

Measuring ecosystem patterns and processes through fractals

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Changes in ecosystems are highly complex, heterogeneous and are extremely difficult to measure through single scale. Fractal geometry has been used to quantitatively estimate the extent of irregularity in ecosystem changes. However, in some cases it has been overly used giving misleading results. To avoid this, other metrics are also being used in studying changes in forest ecosystems. In this article, we review use of fractal geometry in measuring ecosystem components in a range of ecological conditions. Further, case studies from forest fragmentation and soil aggregates stability in different Indian tropical ecosystems with respect to management practices and environmental change have been described using fractal dimension. We have tried to point out some instances where fractals can more appropriately be used in assessing ecosystems properties and where it could not be successfully used. Characterization of ecological situations where fractals can effectively be used in general remains an important issue.

Keywords: Ecosystem complexity, ecosystem patterns, fractal dimension, landscape change, soil processes.

ENVIRONMENTAL heterogeneity is essential for co-existence of species by providing structurally complex habitats with varying micro-habitats and niches^{1,2}. Positive relationship between habitat complexity and species diversity at different scales³⁻⁶ are among the important factors in structuring diverse communities in a variety of ecosystems^{7,8}. Measuring complex ecosystem structure is often difficult using ordinary mathematical formulations.

Fractal dimension (D) has been used for various applications⁹ in characterizing continuous spatial or temporal patterns on the Earth's surface¹⁰⁻¹⁶. In contrast to Euclidian dimensions, D is a non-integer dimension and is a relative measure of complexity of a shape or a process which increases with the complexity of the structure. For instance, the estimated value of D of a lakeshore neither tells us about its actual size or shape, nor helps us in drawing a map of the lake. But, it tells us about the relative complexity of the lakeshore and is an important descriptor

when used in conjunction with other measures. Further, it reflects quantitative measure of the irregular features of a phenomenon or roughness of an object. Variability is common in natural ecosystems, and thus sometimes it is approximated by a stochastic fractal like the model of Brownian motion¹³. Fractal theory has been used to measure different ecological patterns and processes¹⁰⁻¹² more successfully than by the traditional Euclidian geometry.

In this article, we give an overview and application of D in measuring ecosystem patterns and processes. Further, we discuss the use of D and other metrics to explain forest fragmentation and soil aggregation using field data from sites in India.

The notion of D and its measurement

The notion of fractal is based on the principle of scaling laws, where measurement of an irregular object is made by lowering scales to accurately measure the complexity of the object. For example, if we scale down the sides of a square by a factor $r = 1/2$ then 4 small squares would be required to fill up the original square. In other words, the size of the object will be reduced by a factor of $(1/2)^2 = 0.25$. This means that if we scale down by a factor r we need $(1/r)^2$ reduced squares to fill the original square. Likewise, if we do the same with a scale factor r in the case of a cube, then the number of cubes needed to fill the original box is $(1/r)^3$. This shows that the power $1/3$ is directly related to the geometric dimension of the original figure. If r is the scale factor and $m(r)$ shows the number of scaled down self-similar pieces required to fill the original figure, then for both square and cube $m = (1/r)^D$, where D is the dimension (2 for a square and 3 for a cube). This is a reasonable definition of the dimension D of a self-similar fractal which was discussed in detail by Murray¹⁷.

The D of non-self similar objects and box counting dimension

If an object is not self-similar, then a generalized concept like the box counting method is widely used. It involves

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covering an object of measurement with regular square boxes of size r . For example, squares are used for measurements in the plane and cubes for measurements in three-dimensional space. Suppose the original smooth curve has length L then the number of boxes $N(r)$ is dependent on the size of the box, which is dependent on the unit of the scale r . So in this case $N(r)$ is proportional to L/r . In the case of an area A , the number of boxes $N(r)$ is proportional to A/r^2 . Expressing as a power law $N(r)$ is proportional to L/r^D . The fractal box dimension may be defined as

$$D = \text{Limit}_{r \rightarrow 0} \frac{\ln N(r)}{\ln(1/r)} \quad (1)$$

Application of D in ecology

Fractal scaling has been found to be well applicable to measure the natural complexity at every level of ecological organization (from gene to ecosystem). Ecological objects are non-Euclidean, as they exhibit fragmentation or discontinuity in space and time. It has been observed that the fractal geometry is far better than Euclidean geometry in dealing with ecological objects^{11,18}. Most of the ecological objects are scale-dependent and the fractals have profound implications in such cases¹⁰.

In a situation where the goal is to measure the complex surface of the tree bark to model habitat availability for small epiphytes on the tree trunks, fractal geometry is more useful^{15,19}. For a bark with $D = 1.4$, an insect smaller than another perceives a length increase of $10^{D-1} = 10^{0.4} = 2.51$, or a habitat surface area increase of $2.51^2 = 6.31$. In contrast, for a smooth Euclidean surface, $D = 1$ and both the insects travel the same distance. Thus, the higher the value of D , the greater will be the rate of increase in length (or surface area) with decreasing scale.

Surfaces with high D values create an unequal share of available space for animals of different sizes⁸. While on a planar surface, the amount of available space is equal for all animals irrespective of size, an increase in the D value of a surface leads to growing differences in space availability, with a disproportionate bias towards smaller animals²⁰. Habitat complexity has been shown to alter the size-density scaling of species in terrestrial^{18,20,21}, freshwater²² and marine communities^{23–26}. Li²⁷ proposed a theoretical explanation for species-area scaling based on a generalized MacArthur–Wilson model, where the mechanisms of species migration and habitat heterogeneity were interpreted by using D . This model suggests that the observed natural variability in the exponent of species–area relationship is due to the differences in dynamics at the species and habitat levels.

D has been used for measurement, simulation and as a spatial analytic tool in mapping sciences to characterize landscape complexity of natural vegetation^{28,29}. Fractal analysis is frequently used to characterize the data

recorded from remote sensing images³⁰. Changes in the value of D in remote sensing images have implications for changes in the environmental conditions³¹. The value of D of landscape changes according to the type of the land use^{32,33} (e.g. forest exhibits high value of D due to complex shape and agro-ecosystem shows low value of D due to its more regular shape). Consequently, the value of D is negatively correlated with the human disturbances of the landscape^{28,34}.

D is positively correlated to the complexity of a structure. A rough or jagged surface possesses value of D greater than 2 (which is the topological dimension). Theoretically, a very complex surface may become more like a volume, although it is difficult³⁵ to find a natural surface with $D > 2.5$. Fractals have previously been used to describe habitat complexity in marine systems such as coral reefs³⁶, wharf pilings³⁷, marine algae^{23,24} and rocky shores^{5,26,38,39}.

Measurement of D in ecology

There are a number of methods that can be used to determine D ^{15,31} in ecosystem studies and the details about the formal mathematical derivations and proofs have been discussed^{13,40–43}. Empirical methods for estimating D values have also been provided^{11–15,18,19,44–51}. Most of these extensive reviews discussed about patterns and processes in ecological sciences. The variety of approaches for determining D values reflects the differences in objectives and the type of data analysed.

It has to be kept in mind that all ecological objects and processes may not be fractal. Because of highly productive infusion of ideas of fractal geometry in ecology, it has now become a natural tendency to use fractals everywhere, even in situations where the evidences are not so strong¹¹. Some common approaches to estimate D in the ecological objects are described below.

Dividers (compass) method

This method is used to measure D value of a curve (e.g. coastline, landscape edge, etc.). This procedure is analogous to moving a set of dividers of fixed length δ along the curve. The estimated length of the coastline is the product of N (number of times the ruler is used to cover the object) with the scale factor δ . The power-law relationship between the measuring scale δ and the length $L = N\delta$ is

$$L = K\delta^{1-D} \quad (2)$$

D is estimated by measuring the length L of the curve at various scales (δ). This method is not well founded theoretically, as it is used in exact statistically self-similar curves⁵². The value $L = N\delta$ may vary depending on the starting position along the curve.

Box-counting method

Like the dividers method, this procedure can be used to estimate the value of D of a curve⁵³. It may also be applied to overlapping curves⁵⁴ and structures lacking strict self-similar properties such as vegetation²⁰. Fortunately, other dimensions are available, like the Hausdorff and the similarity dimensions, which provide a better grasp of the space-filling characteristics of fractals⁴¹. Formally, the method finds the δ -cover of the object, i.e. the number of pixels of length δ (or circles of radius δ) required to cover the object⁴⁰. A more practical alternative is to superimpose a regular grid of pixels of length δ on the object and count the number (C) of occupied pixels. This procedure is repeated using different values of δ . The defining power-law is as given in eq. (3) below

$$C = K\delta^{-D}. \quad (3)$$

Since slight re-orientation of the grid can produce a different value of C , grid placements should be randomly replicated to obtain a distribution of D -values⁵⁵. Tatsumi *et al.*⁵⁶ have outlined an analogous method for image-processing systems.

Many researchers have discussed the estimation of D in ecological habitats ($2 \leq D \leq 3$) by box-counting method^{10,19,20,57}. They have argued that for estimating D value of a tree branch, a three-dimensional grid system could be superimposed on the branch by varying the size of the counting-cubes but such a procedure is practically impossible to implement in the field. This is an important technical limitation of the method in its present state. Morse *et al.*²⁰ simplified the problem by obtaining a two-dimensional photographic image of the habitat, the D value of which was determined using the box-counting method ($1 \leq D \leq 2$). Though there are limitations in extrapolation to higher dimensions^{58,59}, many researchers have frequently used the procedure to estimate D of habitats^{60,61}.

Perimeter-based dimension

It is a common method of measuring the D values of images in landscape ecology. It has also been used to characterize landscape complexity^{29,33}. The perimeter–area relationship for a set of patches is given by

$$P = kA^{D/2}, \quad (4)$$

where A is the number of pixels making up a given object, the perimeter P is a count of the number of pixel edges, and k is a constant of proportionality. The slope of the log–log area–perimeter plot for a set of objects gives a mean fractal dimension⁶². D may then be derived from eq. (4) as

$$D = 2 \times (\ln P - \ln k) / \ln(A). \quad (5)$$

In this case, the value of D varies between 1 (in landscapes having simplest shapes) and 2 (in landscapes with most complex shapes)³². In fact, this method determines the relative ‘edginess’ of an image. For a single landscape, the perimeter dimension reduces to $D = 2 \ln(P)/\ln(A)$.

Ecologists have been using this perimeter–area relationship with map and image data to characterize complexity of landscape patterns^{28,34}. For landscapes, D has most commonly been calculated by regression methods^{28,32,63}. By this method the natural logarithm of perimeter is regressed against the natural logarithm of area for all patches in the landscape; $\ln(k)$ is the y -intercept and for squares $k = 4$. Thus, D value is estimated as two times the slope of the regression since the slope is $D/2$.

Problems in estimation of D based on area–perimeter

The problems associated with the stability of the D values in characterizing shape complexity for remote sensed or raster data are of two types. The first assumes a power-law relationship with some sort of statistical self-similarity of the patches on the landscape. The second and the important one using perimeter area regression creates a number of problems including goodness of fit, spread of data, calculation of the y -intercept and need for an adequate number of patches³⁴.

Estimation of D through regression

Linear regression is the most common method for estimating D values in landscape ecology²⁸. For estimating D using regression, $\ln(P)$ is plotted against $\ln(A)$ (eq.(5)) for all the patches. D is estimated as two times the slope of the fitted line, $\ln(k)$ being the y -intercept. It is assumed that the perimeter and area relates with each other through a power law.

Raster data structure and D

Because of the problems associated with single scale area regression estimation of D , Olsen *et al.*⁶⁴ proposed a method for computation of D directly from the data. Frohn³⁴ imposed a fixed sampling geometry that limits the possible relationship between perimeter and area. Two different equations are used to calculate D . The first equation sets the constant of proportionality $k = 1$ in eq. (4) or treats it as an unknown to calculate D . Further, the values are scaled in a manner that is difficult to interpret. According to Frohn³⁴, D based on area and perimeter values exceeds the legitimate maximum of 2.0 for any object, which has greater perimeter than area. For example, if the perimeter is 12 and the area is 16 then D will be 2.26. If the perimeter and area both are 16 then D will

be 2. In addition, Frohn³⁴ has shown that D for smaller squares is 2.26 and for the larger squares 2.00. According to conventional interpretation of D , the smaller squares with D value 2.26 would be considered complex shaped. As a square is not complex shaped, many algorithms for regression estimations ignore patches smaller than a threshold number of pixels (normally 4 pixels). As the size of an object increases, the number of pixels making up its area increases rapidly while its perimeter increases more linearly for objects that are not true fractals. Hence, larger patches are much less likely to have a greater number of pixel edges than the total number of pixels.

The problem with the scaling of D can easily be eliminated if the relationship between perimeter and area in the first equation is expressed using the constant of proportionality as in eq. (4). The value of k is determined by the relationship between perimeter and area for a given geometric shape. When $k = 4$, D can be thought of as the amount by which the perimeter of a given object deviates from that of a perfect square of equal area. Frohn³⁴ has calculated D for square tessellation using eq. (6)

$$D_p = 2\ln(P/4)/\ln A. \quad (6)$$

The value of D_p in this equation is constrained to a minimum of 1.0 for any perfect square and cannot exceed 2.0, as the maximum perimeter of raster object is $4A$ (in pixel units). The value of k can suitably be chosen for other equilateral tessellations such as hexagons or equilateral triangles. The value of D obtained by using eq. (6) for a raster image is more appropriate.

Case studies from tropical Indian ecosystems

We analyse and discuss in this part how D captures the extent of anthropogenic pressure on the natural vegetation patches in sacred forests of Western Ghats, India.

Case I: Forest fragmentation

The remote sensing images obtained from four sacred groves of Pune district from the Western Ghats region, India are shown in Figure 1. These four sacred groves are located in Bhimashanker sanctuary (19°03.832'N and 73°31.993'E); Temple area, Velka, Nasrapur, Bhor (18°18.113'N and 73°36.363'E); Varvand Devi Rai, Bhor (18°06.08'N and 73°38.906'E) and Virivachi, Kopre village, Outer range, Junnar (19°20.889'N and 73°52.098'E). Apart from these, three more areas from the Western Ghats region of Pune were identified as control for validation.

Approximately 3.5×3.5 sq. km area from the centre of the sacred grove in the above-mentioned regions (150 pixels \times 150 pixels) was selected. A total of 14 subsets of

the images are created. Using the nearest neighbourhood method, the landscape was classified into forest and non-forest regions applying supervised classification using Linear Imaging Self Scanning Sensor (LISS III) image in January 1998 and January 2004. From the classified subset image, the vector layer of forest and non-forest areas was created in ERDAS image processing software. The vector attributes were exported to Excel for calculation of various landscape metrics.

The area and the perimeter of four segmented images of the forest are given in Table 1. A general decline in the values of area (12.8–14.4%) and perimeter (7.4–20.4%) compared with the values of January 1998 was observed in January 2004; but in Nasrapur the perimeter was slightly (2.5%) increased due to substantial decrease in the forest area (14.4%) as a result of excessive disturbance that has created more fragmentation. The number of discrete forest patches is seen to increase in all the segmented regions and the percentage increase in the number of patches ranged over 9.7–14.7. For Bhimashanker and Nasrapur, D increased by 0.55% and 1.75% respectively, reflecting forest fragmentation. However, D decreased in Varvand and Virivachi by 0.42% and 1.21% respectively because of loss of smaller fragments of forest due to overexploitation.

In general, the forest area decreased in all the sites (in January 2004 from that of January 1998) as a result of continuous pressure mainly due to encroachment for agriculture. The perimeter of the forest patches except for Nasrapur is seen to decrease indicating that the smaller patches of forest in all the study areas have disappeared due to anthropogenic pressure. In Nasrapur area, the perimeter increased slightly due to substantial decrease in the forest area as a result of excessive forest disturbances that has created more fragmentation of the forest area. This is also evident from significant increase in PPU (patch-per-unit area) metric (which indicates porosity of the landscape) of Nasrapur. For quantifying landscape clumping, PPU metric is used. It is defined as $PPU = m/(n*\lambda)$, where m is the total number of patches, n the total number of pixels in the study area and λ is the scaling constant = pixel area.

The D values of the disturbed regions appeared to increase in all the areas facing anthropogenic disturbance (like Bhimashanker and Nasrapur). Both perimeter and area are seen to decrease in Bhimashanker region, and thus, the D value is seen to increase due to increase in porosity and convolution of the boundary of the patches. The lowest value of landscape shape index in Bhimashanker indicates increased forest fragmentation. Among all the landscape metrics mentioned, D is most appropriate to represent the complexity of the patch shape when applied to satellite data at two different time points (1998 and 2004). The highest value of D in the Varvand area shows that this area is most stressed by anthropogenic pressure.

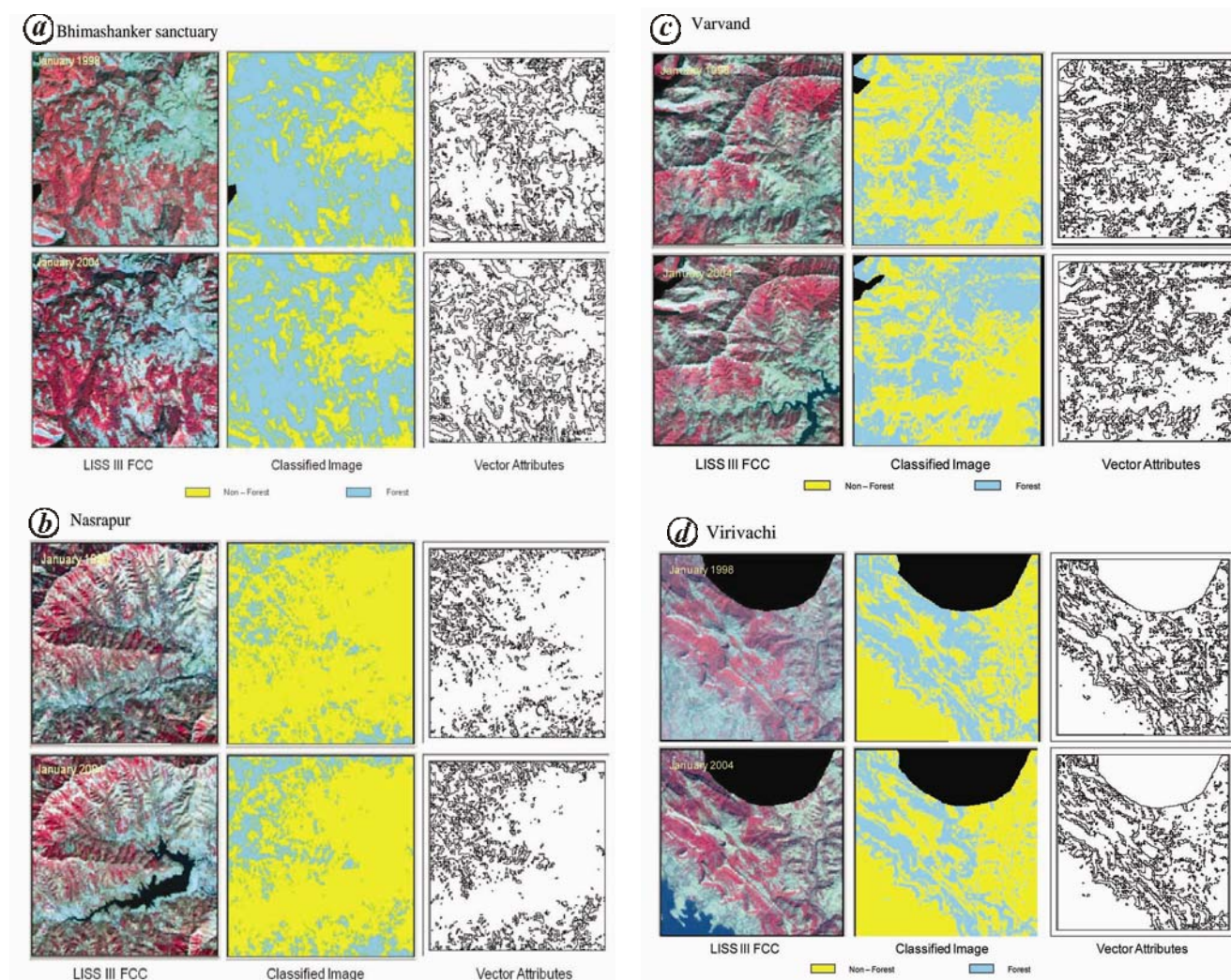


Figure 1 a-d. Use of fractal to identify the level of disturbances at four sites.

Table 1. Changes in the landscape matrices from January 1998 to January 2004 in four groves of Pune district, India

Sites	Parameters		
	Area (m ²)	Perimeter (m)	Fractal dimension (m)
Bhimashanker 1998	53,954	29,824	1.637
Bhimashanker 2004	46,155	27,610	1.646
Nasrapur 1998	53,448	20,748	1.571
Nasrapur 2004	45,770	21,272	1.599
Varivachi 1998	44,872	26,230	1.641
Varivachi 2004	38,567	20,878	1.621
Varvand 1998	52,979	34,482	1.666
Varvand 2004	46,173	29,540	1.659

Case II: Soil aggregation

Soil aggregates are group of primary soil particles, particularly clay that tends to coalesce with each other more strongly than to other surrounding soil particles⁶⁵. Stability of soil aggregates has been found to vary due to different agricultural management practices^{66,67} and also

after the conversion of natural ecosystems to managed ecosystems⁶⁸. Nitrogen (N) and phosphorus (P) additions in dry tropical forest and savanna ecosystems have been reported to affect many ecosystem properties including soil aggregate structure⁶⁹. Scaling of soil and hydrological properties and processes is a burgeoning field responding to increasing need for environmental modelling and prediction⁷⁰. Many workers have reported stability of soil aggregates in terms of mean weight diameter and the percentage by weight of different aggregates greater than some specified sieve-size. So, an index for describing the entire aggregate distribution with a single parameter would be of great importance^{67,71} in assessing the changes in soil fertility due to management practices.

Description of D on measuring soil aggregates stability

Fractals have been used to quantify stability of soil aggregate in various ecosystems. The linear and nonlinear

Table 2. Changes in nonlinear fractal dimension (based on number in water stable soil aggregates), mean weight diameter and geometric mean diameter of soil aggregates in Indian dry tropical forest and derived ecosystems with respect to N and P additions (denoted as N-added and P-added), and agro-ecosystem under different tillage (conventional, CT; minimum, MT and zero, ZT) and residue management (removed, -R; retained, +R) conditions

Ecosystem/ treatment	Mean weight diameter	Geometric mean diameter	Non-linear fractal dimension based on number
Forest			
Control	1.96	0.54	3.24
N-added	2.26	0.75	3.08
P-added	2.10	0.59	3.14
Ecotone			
Control	1.52	0.43	3.29
N-added	1.79	0.57	3.24
P-added	1.75	0.48	3.23
Savanna			
Control	1.57	0.42	2.93
N-added	1.14	0.31	3.30
P-added	1.49	0.39	2.98
Agro-ecosystem			
CT-R	1.40	0.35	3.16
CT+R	2.26	0.56	2.74
MT-R	1.77	0.44	3.10
MT+R	2.78	0.70	2.63
ZT-R	2.08	0.51	2.78
ZT+R	2.39	0.67	2.81

Source: Tripathi *et al.*¹².

D values in soils of different ecosystems show (Table 2) that the value of nonlinear D varies significantly with respect to nutrient addition and management practices¹². D exhibits the changes in stability of soil aggregate more accurately than the other measures of aggregate stability like mean weight diameter. Fractals are not suitable for characterizing a landscape. Other metrics have been developed to deal with this. A few of such metrics are described below.

Probability–density function method

This method was originally developed to analyse point pattern data⁴⁰, but has been frequently used to estimate the D value of a raster image⁷². Discrete habitat islands are not necessary for this method as in the perimeter–area methods. The probability–density function ρ_L is obtained from square ($L \times L$) sampling ‘windows’ successively placed over each ‘on’ pixel. Within each window, a count is made of the number (n) of pixels on. Count frequencies are then expressed as probabilities

$$\sum_{n=1}^{N(L)} \rho_L = 1, \quad (7)$$

where $N(L) \leq L^2$. For a given value of L , the first moment of the probability distribution is given by eq. (8) below and is termed as the mass dimension

$$M(L) = \sum_{n=1}^{N(L)} n \rho_L = 1. \quad (8)$$

This computation is repeated for various values of L . Because each window is centred on a single pixel, L must be an odd number. Voss⁴⁰ showed that the following power law (eq. (9)) holds for fractal images

$$M(L) = kL^D. \quad (9)$$

Value of D can be estimated⁶³ from the log–log plot of the first moment as a function of L .

Milne⁷² compared three artificial landscapes (each half-covered with ‘filled’ pixels) to determine the utility of this method. This method can also be used to estimate the dimension of fractal surfaces, though it gives poor estimates for surfaces with $D > 2.5$. To overcome this problem, Keller *et al.*⁷³ suggested a linear interpolation correction. Under assumptions of isotropy, a toroidal edge correction can also be used to circumvent this problem.

Distribution of areas

Hastings *et al.*⁷⁴ and Mandelbrot¹³ suggested a relationship between persistence (H , that is a parameter of the fractional Brownian motion model, Hurst⁷⁵) and landscape fragmentation (D , that is fractal dimension of patches as determined from the hyper-geometric distribution, Peters⁷⁶). This is important for capturing temporal pattern of ecological processes and it may correlate well with landscape fragmentation. It is useful to predict the changes in the ecological processes in relation to landscape fragmentation as a result of anthropogenic activities. The exact relationship between D and H depends on the model chosen. Sugihara and May¹⁰ stated that increased persistence (more memory in the process) should correspond to smoother boundaries and patches with larger and more uniform areas, whereas reduced persistence corresponds to more complex and highly fragmented landscapes dominated by many small areas. Under certain limiting assumptions¹⁰, the relationship between H and D is $H + D = 2$. This implies that landscapes with many small islands show greater boundary complexity (high D) and are less persistent (low H).

Sugihara and May¹⁰ showed three conditions for the degree of relationship between H and D as: (i) If $H > 0.5$ and $D < 1.5$, then the correlation will be positive and the nature of the process will be persistent, (ii) If $H = 0.5$ and $D = 1.5$, then the correlation will be zero and the nature of the process will be Brownian, (iii) If $H < 0.5$ and $D > 1.5$, then the correlation will be negative and the nature

of the process will be anti-persistent. Hastings *et al.*⁷⁴ used this method to compare cypress (early successional) and broadleaf evergreen (late successional) patches in Okefenokee Swamp. They found that cypress patches had a higher fractal dimension ($D = 1.25$, $H = 0.75$) than broadleaf evergreen patches ($D = 1.0$, $H = 1.0$), implying that the earlier successional vegetation shows greater patchiness and decreased persistence (see also Hastings and Sugihara⁴³). A later study pointed out a number of methodological problems associated with the approach³⁰. While the method may prove useful in remote sensing¹⁰, objective tests are required to determine whether persistence–patchiness relationships developed under limiting assumptions are valid for ecological systems⁵¹.

Distribution of volumes

Stability of soil aggregates in terms of mean weight diameter and the percentage by weight of different aggregates does not give a clear picture about the distribution of the aggregates and much information is lost⁷⁷. Baldock and Kay⁷⁸ gave the power law (eq. (10)) to describe the cumulative percentage of aggregates by weight less than a characteristic linear dimension, x (that is, equivalent diameter or height)

$$W_{<x} = Ax^B, \quad (10)$$

where W is the cumulative percentage weight of aggregate, x the characteristic linear dimension, A and B the regression coefficients. Coefficient B is used as an index of aggregate size distribution as it exhibits maximum variation.

Mandelbrot¹³ has characterized fractals by power-law relation between the number and the size of the objects (eq. (11))

$$N_{>x} = Kx^{-D}, \quad (11)$$

where $N_{>x}$ is the cumulative number of objects greater than x , k is a constant equal to $N_{>x}$ at $x = 1$, and D is the fractal dimension. D varies with the shape of individual objects within the distribution and the extent of aggregate fragmentation. D is scale invariant and thus the shape of the aggregate may be similar in various ranges of aggregate sizes. Turcotte⁷⁹ described the hyper-geometric frequency distribution relation (Rosin's Law) for particle size in soils and other geological material (eq. (12))

$$N = kR_i^{-D}, \quad (12)$$

where N is the number of particles whose radius is greater than R_i , and D is the fractal dimension.

Perfect *et al.*⁸⁰ derived a version of Rosin's Law for use with soil mass data. A higher fractal dimension indicates

greater soil fragmentation and a soil increasingly dominated by small particles⁸¹. If $D = 0$, then it indicates that all soil particles have equal diameter. If $D = 3$, then the number of particles greater than a given radius R_i doubles with each corresponding decrease in particle mass. If $0 < D < 3$, then we say that there will be greater proportion of larger particles than when $D = 3$ (that is sand). If $D > 3$, then there will be greater proportion of smaller particles than when $D = 3$ (that is silt, clay).

The value of D was used to characterize the size distribution of aggregates subsequent to fragmentation^{71,80} and to capture the effect of soil properties and cropping pattern on the size distribution of aggregates as a result of fragmentation^{82,83}. According to Tyler and Wheatcraft⁸⁴ and McBratney⁸⁵, the value of D calculated from mass-size distribution data should not be more than 3. However, Perfect *et al.*⁸⁰ showed that the value of $D > 3$ is theoretically possible if the nature of the fragmentation process is multi-fractal (different regions of an object have different fractal properties). According to Rasiah *et al.*⁸⁶, the value of D obtained using the nonlinear fitting procedure was, in general, smaller and more realistic than that calculated through the linear procedure.

Fractal parameters have been reported to be sensitive to tillage treatment⁸⁷. Applications of fractal geometry in soil studies are getting importance in the recent years^{70,88}. Pirmoradian *et al.*⁶⁷ studied the role of D for estimating stability of soil aggregate influenced by tillage treatments. Tripathi *et al.*¹² have calculated the value of D to assess the soil aggregation in Indian dry tropical forest and modified ecosystems in relation to N and P input and agricultural practices. Nonlinear D could successfully capture the changes in soil aggregation. Tyler and Wheatcraft⁸¹ showed that silt-clay soils have D in the range 3.0–3.5 by using a method suggested by Mandelbrot *et al.*⁸⁹.

Conclusion

Ecosystem properties are highly complex and dynamic in nature, and difficult to measure. Appropriate techniques and suitable metrics to track spatial and temporal changes in ecosystem structure and functions are urgently required. Fractal theory provides suitable method to document such changes in the forests created through anthropogenic disturbances. Fractal dimension can represent changes in soil aggregates structures due to land use change, N and P additions and agricultural practices. In this review, we have outlined some instances where fractals can more appropriately be used in ecological systems. Characterization of ecological situations where fractals can effectively be used in general remains an important issue for more elaborate evaluation.

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