Portfolio mean-variance approach modifications: modulus function, principles of compromise, and 'min–max' approach

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We offer a variant for the problem of portfolio selection, based on the modification of quadratic function. It reduces overestimation of expected returns that arise from large deviations of the market condition. Further, we examine the modified 'min–max' approach to portfolio structure. We obtain analytical expressions to solve the portfolio selection model for a few cases. Finally, we offer certain compromise principles between criterial values of the expected return/risk.

Keywords: Mean-variance, 'min–max' approach, modern portfolio theory, portfolio selection.

THE optimal portfolio selection is part of modern portfolio theory (MPT), developed in 1952 (ref. 1). MPT includes the Harry Markowitz Model[1] and Capital Asset Pricing Model (CAPM)[2–4], as well as other theories and models that allow portfolio selection for meeting the needs and aims of the investor[3,5,6].

The problem of measuring risk has been examined previously[7–15]. Moreover, the behaviour of prices in capital markets has also been examined[2,4,6,16–19]. In addition, the 'min–max' approach to portfolio selection has been studied[20–27]. The application of a principle of compromise for portfolio selection was examined earlier[28,29], regarding the use of a Constrained Compromise Programming Model (CCCP); Bilbao-Terol *et al.*[30] examined the portfolio selection of government bond funds; and González *et al.*[31] studied the concept of social portfolio return, based on the best compromise solution. Hasuike and Katagiri[32] examined the principle of compromise regarding an exact algorithm of an explicit optimal portfolio; and Li and Xu[33] examined the development of a genetic algorithm based on the compromise approach.

Common planning of the portfolio analysis model

Suppose that an investor has capital K, which he intends to invest into a set of M, aimed at a portfolio selection.

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Let A_i, $i = 1, \ldots, M$ denotes the random value of total stock return of type i. By accounting changes in market conditions, the final number of outcomes N of random events can be supposed. Thus, $A_i = (a_{i1}, a_{i2}, a_{ij}, a_{iN})$. Such representation of incoming information is satisfied by data concerning quotation and current payments (percentages, dividends of stocks), for a range of periods (T), while N is supposed to be equal to T. The apparent sequences $\{a_i^{(t)},\ t = 1, 2, \ldots, T;\ i = 1, 2, \ldots, M\}$, within the stated hypothesis, are considered realizations of a set of random values $\{A_i, i = 1, M\}$. Further, return values in the future ($t \geq T + 1$) are supposed by realizations of the same set.

The particular values of the total stock return $a_i^{(t)}$ of type i for period t are calculated using the formula

$$a_i^{(t)} = \frac{\underline{C}_i^{(t+1)} + d_i^{(t)} - \overline{C}_i^{(t)}}{\overline{C}_i^{(t)}} \times 100, \text{ for each } i, t,$$

where $\overline{C}_i^{(t)}$ is the purchase price for the stocks of type i at the beginning of period t, $d_i^{(t)}$ the current profit for period t, and $\underline{C}_i^{(t+1)}$ is the selling stock price at the end of t/beginning of period $(t+1)$. This information is represented in Table 1, where p_j is the probability of result j of the market conditions

$$\sum_{j=1}^{N} p_j = 1,\ p_j \geq 0, \quad j = 1, 2, \ldots, N,\ N \equiv T.$$

In accordance with the described method, suppose that $p_1 = p_2 = \cdots = p_N = 1/N$.

In the last two columns of Table 1, we have

$$m_i = \sum_{j=1}^{N} p_j a_{ij}, \quad D_i = \sum_{j=1}^{N} p_j (a_{ij} - m_i)^2,$$

$$\sigma_i = \sqrt{D_i}, \quad i = 1, 2, \ldots, M.$$

The mean m_i represents the expected value of random variable A_i of the return on investments of type i. The variance D_i characterizes risk.

Table 1. Characteristics of stock sets

	j							
i	p_1	p_2	…	p_j	…	p_N	Mean	Variance (standard deviation)
u_1	a_{11}	a_{12}	…	a_{1j}	…	a_{1N}	m_1	$D_1(\sigma_1)$
u_2	a_{21}	a_{22}	…	a_{2j}	…	a_{2N}	m_2	$D_2(\sigma_2)$
…	…	…	…	…	…	…	…	…
u_i	a_{i1}	a_{i2}	…	a_{ij}	…	a_{iN}	m_i	$D_i(\sigma_i)$
…	…	…	…	…	…	…	…	…
u_M	a_{M1}	a_{M2}	…	a_{Mj}	…	a_{MN}	m_M	$D_M(\sigma_M)$

Analysis of model constructions

The undefined variant of a portfolio's structure (i.e. the combination of the stocks from set numbers) is determined by the set of normalized values (shares) $u_i \geq 0$, $i = 1, ..., M$ (Table 1). The sum of values is equal to one.

Part of a capital K_i, which should be invested into stocks of type i, is set from an elementary ratio $K_i = u_i K$, $i = 1, ..., M$. The total portfolio return corresponding to the undefined variant $v = (v_1, v_2, ..., v_N)$ can be treated as a random value, taken with probabilities p_j the following value

$$v_j = \sum_i a_{ij} u_i, \quad j = 1, 2, \ldots, N.$$

To measure the expected level of (variant) unreliability, variance's value D_v (or the standard deviation (SD) σ_v) should be considered. It is calculated as

$$D_v = \sum_{j=1}^{N} p_j \left(v_j - \sum_{j=1}^{N} p_j v_j \right)^2; \quad \sigma_v = \sqrt{D_v}. \tag{1}$$

The value of the sums $\sum_j p_j v_j = m_v$ characterizes the expected value of a portfolio's return by the evaluated variant. It is clear that D_i is a special case of D_v, where, when $u_i = 1$, $u_k = 0$ at all $k \neq i$.

The variant of portfolio structure with the least risk, is obtained from the model's solution

$$\sum_j p_j (v_j - W)^2 \to \min,$$

$$v_j = \sum_i a_{ij} u_i, \quad j = 1, 2, \ldots, N,$$

$$\sum_i a_{ij} u_i - v_j = 0, \quad j = 1, 2, ..., N,$$

$$\sum_j p_j v_j - W = 0,$$

$$\sum_i u_i = 1, \quad u_i \geq 0, \quad i = 1, ..., M. \tag{2}$$

where W is the average portfolio's return.

This relates to the models of the convex programming. The target function corresponds to D_v from model (1). The selection of the best variant of the combination of stocks is determined by the optimal values u_i^* of variables u_i.

Consider quite a different model assignment to select an optimal portfolio

$$\sum_j p_j (h - v_j)^2 \to \min,$$

$$\sum_i a_{ij} u_i - v_j = 0, \quad j = 1, 2, ..., N,$$

$$\sum_i u_i = 1, \quad u_i \geq 0, \quad i = 1, ..., M. \tag{3}$$

where h denotes any constant value. By transformations, an optimality criterion from model (3) is reduced to

$$h^2 - (2h - m_v)\, m_v + D_v \to \min. \tag{4}$$

As h^2 is constant, it is easy to see that it is represented by two criteria: the minimization of a variance and maximization of an ascending function (in a range of values $m_v \leq h$) of the mean of the portfolio's return. In this criterion, parameter h characterizes the weight, or priority of the summand, containing m_v. Thus, gradually increasing h from $h_0 = W^*$, and consequently solving model (3), we will receive variants $\{u_i^*\}$, $i = 1, ..., M$ of the portfolio's structure. Such variants are characterized by a monotone increase in the mean m_v^* until it is equal to $\max_i m_i$. The variance D_v^* and standard deviation σ_v^* will also increase. Values m_v^*, D_v^* and σ_v^* are calculated by substitution into the corresponding equations of model (3).

Note that a variant $\{u_i^*\}$, $i = 1, ..., M$ at fixed h has a minimal variance that is equal to D_v^* (unlike any other variant ensuring the mean return, which is similar to m_v^*).

From the theory of multi-objective optimization, we know that it is impossible to obtain the only optimal variant from a numerosity of incomparable alternative variants without involving additional expert information concerning the preferences of a decision-maker. Variant $\{u_i^*\}$, $i = 1, ..., M$ with corresponding m_ν^* and D_ν^* is considered as belonging to the numerosity of incomparable variants, if there is no acceptable variant $\{u_i\}$, $i = 1, ..., M$ with m_ν and D_ν, such as $m_\nu \geq m_\nu^*$ and $D_\nu \leq D_\nu^*$. However, at least one of the inequalities is a strict one ($m_\nu > m_\nu^*$ and $D_\nu \leq D_\nu^*$, or $m_\nu \geq m_\nu^*$ and $D_\nu < D_\nu^*$). In accordance with this definition, selection from numerosity should be considered rational.

A range of h values from $h_0 = W^*$ until some h, at which m_ν^* becomes equal to $\max_i m_i$, represents a special scale of preferences, and each point on this scale corresponds to a certain compromise between the particular purpose of the examined model. In these conditions, one of ways to eliminate or reduce the uncertainty of the selection is its execution, based on the preliminary formulation of principles of a compromise. Consider a case, where a model is preliminarily introduced to select a variant of portfolio structure, with a number of stocks types M, equal to two. In this case, models (2) and (3) allow for a solution.

Suppose that Table 1 of the initial data contains the first two lines. An optimal solution $\{u_i^*\}$, $i = 1, 2$ of model (2), in an analytical form, can be obtained using the required conditions of minimum, applicable to the target function of the stated model, by preliminarily expressing it through variables u_i, $i = 1, 2$, and by substituting $u_1 = 1 - u_2$.

By solving an equation corresponding to a stationary point, taking into account the condition $0 \leq u_2 \leq 1$, we get the following model

$$u_2^* = \begin{cases} 0, & \text{if } u_2' < 0, \\ u_2', & \text{if } 0 \leq u_2' \leq 1, \\ 1, & \text{if } u_2' > 1, \end{cases} \tag{5}$$

where

$$u_2' = \frac{m_1 b - \sum_j p_j a_{1j} \beta_j}{\sum_j p_j \beta_j^2 - b^2}; \quad \beta_j = a_{2j} - a_{1j},\ b = \sum_j p_j \beta_j.$$

As such, $u_1^* = 1 - u_2^*$. A denominator of an expression determining u_2' is equal to zero only when $\beta_1 = \beta_2 = \cdots = \beta_N$. However, in this case, any combination of the two stocks in one portfolio is irrational (provided that $\beta_j \neq 0$).

Similarly, for model (3), we will have

$$u_2^*(h) = \begin{cases} 0, & \text{if } u_2'' < 0, \\ u_2'', & \text{if } 0 \leq u_2'' \leq 1, \\ 1, & \text{if } u_2'' > 1, \end{cases} \tag{6}$$

where

$$u_2'' = \frac{\sum_j p_j \beta_j \alpha_j}{\sum_j p_j \beta_j^2} \text{ or } \frac{\sum_j \beta_j \alpha_j}{\sum_j \beta_j^2},$$

at $p_1 = p_2 = \cdots = p_N = 1/N$, and $\alpha_j = h - a_{1j}$.

By setting definite values u_2 $(0 \leq u_2 \leq 1)$, it is easy to find the corresponding h

$$h = \frac{u_2 \sum_j p_j \beta_j^{\,2} + \sum_j p_j \beta_j a_{1j}}{\sum_j p_j \beta_j}. \tag{7}$$

Model (7) has a solution when $m_1 \neq m_2$. Otherwise, the denominator in (7) can vanish, since from (5)

$$m_1 = \sum_j p_j a_{1j}, \quad m_2 = \sum_j p_j a_{2j}.$$

Note that if $m_1 = m_2$, then as it follows from model (5), $u_2^*(h)$ will be equal to u_2^* at any $-\infty < h < \infty$.

As noted above, the fixation of some point $h[h_0, \hat{h}]$ and, accordingly, the selection of a variant $u_2^*(h)$ from model (6) as the final one requires the involvement of any (out of the multi-objective model) information content. Moreover, such information requires expert assistance. In any common case, we have $\{u_i^*(h)\}$, $i = 1, ..., M$ from (3).

Such necessity is associated with the non-comparability of effective variants against one another, by determined criteria: if one has a higher value for the expected return than the other does, then the variance level is also higher, and vice versa. Further, the problem of improving the best solution can be examined in a higher-level model.

For example, the return values, by the direction of investments, are characterized by the following data (Table 2).

Consider a variant of the investment structure, differing by the least variability (variance). To do so, calculate u_2' as

$$\beta_1 = 25 - 20 = 5,\ \beta_2 = -8,\ \beta_3 = 3,\ \beta_4 = -4;$$

$$b = 0.25 \times (5 - 8 + 3 - 4) = -1;\ m_1 = 24;$$

$$u_2' = \frac{-24 + 35.75}{27.5} \approx 0.4273.$$

As per model (5), $u_2^* = 0.4273$ and $u_1^* = 0.5727$ (i.e. investments should be divided into proportions of 42.73%

Table 2. Initial information for an example

i	1	2	3	4	m_i	D_i (σ_i)
U_1	20	25	23	28	24	8.5 (2.915)
U_2	25	17	26	24	23	12.5 (3.535)

$M = 2$, $N = 4$, $P_1 = P_2 = P_3 = P_4 = 1/4$.

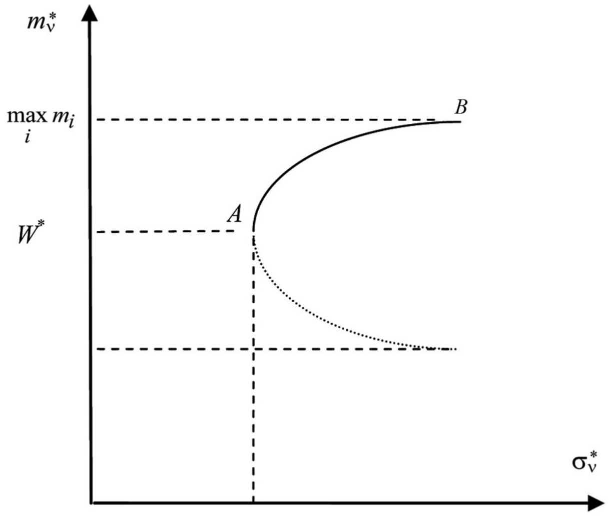

Figure 1. Efficient frontier $f(\cdot)$

and 57.27% respectively). At this expected return, $m_v^* = W^*$ is 23.573%, and SD σ_v^* = 1.865%.

All possible effective variants of a combination of these two directions of capital investments are within a range $u_2 = 0.4273$ to $u_2 = 0$. The scale of preferences, in terms of parameter h, has a range $h_0 \div \hat{h}$, where $h_0 = W^* = 23.5727$, and in accordance with model (7)

$$\hat{h} = \frac{0 \times 28.5 - 35.75}{-1} = 35.75.$$

Moreover, graph-based mapping of the efficient frontier proves to be useful.

f: $m_v^*(h) \to \sigma_v^*(h)$ (Figure 1).

The upper part of the graph is of particular interest. Within the graph, a point that determines the shares of stocks in the portfolio should be selected. Here, the application of formal methods is possible (e.g. in a part of their formulation, using the principles of compromise between the criterial values of the expected return and risk, like axioms of fair selection).

Note that the expression $(2h - m_v)m_v$ from model (4) is an increasing function from m_v for $m_v \le h$. As such, h as the priority parameter m_v should not be set to less than the maximum m_i.

The function $\sigma_v^* = f(m_v^*)$, in an explicit form, is unknown; however, it can be approximated with decent accuracy by several points within a section of interest, using any simple analytical expression, like a 2nd–4th degree polynomial. Consider the following two principles of compromise, the most rational from our point of view.

(1) A compromise at which the reduction of the expected return does not exceed the corresponding standard deviation reduction is considered fair

$$\sigma_v^*(\hat{h}) - f(m_v^*) \ge m_v^*(\hat{h}) - m_v^*,$$

$$W^* \le m_v^* \le \max_i m_i = m_v^*(\hat{h}).$$

(2) A compromise with maximum difference between the standard deviation reduction and reduction of the expected portfolio return is considered fair

$$m_v^* - f(m_v^*) \to \max, \; W^* \le m_v^* \le \max_i m_i.$$

The variants of a portfolio's structure, corresponding to the calculated values of m_v^*, can be obtained from the solution of model (3). To do so, it is necessary to use the dependency $h = \varphi(m_v^*)$. In an explicit form, a function φ is unknown, but calculations have shown that it is strongly described by a linear dependency.

The variant of a model assignment is to select an optimal portfolio structure that can be obtained from model (2), at the fixation of an expected value of the portfolio's return m_v, in a level that is acceptable for an investor m_0. Taking into account that

$$m_v = \sum_{i=1}^{M} m_i u_i,$$

as well as expressing an objective function through variables u_i, $i = 1, ..., M$, we get

$$D(u) = \sum_{j=1}^{N} p_j \left(\sum_{i=1}^{M} \hat{a}_{ij} u_i \right)^2 \to \min,$$

$$\sum_{i=1}^{M} m_i u_i \ge m_0,$$

$$\sum_{i=1}^{M} u_i = 1, \quad u_i \ge 0, \; i = 1, ..., M.$$

where $\hat{a}_{ij} = (a_{ij} - m_i)$, $i = 1, 2, \ldots, M$; $j = 1, 2, \ldots, N$.

Having made a replacement in a criterion of optimality

$$\left(\sum_{i=1}^{M} \hat{a}_{ij} u_i\right)^2$$

into the modulus function

$$\left|\sum_{i=1}^{M} \hat{a}_{ij} u_i\right|$$

for each j, and having executed linearization, we obtain a model of the linear programming of a certain type

$$\Phi(m_0) = \min_{u(m_0)} \sum_{j=1}^{N} p_j (Y_j + Z_j),$$

$$\sum_{i=1}^{M} \hat{a}_{ij} u_i - Y_j + Z_j = 0, \quad j = 1, \ldots, N,$$

$$\sum_i m_i u_i \geq m_0, \quad \sum_i u_i = 1, \quad u_i, Y_j, Z_j \geqslant 0, \text{ for } \forall i, j. \tag{8}$$

In this case, for a purposeful portfolio selection of the effective variants of a portfolio's structure, an analysis of a function of a minimum $\Phi(m_0)$ of a linear form of model (8) can be useful.

By examining model (8) as bi-criterial (the second criterion is $\sum_{i=1} m_i u_i \to \max$) and combining these criteria, we can obtain one more model

$$\sum_{i=1}^{M} m_i u_i - k \sum_{j=1}^{N} p_j (y_j + z_j) \to \max,$$

$$\sum_{i=1}^{M} \hat{a}_{ij} u_i - y_j + z_j = 0, \quad j = 1, \ldots, N,$$

$$\sum_{i=1}^{M} u_i = 1, \qquad u_i, y_j, z_j \geq 0, \forall i, j; k > 0. \tag{9}$$

Having changed k, it is possible to get Pareto-optimal variants of a portfolio's structure, from which a final selection can be made.

Modified 'min–max' approach to portfolio selection

In the previous sections, the evaluation of 'mean-variance' approach was used to characterize the supposed level of aggregation of a portfolio's return, as well as the security level of the capital invested into it. At the same time, another approach at which a certain variant of a portfolio's structure is selected, pending its guaranteed returns, have some merits. Of course, a guarantee should be understood in direct dependency of accuracy, and the completeness of the statistical description of the investment conditions, in the form of initial information, available at the moment of making a decision.

Examine a particular case with a number of stock types M, which is equal to two. The task is to find a vector $u = (u_1, u_2)$, maximizing

$$Z(u) = \min_{1 \leq j \leq N} \{a_{1j} \cdot u_1 + a_{2j} \cdot u_2\} \quad \text{at } u_1 + u_2 = 1;\ u_1, u_2 \geq 0.$$

After the replacement of $u_1 = 1 - u_2$, we will get

$$Z(u) = \min_j \{(a_{2j} - a_{1j}) u_2 + a_{1j}\}.$$

Thus, $Z(u)$ is the minimum N of the linear functions of a variable u_2. It is easy to plot the graphs of these functions, by ensuring that they pass through points $(0, a_{1j})$ and $(1, a_{2j})$, and then maximizing their minimum $Z(u)$, using a graph-based method.

Typically, a linear model should be solved as

$$Z \to \max, \qquad \sum_i a_{ij} u_i \geq Z, \quad j = 1, \ldots, N,$$

$$\sum_i u_i = 1,\ u_i \geq 0,\ i = 1, \ldots, M. \tag{10}$$

In essence, the described model represents one of the variants of technical analysis, using the calculation and analytical procedures, based on the statistical processing of time series.

Conclusion

The variant offered to solve a model of portfolio selection is based on the use of absolute deviations and the modulus function. This allows reductions in the re-evaluation of a contribution of large (marginal) deviations of the market conditions from the mean value, as well as for a reduction in time consumption to prepare the initial information, and to simplify the procedure for solving and analysing a model (e.g. a dual estimation problem of linear programming can be used, including finding a compromise variant).

This article contains certain principles of compromise, offered by the authors, between the criterial values of an expected return and risk (i.e. axioms of fair selection). Further, the modified 'min–max' approach to portfolio selection has been described.

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