

2023 Abel Prize

The Norwegian Academy of Science and Letters in Oslo, Norway, has awarded the Abel Prize for 2023 to Luis Ángel Caffarelli, USA. He currently holds the Sid Richardson Chair at the University of Texas at Austin, Austin, USA. The prize has a cash award of 7.5 million Norwegian kroner (USD 716,000). The citation is as follows: ‘for his seminal contributions to regularity theory for nonlinear partial differential equations including free-boundary problems and the Monge–Ampère equation.’ The award ceremony took place on 23 May in Oslo, the capital of Norway, under the auspices of His Majesty King Harald.

Caffarelli was born in Buenos Aires, Argentina in 1948. He received his Ph.D. in mathematics from Universidad de Buenos Aires in 1972 under the guidance of Calixto Calderón. In 1973, he joined the University of Minnesota, in the Twin Cities of Minneapolis and Saint Paul, Minnesota, USA on a postdoctoral fellowship and by 1979, he became a professor there. He served as a professor at the University of Chicago, Chicago (1983–1986), the Courant Institute of Mathematical Sciences, New York University, New York City (1980–1982 and 1994–1997) and the Institute for Advanced Study, Princeton, New Jersey (1986–1996). Since 1997, Caffarelli has been a professor in the Department of Mathematics and the Institute for Computational Engineering and Science at the University of Texas at Austin, Austin holding the Sid Richardson Chair.

Caffarelli’s work encompasses partial differential equations. The study of differential equations dates back to Sir Isaac Newton (1642–1727) and Gottfried Wilhelm Leibniz (1646–1716) and is a ubiquitous tool in describing not only the physical world but even in diverse areas of pure mathematics. It is impossible to find an area of real life that does not involve differential equations. Very few differential equations can be solved exactly – that is, one can find an exact formula for their solutions – but this is very much the exception rather than the rule. Instead, one has to be content with being able to show that solutions exist, and with being able to say something about how they behave. The field of differential equations is vast, which is evident by the fact that there are over a dozen journals contain-

ing the words ‘differential equations’ in their titles. Nevertheless, fundamental problems regarding the existence, uniqueness, regularity, and stability of solutions to several of the key equations remain unanswered despite significant efforts made by various mathematicians over the centuries. This vast subject of partial differential equations heavily relies crucially on establishing deviously clever estimates of relevant quantities. One of its best practitioners is Caffarelli. According to Helge Holden, Chair of the International Abel Prize Committee, ‘Caffarelli’s theorems have radically changed our understanding of classes of nonlinear partial differential equations with wide applications. The results are technically virtuous, covering many different areas of mathematics and its applications.’ He has repeatedly made seminal breakthroughs using geometric insights. In particular, he has made important advances to regularity theory for more than four decades. In numerical calculations, the regularity (or smoothness) of solutions is crucial, and the lack of regularity is a gauge of how erratically nature can behave. His 1977 paper on the regularity of free boundaries in higher dimensions demonstrated his extraordinary talent and intuition¹.

Caffarelli revolutionized the study of fully nonlinear elliptic partial differential equations, which are partial differential equations with nonlinear terms. Such equations arise for instance in the study of gravitation and fluid dynamics. Such equations are challenging to examine since there are essentially no universal methods that apply to all such equations, and often each equation has to be looked at as a unique case. He proved several regularity results for fully nonlinear elliptic equations. One of these results concerns the Monge–Ampère equation arising in differential geometry². Another fundamental contribution by Caffarelli is his joint work with Robert V. Kohn and Louis Nirenberg (2015 Abel Prize Laureate) on the partial regularity of solutions of the incompressible Navier–Stokes equation arising in fluid dynamics^{3,4}.

Caffarelli is a recipient of many awards including Bôcher Memorial Prize by the American Mathematical Society (1984), Rolf Schock Prize (2005), Leroy P. Steele

Prize by the American Mathematical Society (2009), Wolf Prize (2012) and Shaw Prize (2018). The two books co-authored by Caffarelli are widely used^{5,6}.

The Abel Prize was instituted in 2001 in honour of the Norwegian mathematician Niels Henrik Abel (1802–1829). Along with the Fields Medal (first given in 1936), the Abel Prize is considered to be the highest award in mathematics. The inaugural prize was given to Jean-Pierre Serre in 2003. Among the twenty-six recipients, Karen Keskulla Uhlenbeck is the only woman laureate, who received the honour unshared in 2019. Sathamangalam Ranga Iyengar Srinivasa Varadhan is the only recipient of Indian (and Asian) origin to have won the prize (unshared in 2007)⁷. Six of the twenty-six Abel Laureates are Fields medalists (Jean-Pierre Serre, Sir Michael Francis Atiyah, John Griggs Thompson, John Willard Milnor, Pierre Deligne and Grigory Aleksandrovich Margulis). The deadline for nomination is the first of September each year (<https://www.abelprize.no/>).

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