

**Entropy for Smart Kids
and
their Curious Parents**



Arieh Ben-Naim

Entropy for Smart Kids and Their Curious Parents. Arieh Ben-Naim. Cambridge Scholars Publishing, Lady Stephenson Library, Newcastle upon Tyne, NE6 2PA, UK. 2019. xvi + 336 pages. Price: £64.99 (hardback)/£20 (paperback).

Entropy is a fascinating concept! It usually goes hand in hand with one of the most widely applied scientific laws, the so-called Second Law of Thermodynamics, which is about spontaneous processes occurring in nature. For many, entropy is an enigma; they have heard about it playing a role in every change that takes place, yet don't have a conceptual understanding of it. Entropy is often equated to disorder and sometimes to a mysterious force that causes disorder. These notions, although popular, are not really accurate. So, if you are wondering, 'What is entropy, if not disorder?', the book, *Entropy for Smart Kids and their Curious Parents* by Arieh Ben-Naim is just right for you.

This book, written for a general audience, gives a scientifically rigorous introduction to the concept of entropy, following an approach which is not very conventional (more on that below). It also gives an intuitive picture of the origin of the second law of thermodynamics. Although the title and front cover may suggest that the book is for small children, it will be, in our opinion, accessible to kids who are at least in high school. At the same time, the book is not *just* for kids and it can be of great interest to grownups. As the author clarifies in the preface of the book ('what it [the book's title] actually means is that even Smart Kids can understand the concept of entropy'), the cognitive ability or the academic preparation of an adult is not

necessary to understand entropy. Essential to understanding the contents of the book is interest and serious engagement. The book has a friendly large font and double spacing, which makes it inviting to read. It however has several typographical and editing errors especially in the early part of the book (details below), which can be confusing for the reader who has no prior knowledge of the subject.

The goal of the book is to demystify the concept of entropy. It explains how entropy of a system is characterized by the distribution of the positions and the distribution of the momenta of its particles. Entropy is only defined for a system when it is in thermodynamic equilibrium (that is, when the macroscopic or bulk properties of the system like temperature, pressure, mass and density are not changing with time). Central to the book, is the discussion of a quantity characterizing these distributions of positions and momenta called the Shannon's measure of information (SMI). It is shown that entropy is just a multiplicative factor times a special case of SMI. While this is not the traditional way to define entropy, the author argues with reasons that 'this is the simplest and the only valid and proven interpretation of entropy'.

Let us consider some background about entropy, which will be useful for discussing the book further. Specifically, we will discuss briefly the history of entropy and how entropy is taught in undergraduate physics and chemistry courses. This can then be contrasted with the approach taken in the book.

The word entropy was coined by German physicist Rudolf Clausius in 1865 to describe a newly discovered property of a system. Clausius formulated a procedure to calculate the change in entropy: $dS = dQ/T$, where dS is an infinitesimal change in entropy which happens when dQ , an infinitesimal quantity of heat, flows into the system at the absolute temperature T . Note that Clausius did not provide a way to calculate the absolute entropy of a system, but only changes in it. Around 1877, Austrian physicist Ludwig Boltzmann gave a definition of absolute entropy, which was based on the total number of microstates (that is the number of different positions and momenta of the particles) of a system having certain well-defined macroscopic or bulk properties, namely number of particles N , volume V and energy E . Boltzmann's formula for entropy is

$S = k_B \ln W$, where k_B is a constant and W is the total number of microstates. The above two pictures of entropy are taught in undergraduate physics and chemistry courses. Clausius' picture is taught in a course on Classical Thermodynamics and Boltzmann's picture is taught in a course on Statistical Thermodynamics.

The book takes a different approach to define and explain entropy. It defines entropy as a special case of SMI. It makes a case that this definition is superior to both the Clausius and Boltzmann definitions and provides a clear, simple and intuitive interpretation of entropy. The definition is statistical like the Boltzmann definition, but the author argues that it is more general. The book is organized in three chapters. Each chapter builds on the previous one. The first chapter discusses the concept of probability. The second chapter discusses the concept of SMI. The third chapter defines entropy as a special case of SMI and further, it formulates the Second Law of Thermodynamics in terms of probability. Finally, there is an epilogue titled, 'Misinterpretations and over interpretations of entropy'.

The first chapter begins with a simple game that the reader is invited to participate in. It involves throwing 100 times, a dice whose six faces are coloured either red or blue. The reader is asked to examine the dice, that is to see how many faces are red and how many blue, and then choose a colour. On throwing, if upper face is of the chosen colour, the reader gets \$1 and if not has to pay \$1. The goal is to maximize one's earnings. The choice of the winning colour is usually obvious except when the number of red and blue faces are equal. This game is used to demonstrate that everyone has a probability sense or an intuitive feeling for probability. It also introduces the concept of average uncertainty or missing information about outcomes of an experiment. When thrown, the dice with equal red and blue faces has the most uncertainty in the outcome, while the dice with 6 faces of the same colour has zero uncertainty. Several such simple examples of probabilistic events or experiments are discussed throughout the book to concretize new ideas as they are introduced. The first chapter goes on to discuss various concepts in probability like conditional probability, children's perception of probability and some common probability distributions like the

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uniform distribution and the normal distribution.

The second chapter is about a property of probability distributions called SMI. For context, some familiar examples of properties of distributions are average and variance. The concept of SMI was developed by American engineer Claude Shannon in 1948 to quantify the amount of information in a message that a sender desires to transmit or communicate to a receiver. It is important to note that SMI is a measure of certain types of information, that is, not every type of information has an associated SMI. The book uses several examples to familiarize the reader with the idea of SMI. It uses the '20 questions' game (which it introduces to those not familiar with it) to give an idea of the amount of information associated with a probability distribution. It provides several interpretations of SMI. One of these is the average uncertainty associated with the distribution. Another is the amount of information that one needs, so as to be certain of the outcome of an experiment, where the outcome follows the specific probability distribution.

The third chapter shows that entropy is a special case of SMI. It concretizes the connection between SMI and entropy using the example of an ideal gas. The SMI is associated with the distributions of locations and momenta of the particles of this gas. The gas has an SMI at all times, even when it is not in thermodynamic equilibrium. However, when the gas is in equilibrium, the distributions of the locations and momenta are such that it maximizes the corresponding SMI. This maximum SMI turns out to be identical (up to a multiplicative constant) to Boltzmann's entropy of the gas. Quoting the book: 'This is the most amazing result. Starting with a quantity defined by Shannon in *communication theory*, a quantity which has nothing to do with physics, we got the *entropy* of a system defined in thermodynamics'.

The third chapter also asserts that it is not necessary to cast the second law of thermodynamics in terms of entropy. While emphasizing that the entropy formulation of the second law is only applicable to isolated systems, it formulates the second law in terms of probability, which is applicable to *any* well-defined thermodynamic system. When a constraint on a system in equilibrium is removed, it shows how the probability of the position and momentum distribution

of the final equilibrium state is overwhelmingly greater than that of the initial state, and that is the reason why the system evolves to the new equilibrium state. In other words, events which are more probable will occur, which is just common sense.

In our opinion, the significance of the book is that it connects entropy to SMI, a concept in information theory, and this provides important insights about the meaning of entropy. For a reader familiar with the statistical formulation of entropy based on the Boltzmann formulation, this book can solidify the understanding of the concept of entropy. It can help make connections and clarify misunderstandings and we would highly recommend this book for such a reader. The author is clearly a master of the subject and there is a lot to learn about the subtleties surrounding entropy and the second law from this book. However, for a completely uninitiated reader, this book can be challenging because of two quite different reasons. The first has to do with the approach itself and the second with the presentation of the approach in this book. We will discuss these briefly.

The approach based on SMI involves first understanding new concepts about information (which are quite different from our regular associations with information) and how certain very specific type information is measured. This is intimately intertwined with understanding probability distributions. One has to understand what SMI tells us about a probability distribution, and the relation between SMI and super probability, i.e. probability of finding a specific probability distribution. If these ideas are understood, entropy follows easily. In contrast, the traditional Boltzmann approach requires ideas of probability, but relatively preliminary ones. However the Boltzmann definition is applicable to only isolated systems and to extend that to all types of systems, one requires to understand a new concept, that of 'ensembles'. The approach of the book and the traditional approach to understand entropy, in our opinion, are of a similar level of conceptual challenge and one approach is not clearly simpler than the other, unlike what the book claims.

The ideas in the book are very precise and scientifically accurate, but the presentation is such that, it is likely to be difficult to read for a person who has no previous knowledge of the statistical pic-

ture of entropy or of information theory. The book spends a lot of time on relatively simple ideas (e.g. playing a game with dice with different number of coloured faces) but moves quickly and makes assumptions while going through more complex concepts (e.g., quantifying average uncertainty, the meaning of asking less than one question on average to know the outcome corresponding to SMI <1). Moreover, there are several typographical and editorial errors in the book. For example, the fifth page explains the rules of a game, based on which a discussion follows for the next 10 pages. There is a significant error in the key sentence explaining the rules and this makes the entire discussion confusing. The narrative also relies heavily on 'Notes' or extended explanations of ideas, but it is not mentioned (either in the preface or the first time a Note appears) how these Notes are referenced. It takes a while to figure out that the tiny superscript numbers appearing next to words in the text are actually references to Notes which are at the end of the book. Notes are sometimes incorrectly referenced. The first chapter also meanders into ideas which have nothing to do with the primary narrative and are an unnecessary distraction for the reader. For example, the sections on the axiomatic approach to probability (Section 1.4, 7 pages) and probability sense of animals (Section 1.10, 9 pages) are of this type.

Notwithstanding such shortcomings, the book is an extremely valuable and welcome contribution to the popular literature on the concept of entropy. Without any compromise on scientific rigor, it makes the connection between information and entropy for a non-expert, and in that sense we think it may be one of the first books of its kind. It demonstrates that entropy is not the same as disorder and addresses misuses and exaggerations of the 'power' of entropy that have been propagated by writers on entropy. The book provides plenty of food for thought and will surely leave you stimulated.

ANIRBAN HAZRA*
ARNAB MUKHERJEE*

*Department of Chemistry,
Indian Institute of Science Education
and Research Pune,
Dr Homi Bhabha Road,
Pune 411 008, India
*e-mail: ahazra@iiserpune.ac.in;
arnab.mukherjee@iiserpune.ac.in*