145–1164.
18. Alcrudo, F., Garcia-Navarro, P. and Saviron, J. M., Flux difference splitting for 1D open channel flow equations. *Int. J. Numer. Meth. Fluids*, 1992, **14**, 1009-1018.
19. Kassem, A. M. and Chaudhry, M. H., Comparison of coupled and semicoupled numerical models for alluvial channels. *J. Hydraul. Eng.*, 1998, **124**(8), 794–802.
20. Tayfur, G. and Singh, V. P., Kinematic wave model for transient bed profiles in alluvial channels under non-equilibrium conditions. *Water Resour. Res.*, 2007, **43**(12); doi:10.1029/2006WR005681.
21. Tayfur, G. and Singh, V. P., Transport capacity models for unsteady and non-equilibrium sediment transport in alluvial channels. *Comput. Electron. Agric.*, 2012, **86**, 26–33; doi:10.1016/j.compag.2011.12.005.
22. Goutiere, L., Soares-Frazao, S., Savary, C., Laraichi, T. and Zech, Y., One-dimensional model for transient flows involving bed-load sediment transport and changes in flow regimes. *J. Hydraul. Eng.*, 2008, **134**(6), 726–735.
23. Schippa, L. and Pavan, S., Bed evolution numerical model for rapidly varying flow in natural streams. *Comput. Geosci.*, 2009, **35**(2), 390–402.
24. Rahuel, J. L., Holly, F. M., Chollet, J. P., Belludy, P. and Yang, G., Modelling of river bed evolution for bed load sediment mixtures. *J. Hydraul. Eng.*, 1989, **115**(11), 1521–1542.
25. Holly, F. and Rahuel, J., New numerical physical framework for mobile-bed modelling, part I: numerical and physical principles. *J. Hydraul. Res.*, 1990, **28**(4), 401–416.
26. Holly, F. and Rahuel, J., New numerical physical framework for mobile-bed modelling, part II: test applications. *J. Hydraul. Res.*, 1990, **28**(4), 545–564.
27. Correia, L. R. P., Krishnappan, B. G. and Graf, W. H., Fully coupled unsteady mobile boundary flow model. *J. Hydraul. Eng., Proc.*, 1992, **118**(3), 476–494.
28. Saiedi, S., Coupled modeling of alluvial flows. *J. Hydraul. Eng.*, 1997, **123**(5), 440–446.
29. Park, I. and Jain, S. C., River bed profiles with imposed sediment load. *J. Hydraul. Eng.*, 1986, **112**(4), 267–279.
30. Wu, W., Depth-averaged two-dimensional numerical modelling of unsteady flow and nonuniform sediment transport in open channels. *J. Hydraul. Eng.*, 2004, **130**(10), 1013–1024.
31. Saiedi, S., Experience in design of a laboratory flume for sediment studies. *Int. J. Sediment Res.*, Beijing, China, 1993, **8**(3), 89–101.
32. Yen, C.-L., Chang, S.-Y. and Lee, H.-Y., Aggradation-degradation process in alluvial channels. *J. Hydraul. Eng.*, 1992, **118**(12), 1651–1669.
33. Foster, G. R., Modelling the erosion process, in hydrologic modelling of small watersheds (eds Haan, C. T., Johnson, H. P. and Brakensiek, D. L.) Am. Soc. of Agric. Eng. Monograph No. 5, St. Joseph, Michigan, 1982, pp. 295–380.
34. Tayfur, G. and Singh, V. P., Simulating transient sediment waves in aggraded alluvial channels by double-decomposition method. *J. Hydrologic Eng.*, 2011, **16**(4), 362–370.
35. Adomian, G., A new approach to nonlinear partial differential equations. *J. Math. Anal. Appl.*, 1984, **102**, 420–434.
36. Adomian, G., A review of decomposition method in applied mathematics. *J. Math. Anal. Appl.*, 1988, **135**, 501–544.

37. Rahman, M. A. and Matin, M. A., Numerical modelling of bed level changes of alluvial river. *J. Civil Eng. (IEB)*, 2010, **38**(1), 53–64.
38. Colby, B. R., Discharge of sands and mean–velocity relationships in sand bed streams. U.S. Geological Survey Professional Paper 462-A, US Geological Survey, Washington, DC, 1964.
39. Begin, Z. B., Meyer, D. F. and Schumm, S. A., Development of longitudinal profiles of alluvial channels in response to base-level lowering. *Earth Surf. Processes Land Forms*, 1981, **6**(1), 49–68.
40. Meyer, P. E. and Muller, R., Formulas for bed load transport. In Proceeding 2nd Congress of IAHR, Appendix-2, Stockholm 7–9, Sweden, 1948, pp. 39–64.
41. Tayfur, G. and Singh, V. P., Kinematic wave model of bed profiles in alluvial channels. *Water Resour. Res*., 2006, **42**(6), 1–13.
42. Guy, H. P., Simons, D. B. and Richardson, E. V., Summary of alluvial channel data from flume experiments. 1956–1961, US Geol. Surv. Prof. Pap., 462–I, 1966, p. 96.
43. Zhang, H. and Kahawita, R., Nonlinear model for aggradation in alluvial channels. *J. Hydraul. Eng*., 1987, **113**(3), 353–369.
44. Cao, Z., Pender, G., Wallis, S. and Carling, P., Computational dam-break hydraulics over erodible sediment bed. *J. Hydraul. Eng*., 2004, **130**(7), 689–703.
45. MacCormack, R. W., An efficient numerical method for solving the time dependent compressible Navier-Stokes equations at high Reynolds number. Computing in Applied Mechanics; A77-46133 21–59, ASME, 49–64; In Proceedings of the Winter Annual Meeting, New York, 1976.
46. Garcia–Navarro, P., Alcrudo, F. and Saviron, J. M., 1-D Open-channel flow simulation using TVD – MacCormack scheme. *J. Hydraul. Eng*., 1992, **118**(10), 1359–1372.
47. Ferreira, R. M. and Leal, J. G., 1-D mathematical modeling of the instantaneous dam-break flood wave over mobile bed: application of TVD and flux splitting schemes. CADAM Project meeting, Munich, Germany, 1998.
48. Alcrudo, F. and Garcia-Navarro, P., A high resolution Godunov-type scheme in finite volumes for the 2D shallow-water equations. *Int. J. Numer. Meth. Fluids*, 1993, **16**(6), 489–505.
49. Einstein, H. A., The bed-load function for sediment transport in open channel flows. US Dept. of Agric., Soil Cons. Serv., Tech. Bull. No. 1026, 1950.
50. Brown, C. B., *Sediment Transportation*, Chapter XII Engineering Hydraulics (ed. H. Rouse), John Wiley and Sons, New York, 1950.
51. Ashida, K. and Michiue, M., Hydraulic resistance of flow in an alluvia bed and bed load transport rate. In Proceedings of Japan Society of Civil Engineers, 1972, p. 206.
52. Ackers, P. and White, W. R., Sediment transport-new approach and analyses. *J. Hydr. Div.*, 1973, **99**(11), 2041–2060.
53. Engelund, F. and Hansen, E., A monograph on sediment transport in alluvial streams. Report by Teknisk Forlag, Skelbrekgade 4, Copenhagen V, Denmark, 1967.
54. Brownlie, W. R., Unsteady Sediment Transport Modeling. In Proceedings of the Water Forum. *ASCE*, 1981, **81**(II), 1193–1200.
55. Karim, M. F. and Kennedy, J. F., Computer based predictors for sediment discharge and friction factor of alluvial streams. In Proceedings of 2nd International Symposium on River Sedimentation, Cot.-Nov., Nanjing, China, 1983.
56. Misri, R. L., Garde, R. J. and Ranga Raju, K. G., Bed load transport of coarse nonuniform sediment. *J. Hydraul. Eng*., 1984, **110**(3), 312–328.
57. Van Rijn, Sediment transport Part-I; bed load transport. *J. Hydraul. Eng*., 1984, **110**(10), 1431–1456.
58. Samaga, B. R., Ranga Raju, K. G. and Garde, R. J., Bed load transport of sediment mixtures. *J. Hydraul. Eng*., 1986, **112**(11), 1003–1018.
59. Samaga, B. R. Ranga Raju, K. G. and Garde, R. J., Suspended load transport of sediment mixtures. *J. Hydraul. Eng*., 1986, **112**(11), 1019–1035.
60. Hanes, D. M., Grain flows and bed-load sediment transport: review and extension. *Acta Mech*., 1986, **63**(1–4), 131–142.
61. Nielsen, P., Coastal bottom boundary layers and sediment transport. World Scientific, 1992, **4**.
62. Wong, M. and Parker, G., Reanalysis and correction of bed load relation of Meyer–Peter and Muller using their own database. *J. Hydraul. Eng*., 2006, **132**(11), 1159–1168.
63. Mutreja, K. N., *Applied Hydrology*, First Edition, McGraw-Hill Publishing Company Ltd, New Delhi, 1986.
64. Velikanov, M. A., Gravitational theory of sediment transport (in Russian). *J. Sci. Soviet Union*, 1954, **4**.
65. Bridge, J. S. and Dominic, D. F., Bed load grain velocities and sediment transport rates. *Water Resour. Res*., 1984, **20**(4), 476–490.
66. Dietrich, W. E., Settling velocity of natural particles. *Water Resour. Res*., 1982, **18**(6), 1615–1626.

ACKNOWLEDGEMENTS. We are grateful to the Department of Science and Technology (DST), New Delhi, for financial support as part of the project for installing the flume facilities through the Research Project 'Erosion of non-uniform unimodal and bimodal sediments' with project Grant No. SR/S3/MERC/015/2009.

Received 5 December 2016; revised accepted 21 November 2017

doi: 10.18520/cs/v114/i08/1697-1708