The early days of general relativity

Jayant V. Narlikar*

Inter-University Centre for Astronomy and Astrophysics, Ganeshkhind, Pune 411 007, India

This account deals with the progress of Einstein's general relativity (GR) theory first published in 1915. It will discuss the 'bending of light' experiment planned and executed by Eddington in 1919 and then concentrate on the development of GR in India. For, it will be argued that despite the reputation of GR as an obtuse theory it did find fertile soil to grow in India. This account will go as far as the time of India's independence.

Keywords: Bending of light, general relativity, unified field theory.

Introduction

I begin with a stanza in a verse written by a member of the American Mathematical Society. The different stanzas pay tributes to various famous mathematicians. The last one is about Einstein:

To Einstein, hair and violin,
We give our final nod;
Though understood by just two men,
By himself and sometime by God.

This stanza recalls the early phase soon after GR was published as a theory. To physicists it appeared very weird in the sense that it did away with gravity as a force but slipped it back in as a manifestation on non-Euclidean spacetime geometry. To mathematicians, accustomed to thinking of non-Euclidean geometries as notebook exercises with no contact with reality, the GR claimed to produce measurable effects.

Thus the acceptance of the newly created GR required a cautious approach. Ten years earlier, Einstein's special relativity (SR) theory had evoked considerable reaction from physicists as well as laymen. That may have been because the concepts introduced by SR, though revolutionary, could be related to real life experiences and doable experiments. The GR in comparison suffered from difficult concepts like curved spacetime and lack of many experiments to test the theory.

The bending of light

It was against this background that Eddington thought of the bending of light experiment to test the Einstein hypothesis against the Newtonian prediction. The basic idea is simple.

For a Sun-like massive body of mass M, the ray of light coming from a star S, would be slightly bent by the star's gravity so that it enters the viewer's telescope at a slightly different angle than when no such massive body played a role of 'light bender'. The star image would shift by a calculated angle

$$\delta\alpha_E = \left[\frac{GM}{c^2R}\right] \cong 1.7\left(\frac{M}{M_\odot}\right)\text{arcsec.}$$

What does Newtonian theory say? When quizzed on whether massive objects can bend light rays, Newton refused to answer because he had always been reluctant to speculate. *Non fingo hypothesis* (I do not speculate) is what he would say in reply. He listed the question amongst his collection of unsolved queries.

But others following him with specific assumptions got a definitive answer. For example, if we take a light quantum of frequency ν, its energy is $h\nu$ and by SR-type argument its mass is $h\nu/c^2$. Such a quantum subject to Newton's laws of motion will give a bending by the angle

$$\delta\alpha_N \cong 0.5\left(\frac{M}{M_\odot}\right)\text{arcsec.}$$

That is, the Newtonian value is half the Einstein one.

Thus we had here theoretical predictions from GR and (amended!) Newtonian theory with the former twice the latter. Which one, if any will survive a test to measure bending of light?

This provided a potential test provided one could photograph stellar images close to the solar disc. There is only one situation when one can photograph stars in the sky with the Sun around: the occasion of total solar eclipse. Eddington, the mathematician and astronomer was one of the very few people who really understood GR and felt the need for carrying out such a test. The 1919 total solar eclipse provided the necessary background for this experiment.

For details of this experiment and its aftermath see references 1–3. At a joint meeting of the Royal Society and the Royal Astronomical Society on 6 November 1919, Eddington announced the findings of his team and argued that the data favoured Einstein's rather than Newton's value[1].

*e-mail: jvn@iucaa.ernet.in

This experiment has been repeated several times at different eclipses. Although progressively the observational techniques improved, all optical measurements carry a number of error-bars. As radio and microwave technologies became available the experiment could be performed with very small errorbars. Two such experiments are described in refs 2 and 3, using radio and microwaves respectively.

The 1919 experiment though, being a pioneering one, had a tremendous influence on the reception accorded to GR and to Einstein. The experiment showed GR to be a testable scientific theory despite its obtruse formulation. It raised Einstein from the general level of a distinguished scientist to an all-time genius of the rank of Newton.

Looking back, though, one finds several 'holes' in the way the 1919 experiment was conducted and how the data were reduced. For a graphic account of the experiment as reviewed from modern times, see the article by Peter Coles[4].

For the Indian angle it is worthwhile to recall that the Kolkata paper *Statesman* published a detailed account of what the experiment was about. This was written by a young physicist, Meghnad Saha who later became famous for his astrophysical contributions.

It is a measure of the importance the media attached to this experiment that the *Statesman* had a special article on it followed by reader's correspondence. The article by Saha is a good one and shows that writing on science at a popular level was not unheard of, if not very common.

Two schools

In the year 1911, three bright young scientists started their careers in mathematics and physics, more specifically, applied mathematics and theoretical physics. These were the first three on the merit list of the intermediate examination in Bengal (equivalent to Std XII today). The topper was Satyen Nath Bose, who went afterwards for particle physics, second was Meghnad Saha (who wrote the article on bending of light) and the third was Nikhil Ranjan Sen. Saha went for astrophysics and Sen opted for applied mathematics.

Bose was destined to play a key role in the developing field of quantum mechanics. His work on quantum statistics resulted in a whole family of particles being named 'bosons'. Saha's work on ionization equation started a major new area of work on stellar atmosphere. Indeed Eddington's classic work on stellar structure could be undertaken because of Saha's equation providing the vital details of the surface conditions for stars.

It was the third boy who opted for applied mathematics; N. R. Sen went to Germany for Ph D and higher research, worked with Von Laue on GR and Cosmogony at Berlin University and returned to India with great enthusiasm for the newly emergent GR. Interested in a wide range of subjects within applied mathematics, Sen had special interest in GR. As research in GR started to grow around him, Kolkata in the late twenties acquired a reputation of hosting the first school on general relativity in India.

A few years later, in 1932 a second school on GR began to develop around Vishnu Vasudeva Narlikar (V.V.N.) at the Banaras Hindu University (BHU).

V.V.N. had been studying in Cambridge. After a brilliant career at the Mathematical Tripos, he continued at Cambridge as an Issac Newton student. This was a research scholarship usually given to the topmost astronomy student passing the final part of the Tripos. By virtue of his research interests V.V.N. had good interaction with famous figures at Cambridge – in particular Eddington, astronomer W. M. Smart and mathematical physicist Joseph Larmor. While a research student V.V.N. distinguished himself by winning the Rayleigh Prize.

His laurels spread and they attracted the attention of Pandit Madan Mohan Malaviya, who had founded the Banaras Hindu University in 1916. Ever on the lookout for highly talented staff and students for this budding seat of learning he took advantage of the Round Table Conference in London to come over and meet V.V.N. At the meeting Malaviya made a promise of giving him a senior post so that he could develop mathematics at the BHU.

In 1932 V.V.N. was due to go to USA for a year's work at the Mt Wilson Observatory of Caltech. Prior to that he had planned to spend 2/3 months in India. While on this long vacation he decided to visit BHU and see how the organization was functioning. Malaviya welcomed him and showed him round. Thereafter he made an offer that V.V.N. could not refuse. He straight away joined as Professor and Head of the Department of Mathematics.

This was shortly to grow into the second thriving school of GR in India.

Sen and the Kolkata School

Being a theoretician with strength in applied mathematics, N. R. Sen (NRS) looked for solutions of Einstein's equations with mathematically significant properties. We briefly describe a few of such solutions.

Static and spherically symmetric solution: Sen obtained a solution which showed a static but spherically symmetric system in its most general form, as seen in ref. 5. This is based on Einstein's solution of a cluster of particles each moving in a circular orbit, with these particles distributed so that each moves in the field of others.

The list prepared by Andrzej Krasinski of important papers in GR ('Golden Oldies') includes Sen's article on stability of cosmological models (see ref. 6).

Another rather curious result by Sen describes a spherical shell of matter in an otherwise empty space. A coordinate transformation of the solution *inside* the shell leads to the de Sitter spacetime!

As an excursion into the area wherein electro-magnetic energy tensor produced by charged particles acts as a source of gravity, Sen obtained the equilibrium condition for a charged particle with definite spherical boundary[7].

Sen found that for any charge distribution in the spherical volume, three-fourths of the total energy of the particle are electrical and one-fourth gravitational, provided the charge distribution is describable by an analytical function.

In the early 1930s, carried over by the general excitement of the 'expanding universe', Sen looked at models not perfectly spherically symmetric. He showed that it is possible to have static universe models in equilibrium *provided* the total mass of such a universe exceeds the mass of the static Einstein universe. This conclusion could be linked with the question, why the universe is expanding.

While Sen guided a number of students in GR to their Ph D degree, there appears to be one loner whose work we describe next.

Who was B. Datt?

While reading the Landau–Lifshitz text book *Classical Theory of Fields* (2nd edn), I came across a reference to the paper by one 'B. Datt'. As a post-doc working on gravitational collapse, I found Datt's approach quite general. In fact, as I discovered, the Landau–Lifshitz text followed the method used by Datt. This work of Datt[8] was published in 1938.

A year later Oppenheimer and Snyder wrote a paper[9] with similar material content so far as the technique of handling the GR equations goes. The Oppenheimer–Snyder paper is generally regarded as the pioneering work on spherical massive objects contracting with increasing inward speed. Datt, however, kept his approach general; thus giving solutions not only of contraction but other motions too. More importantly, he had seen the significance of comoving coordinates in solving such problems.

At this stage one may very well ask, what later work by Datt is found in GR literature. The answer, *prima facie*, is *nil*. Indeed, my enquiries with relativists from Kolkata belonging to Datt's generation drew a blank. It was more recently reported by Somak Raychaudhury that Datt belonged to the famous Presidency College (now elevated to the university status) and was a favourite student of NRS. As to why no later work by him is reported, the answer is tragic: around 1940 he died in the course of a surgery that went wrong.

One cannot help but recall two distinguished GR workers who died at a young age after producing important work: Karl Schwarzschild and Alexander Friedmann. The former took part in World War I and while serving on the German front in Russia, contracted a rare but painful skin disease called *pemphigus* which may have led to his demise in 1916. Friedmann died of typhoid which he caught during a holiday in Crimea.

Unified field theory

Einstein regarded GR as a stepping stone towards a more comprehensive theory that, ideally, would include *all* basic physical interaction under one umbrella. As a starting point he attempted bringing the electromagnetic theory and gravitation together as a 'Unified Field Theory'.

Several scholarly scientists were tempted to join the search for a unified theory. One finds such names as S. N. Bose, Gangopadhyaya, Mahadeo Dutta, etc. from the Kolkata, region with occasional contributions by V.V.N. from the Banaras school. In particular, V.V.N.'s review talk as President of the mathematical section of the Indian Science Congress 1947 gives an updated version of the unified theories then under discussion[10].

However, as is well known, attempts abroad or in India to find a unified field theory did not succeed. Although disappointing in a way, the ideas like Kaluza-Klein theory[11,12] which were the outcome of use of higher dimensions for a unified field theory have found use in modern theoretical cosmology.

V.V.N. and the Banaras School

We now come to the Banaras School started by V.V.N.

In a private communication, V.V.N. has described an incident in Cambridge involving Eddington and himself. Early in the 1930s, V.V.N. solved Einstein's equations with as well as without the λ-term to generate models of the expanding universe. At the time Hubble's observations had indicated an expanding universe. V.V.N. had simplified the problem by assuming the space to be homogeneous and isotropic. When he showed his calculations to Eddington (who was one of his research advisers). Eddington was very impressed with the work and offered to communicate it to an astronomy journal.

However, while the paper was getting ready for submission, Eddington received a letter and a paper by Abbe' Lemaitre. The letter requested Eddington to arrange publication of an English translation of the attached paper in French which was published in 1927. See ref. 13.

When Eddington read the French paper he realized that Lemaitre had already (in 1927) done the same work which V.V.N. had recently reported to him. So he called V.V.N. to explain that the work in question had already been done and published in 1927. Being in French and in a journal not very well known he had missed it. He therefore could not communicate V.V.N.'s paper although he regretted the extra work V.V.N. was put to in writing it. As a post-script, I may add that even Lemaitre's paper had been 'anticipated' by a couple of papers by Alexander Friedmann[14,15].

When V.V.N. settled down in the campus of BHU he carried on his research in GR. Some of the work done by the Banaras School (V.V.N. and his student) is highlighted next.

The GR being a nonlinear theory with complicated set of partial differential equations, there are very few exact solutions known. This area of *exact solutions* therefore interests the mathematicians who like their models to be precise and not approximate.

Some BHU workers did research on the unified field theory of Einstein and Schrödinger[16]. In terms of worldwide perception unified field theories became increasingly isolated as most physicists believed that unification should proceed in another order:

em theory $\rightarrow$ em + weak theory (electroweak theory) $\rightarrow$ electroweak + strong theory (grand unified theory) $\rightarrow$ grand unification + gravity $\rightarrow$ complete unification.

Thus gravity comes in the end rather than in the beginning and also the present approach requires gravity to be quantized – a stage not yet reached even today.

Hightlights of work from BHU

Some of the relevant work from the Banaras school may be described as below:

In 1922 the noted mathematician, T.Y. Thomas had proved that in Riemannian manifold of 4 dimensions only 14 independent curvature invariants can be constructed. But the explicit construction of these 14 invariants using the curvature tensor and the Weyl tensor was first given by Narlikar and Karmarkar[17]. However, this work was published in the *Proceedings of the Indian Academy of Sciences*, a journal which did not have much circulation outside India. Unaware of this work therefore, several years later Geheniau and Debever in 1956 did the same work for which they were given credit[18]. This was noticed by A. R. Prasanna, a student of Narlikar at Pune who pointed out to Gehenau this fact when they met in 1972, at the Dirac Symposium at Trieste. Gehenau readily agreed that these invariants should be called 'Narlikar–Karmarkar invariants'.

These invariants are important in deciding if a spacetime manifold has singularities. The question of singularities became relevant to reality by the discovery of collapsed massive objects in the form of quasars in 1963. Will the spacetime in the neighbourhood of such massive objects develop a singularity? If so how to spot it in a coordinate–invariant fashion? This is where the curvature invariants become important.

Work of a more mathematical nature came out of the studies of Narlikar and his students Ramji Tiwari and Kamala Prasad Singh. Tiwari was concerned with the unified field theory proposed by Einstein in the late 1940s and examined in detail the interaction between gravitation and electromagnetism[19]. Singh on the other hand worked on metric invariants. His work on the Christoffel symbols is of interest in bringing out the role of coordinate transformations that lead to indeterminateness[20].

General relativity has the unique feature that it contains the equations of motion of the gravitational sources and the method of deriving them was indicated by Einstein, Infeld and Hoffmann[21]. Narlikar's student, B. R. Rao worked on the details of this problem and pointed out some corrections to the EIH work. This was recognized by Infeld and Hoffmann. The Narlikar–Rao paper[22] appeared in print in the year following Einstein's death.

The Vaidya solution

P. C. Vaidya (P.C.V.) started his research career as a student of V.V.N. Himself a postgraduate of Bombay University, Vaidya enrolled himself as an external research student of Narlikar in BHU in 1942–43. Essentially living on his savings he made them stretch out to last for this period (during which he also had to support a family of wife and child). Yet during those two years P.C.V. was able to produce work that was to prove to be of very special interest to relativistic astrophysics about 25 years later.

Basically the 'Vaidya solution' is a generalization of the classical Schwarzschild solution, the main difference between the two being that while the exterior of the gravitating sphere in the Schwarzschild solution is empty, the sphere in the Vaidya solution is radiating. Evidently, the situation described in the Vaidya solution is time-dependent; not static. We summarize this work below: for details of this solution see refs 23, 24.

The 1950 paper quotes V.V.N.[23]: 'If the principle of energy is to hold good, that is, combined energy of the matter and the field is to be conserved, the system must be an isolated system surrounded by flat space-time. A spherical radiating mass would probably be surrounded by a finite and non-static envelope of radiation with radial symmetry. This would be surrounded by a radial field of gravitational energy becoming weaker and weaker as it runs away from the central body until at last the field is flat at infinity. It has to be seen whether and how this view of the distribution of energy is substantiated by the field equations of relativity.' This conjecture was borne out by the Vaidya solution.

To start with, take the four spacetime coordinates as $x^0 = t$, $x^1 = r$, $x^2 = \theta$, $x^3 = \phi$. A star of mass M and radius r_0 is supposed to start radiating at time t_0 and as time goes on, the zone of radiation increases in thickness, its outer surface at time $t = t_1 > t_0$ being given by $r = r_1 > r_0$. For $r_0 \le r \le r_1$ and $t_0 \le t \le t_1$, the line element is given by

$$ds^2 = e^{\nu}\,dt^2 - e^{\lambda}dr^2 - r^2\,(d\theta^2 + \sin^2\theta\,d\phi^2).$$

where both λ and ν are functions of r, t only. The outflowing radiation is described by the energy momentum tensor

$$T^{ik} = \rho v^i v^k,$$

where ρ is the density of radiation and v^i is the null vector representing its flow direction. For radial flow, of course we have $v^2 = v^3 = 0$.

The field equations then give (with $G = 1$, $c = 1$)

$$e^{-\lambda} = 1 - \frac{2m}{r}, \; m = m(r, t),$$

and

$$e^{\nu/2} = -\frac{\dot{m}}{m'}\left(1 - \frac{2m}{r}\right)^{-1/2},$$

where m satisfies the relation

$$m'\left(1 - \frac{2m}{r}\right) = f(m).$$

The dot and dash denote differentiations with respect to t and r respectively.

The function $f(m)$ is so far arbitrary but needs to be specified by the physical conditions that lead to the radiation from the star, whose mass m decreases at a rate determined by the amount of energy radiated by it. The radiation envelope of the star is described by the line element

$$\mathrm{d}s^2 = \frac{\dot{m}^2}{f^2}\left(1 - \frac{2m}{r}\right)\mathrm{d}t^2 - \left(1 - \frac{2m}{r}\right)^{-1}\mathrm{d}r^2$$

$$-r^2(\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\phi^2).$$

The energy conservation relation is described by the condition

$$\frac{\mathrm{d}m}{\mathrm{d}\tau} \equiv v^0\frac{\partial m}{\partial t} + v^1\frac{\partial m}{\partial r} = 0.$$

Of course, this equation is automatically satisfied if all field equations are satisfied.

This formalism has been useful in the context of bright radiating objects in astrophysics, such as quasars, active galactic nuclei, gamma ray bursts, etc.

P.C.V. has recorded that the problem as such was posed by V.V.N. as a research area for P.C.V. Having stated the problem V.V.N. proceeded to solve it. There were three equations of which V.V.N. solved the first one, leaving the remaining two to be solved by P.C.V.

At this stage there was a gap of a few days when V.V.N. was out of BHU on some official work. It was his practice both in teaching and research to pause halfway in his work and leave a time gap for the student to think for himself and proceed on his own if possible. In this particular case when V.V.N. came back P.C.V. was ready with the solution.

P.C.V. also mentions that he had put the names V.V.N. and P.C.V. as joint authors of this work. However, V.V.N. overruled him and stated P.C.V. as the sole author.

Concluding remarks

V.V.N. had occasional correspondence with the astrophysicist S. Chandrasekhar (Chandra) with whom he had overlapped for two years at Cambridge. After Vaidya's work, V.V.N. asked Chandra if there were any areas in astrophysics where GR could be profitably applied. Chandra replied in the negative with his conjecture that gravity would not be strong enough in astrophysical situations to demand GR!

This concludes our account of GR in its early stages starting from its creation to the contributions made by Indian scientists in British-ruled India. This period may be considered (worldwide) as an era of understanding what GR really means. Subsequent to Einstein's death in 1955, events happened which led to a diversification of the menu of problems tackled including gravitational radiation, topological and structural problems of space-time, Mach's principle, cosmological models, etc. And quasars brought in GR specialists to work on relativistic astrophysics, contrary to Chandra's expectations. This era will be covered in another paper.

1. Dyson, F. W., Eddington, A. S. and Davidson, C., A determination of the deflection of light by the Sun's gravitational field, from observations made at the total solar eclipse of 29 May 1919. *Philos. Trans. R. Soc*., 1920, **A220**, 291.
2. Counselman III, C. C., Kant, S. M., Knight, C. A., Shapiro, I. I. and Clark, T. A., Solar gravitational deflection of radio waves measured by very long baseline interferometry. *Phys. Rev. Lett*., 1974, **33**, 1621.
3. Fomalont, E. B. and Sramek, R. A., A confirmation of Einstein's general theory of relativity by measuring the bending of microwave radiation in the gravitational field of the Sun. *Astrophys. J*., 1975, **199**, 749.
4. Coles, P., Einstein, Eddington and the 1919 eclipse. In *The Historical Development of Modern Cosmology* (eds Martinez, V. J., Trimble, V. and Pons-Borderia, M. J.), ASP Conference Series, 2001, vol. 252.
5. Sen, N. R. and Roy, T. C., On a steady gravitational field of a starcluster free from singularities. *Z. für Astrophys*., 1954, **34**, 84.
6. Sen, N. R., On the stability of cosmological models. *Z. Astrophys*., 1934, **9**, 215.
7. Sen, N. R., On the electric particle according to Einstein's field theory. *Z. Phys*., 1927, **40**, 667.
8. Datt, B., On a class of solutions of the gravitation equations of relativity. *Z. Physik*., 1938, **108**, 314.
9. Oppenheimer, J. R. and Snyder, H., On continued gravitational contraction. *Phys. Rev*., 1939, **56**, 455.

10. Narlikar, V. V., From general relativity to a unified field theory. Presidential Address to Mathematics Section at the Lucknow Indian Science Congress, 1953.
11. Kaluza, T., The unification problem in physics. *Sitzungsber. Preuss. Akad. Wiss* (*Berlin*), 1921, p. 966.
12. Klein, O., The atomicity of electricity as a quantum theory law. *Nature*, 1926, **118**, 516.
13. Abbe' Lemaitre, G., A homogeneous universe of constant mass and increasing radius accounting for the radial velocity of extragalactic nebulae. *Ann. Soc. Sci. Bruxelles*, 1927, **XLVII A**, 49.
14. Friedmann, A., Uber die krummung de Reumes. *Z. Phys.*, 1922, **10**, 377.
15. Friedmann, A., Uber die krummung de Reumes. *Z. Phys.*, 1924, **21**, 326.
16. Schrödinger, E., *Space–Time Structure*, Cambridge University Press, 1950.
17. Narlikar, V. V. and Karmarkar, K. R., On the algebraic invariants of the four-dimensional Riemann tensor. *Proc. Indian Acad. Sci.*, 1949, **29**, 91.
18. Geheniau, J. and Debever, R., Invariants of general relativity. *Bull. Acad. Belg.*, 1956, **42**, 114.
19. Narlikar, V. V. and Tiwari, R., *Proc. Nat. Inst. Sc*., 1949, **14**, 73.
20. Narlikar, V. V. and Singh, K. P., *Proc. Nat. Inst. Sc*., 1951, **17**, 311.
21. Einstein, A. Infeld, L. and Hoffmann, B., The gravitational equations and the problem of motion. *Ann. Math*., 1938, **39**, 65.
22. Narlikar, V. V. and Rao, B. R., The problem of motion in general relativity. *Proc. Nat. Inst. Sci.*, 1956, **A21**, 416.
23. Vaidya, P. C., The external field of a radiating star in general relativity. *Curr. Sci.*, 1943, **12**, 183.
24. Vaidya, P. C., The gravitational field of a radiating star. *Proc. Ind. Acad. Sci.*, 1950, **33(**5), 264.

doi: 10.18520/v109/i12/2214-2219