Advances in classical general relativity

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The year 2015 is the centenary of Einstein's creation of general relativity. Over the century general relativity has gradually increased its footprints on mainstream physics and this article highlights advances in the classical aspects of general relativity since its creation.

Keywords: Black holes, centenary, cosmology, general relativity, gravitational waves, ISGRG conference.

Introduction

EINSTEIN arrived at General Relativity (GR), his relativistic classical theory of gravitation on 25 November 1915 (ref. 1). The prime motivation was to have a theory of gravity compatible with special relativity (SR) that agreed with Newton's theory in the appropriate limit. Unlike SR that came in its final form over a year in 1905 and involved only himself, GR went through manifold 'tinkering phases' over eight long years and involved collaboration on mathematical aspects with Marcel Grossman and Michele Besso. As was later shown by Lovelock[2], if one allows only second order equations and a single 4-dimensional ST metric, GR is the unique gravitation theory based on Riemannian geometry. Beyond the new mathematical description was also a profound physical insight. The geometry of spacetime (ST) was no longer a fixed backdrop but a dynamical physical entity determined by the matter-energy content and nonlinear equations. The geodesics of ST determined the paths of light and freely falling particles. The geodesic deviation equation determines the curvature and the tidal forces; the curvature is determined by the matter and motion content of ST. The metric in the non-relativistic limit is related to the gravitational potential. Standard physics based on linearity is not adequate in general and the search for exact solutions using tensor calculus, covariant equations, structures in non-Euclidean geometry and coordinate-free methods, the way forward. Generalizations of GR do exist. They include scalar tensor theories, theories with higher derivatives or torsion, bimetric theory, unimodular theories and theories in higher dimensions[3].

Not only is GR universally acknowledged to being the epitome of mathematical elegance and conceptual depth but importantly for over a century demonstrated remarkable observational success. Being very nonlinear, it has collaterally led to many developments in analytic and numerical techniques. GR is mathematically a complicated nonlinear theory. So it was surprising to Einstein himself that an exact solution could be found so quickly: the Schwarzschild solution[4] that describes the gravitational field exterior to a spherically symmetric body as also the Schwarzschild constant density interior solution. Other interesting solutions included the linearized gravity solutions describing gravitational waves in analogy to electromagnetic waves, solutions corresponding to the Einstein static universe[5], static de Sitter[6], Friedman[7,8] and Lemaître[9] expanding models[10] describing the gravitational field of the whole universe, Vaidya metric[11] representing a spherically symmetric radiating solution and Majumdar–Papapetrou solution representing system of charged black holes in equilibrium under their gravitational and electrostatic forces[12]. For a long time this inspired mathematical research in GR to seek exact solutions of Einstein's equations (EE). The complexity of EE made this an interesting mathematical challenge even if the physical interpretation of some of these solutions was not very obvious. It led to interesting developments in algebraic computing for long computations in tensor calculus typical of GR and later means to classify solutions of EE and recognize equivalent solutions that appeared new due to a different choice of coordinates. The use of symmetry groups to simplify the system of equations, the classification of exact solutions on the basis of symmetries, the search for techniques to generate new solutions from old constituted a major area of research in GR for many years[13]. It naturally led to the use of coordinate free methods, tetrad formalism, use of null tetrads, the Newman Penrose formalism[14] and methods to investigate kinematical properties of null and timelike vector fields describing radiation and matter respectively. In spite of these theoretical developments, in its first half century of its existence, GR was outside of mainstream physics in contrast to the following fifty years with increasing profound applications in astronomy, astrophysics and cosmology. Though classical differential geometry was an adequate starting point for the initial studies, later developments required a careful understanding of the global structure of spacetime, singularities and asymptotics to interpret these solutions[15] and formalisms to disentangle physical effects from coordinate or gauge-dependent ones.

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In 1939 Oppenheimer and Snyder[16] showed that collapse of matter (greater than Chandrasekhar mass limit) due to gravitational self-attraction leads to astrophysical black holes (BH). Yet BH were not considered physical up to seventies due to twin concerns that the above result was based on exact spherical symmetry and used as source dust of uniform density. Insight into distinction between the coordinate and physical singularities led to the correct understanding of the Schwarzschild singularity as a coordinate singularity[17,18], the distinction of notions of infinite red shift surface and one way membrane or event horizon and the Schwarzschild black hole. Penrose pointed out that higher multipoles would be radiated away and hence generic collapse of massive stars also lead to BH[19]. This was confirmed by Price's perturbative calculations later. Using global methods Penrose showed that if appropriate causality conditions hold and matter and fields satisfy suitable energy conditions (satisfied by normal matter but violated by scalar fields leading to inflation) then closed trapped surfaces will form leading to singularities in gravitational collapse. Closed trapped surface occurs because the gravitational field is so strong that outgoing null rays from a two-sphere cannot escape. The discovery of quasars brought about the transformation of BH from just mathematical curiosities to physical entities. Dramatic theoretical developments followed. The discovery of the Kerr solution[20] which was realized to be a generalization of the Schwarzschild solution, its systematic global analysis by Carter[21] and Boyer and Lindquist[22], the critical distinction between the event horizon and infinite redshift surface[23] leading to the notion of the ergosphere, possibility of energy extraction, uniqueness theorems for BH, study of perturbations of BH, their stability[24] and quasi-normal modes[25], the amazing discovery of the laws of BH mechanics and their close analogy to laws of thermodynamics[26,27]. Not just for BH but more generally, relativistic effects play an important role in the stability analysis of neutron stars and thus in the case of neutron stars lead to constraints on equations of state of nuclear matter at high densities[28].

Hawking proved singularity results in the cosmological case leading on to the fundamental question of whether these were only a consequence of the high symmetry of these spacetimes or a feature of more generic situations. Direct analyses of the field equations was not adequate to address this question and an alternative characterization of singularities by geodesic incompleteness more effective[15]. Focusing or defocusing of geodesics is related to spacetime curvature and the trace of geodesic equation for timelike geodesics is the famous Raychaudhuri equation[29] which relates the kinematical and dynamical properties of timelike and null vector fields to their acceleration, expansion, shear, and rotation and plays a key role in discussions of the singularity theorems. Going beyond is the question of the stability of the solutions of EE since it is not obvious whether small gravitational perturbations of Minkowski spacetime will fade away leading to its stability or grow to form a BH leading to instability. A mathematical opus is the Christodoulou-Klainerman[30] proof of nonlinear stability of Minkowski ST. One also has Friedrich's proof of the stability of De Sitter[31]. Two open questions in mathematical relativity are the Strong Cosmic Censorship (SCC) conjecture and the Weak Cosmic Censorship (WCC) conjecture. The former relates to the generic nature of solutions of EE that are geodesically incomplete and the latter whether solutions of EE in gravitational collapse generically leads to formation of BH and not a naked singularity[32]. The stability of Kerr is still an open question.

GR is a mathematically elegant geometric description of gravitation. The technical structure of this picture as a partial differential equation system on the other hand is very complex. It is not merely nonlinear but unlike other familiar equations of physics not generally classifiable as a wave-like, potential-like or heat-like system. Choquet-Bruhat[33] first proved that EE is a well-posed Cauchy problem demonstrating that GR is like other physical theories and its initial configurations and motion determine its future evolution. This well-posedness makes possible numerical simulations to accurately model phenomena in strong field regime by splitting the problem into two independent parts: 4 constraint equations to characterize the initial gravitational field and its rate of change, 6 equations to compute the evolution of the gravitational field and construct ST and its associated geometry. Unlike in EM, constraints of EE are harder to handle. In the popular conformal method one has seed data that can be freely chosen and determined data obtained by solving constraint equations using the chosen seed data. This leads to an effective parametrization of the degrees of freedom of the gravitational field enabling one to develop initial data sets corresponding to the physics one is trying to describe. Important techniques include conformal thin sandwich method, simple connected sum gluing techniques and more recently Corvino gluing[34] that proves how very general interior gravitational configurations can be smoothly glued to Schwarzschild exterior. A very important development was the Arnowitt, Deser, Misner (ADM) formalism[35] that delineates the initial data required and the constraints it satisfied for well-defined ST development. The existence and uniqueness of maximal Cauchy development is studied by using functional analysis methods based on Sobolev spaces. EE has turned out to be an interesting and important system in partial differential equations (PDE) theory and geometric analysis. These led to the following significant results of mathematical relativity like the Positive energy theorem (Schoen, Yau, Witten)[36,37] and the the Penrose inequality theorem[38]. For physical fields in Minkowski ST total energy-momentum is causal and future directed. In the presence of a gravitational field the result is not obvious because gravitational potential energy is negative.

Positive energy theorem shows that if matter sources have future directed causal 4 momentum density, both the total ADM 4 momentum and Bondi 4 momentum (4-momentum at any retarded instant) are also future directed and time-like. This led to a new invariant for asympotically flat Riemannian manifolds of interest to mathematicians. This work exposed general relativists to the powerful techniques at the interface of geometry and PDE (nonlinear geometric analysis) and attracted a new generation of mathematicians to unexplored profound global problems in GR.

From its preliminary exploratory studies in the seventies, numerical relativity came of age in the last decade with the discovery by Choptiuk[39] of critical phenomena in gravitational collapse providing unforeseen insights into full nonlinear regime of GR. He showed that in the evolution of a one parameter family of initial data set solutions with large values collapse to BH and those with small values disperse. The transition value solution has special features including time symmetry described by scaling laws. There is universality in that it is seen for all choices of one parameter families. No general mathematical proof of its existence is known in GR. More recently, by a combination of geometric analysis and numerical methods it has been discovered that though Anti De Sitter is linearly stable it is nonlinearly unstable[40].

The GR problem of motion goes back to the 1916 to the works of Einstein, Droste and De Sitter. They introduced the post Newtonian (PN) approximation method that combines a weak-field expansion, slow-motion expansion and a near-zone expansion. The 1PN corrections to Newtonian Gravity for describing the dynamics of N-extended bodies had problems related to treatment of the internal structures of the bodies and only in the famous 1938 work of Einstein, Infeld and Hoffmann (EIH)[41] did the GR N-body problem reach its first stage of maturity as expounded in the books of Fock *et al.* and of Landau–Lifschitz[42]. In the 1916 paper exploring physical implications of GR, Einstein[43] proposed the existence of gravitational waves (GW) as one of its important consequences. Soon after, Einstein[44] calculated the flux of energy far from source; the famous quadrupole formula and discussed in analogy with EM the related radiation reaction or radiation damping, distinguishing between energy carrying waves in contrast to non-energy carrying wave-like coordinate artefacts. In 1922, Eddington pointed inapplicability of the above derivation for self-gravitating systems[45]. Landau and Lifshitz and Fock extended the quadrupole formula to weakly self-gravitating systems and these constitute two different approaches to GW generation today. The complication for the self-gravitating case is fundamental since higher order PN calculations require dealing with higher order non-linearities of EE. The physical reality of GW remained in dispute for decades because of issues related to delineating physical degrees of freedom from coordinate or gauge effects. Pirani[46] by focussing on the effect of GW rather than its generation showed that GW are Weyl tensor waves. In the sixties Bondi *et al*.[47] proved that far from the source one can define a News function (derivative of shear) to describe energy carried away by GW. Chandrasekhar[48] was first to show conceptually that radiation reaction problem could be solved for continuous systems. Energy and angular momentum radiated as GW was correctly balanced by the loss of mechanical energy and angular momentum. His work gave astrophysicists confidence that GR was physically reasonable and well behaved. However, in the gauge he used, some terms at 2PN were divergent raising doubts for more mathematically demanding relativists[49]. The discovery in 1974 by Hulse and Taylor[50] of the binary pulsar 1913 + 16 was a watershed event. Radio pulsar timing observations allow one to reconstruct the orbit and measure the related inspiral of this system due to emission of GW. This provides high quality data that is proof that GW exist leading to a Nobel Prize in 1993 to Hulse and Taylor[51]. The prospects of testing theory against the Hulse-Taylor system once again revived more critical questions regarding existing treatments of GW, forcing a revisit to approximation methods in GR to remedy their mathematical shortcomings[52]. It mandated improved approaches to the N-body problem: modern versions of EIH going beyond 1PN relativistic effects to 2.5PN EOM, i.e. inclusion of terms of $\mathcal{O}(v^5/c^5)$ beyond the leading Newtonian acceleration. This involved careful control of ultraviolet (uv) divergences arising from the use of delta functions to model point particles in a nonlinear theory. The discovery of similar binary neutron star systems implies the existence of binary neutron star population emitting GW for hundreds of million years before coalescing spectacularly in sensitivity bandwidth of GW detectors like LIGO and Virgo. Even GW from such strong systems are weak signals buried in the noise of the detector requiring techniques like matched filtering both for detection and later for characterization[53]. Matched filtering requires the best possible model of the gravitational waveform favoring sources like coalescing compact binaries (CCB) (NS-NS, BH-BH, NS-BH) over unmodelled sources like supernovae. This has led to spectacular theoretical progress in 2-body problem in GR[54–56] complementing spectacular experimental progress in GW detection endeavours. Many approximation methods have been employed in this quest. The ADM approach has been effective for computations of the EOM while the MPM-PN approach of Blanchet *et al.*[55] has been also crucial in the computation of the far zone fluxes including the hereditary contributions to them. Other methods include the strong field point particle approach[56], the Effective Field Theory[57], Direct Integration of the relaxed Einstein equation (DIRE)[58,59] and the perturbation approach in the test particle limit[54]. The conservative terms in the two-body problem is computed to 4PN order beyond the Newtonian

Table 1. Breakup of different areas of GR as represented in the triennial meetings of the International Society of General Relativity and Gravitation (ISGRG) as documented in the available proceedings. Beyond 1983 the notation a + b represents plenaries (a) and workshops (b). In the earlier meetings the proceedings are not as uniform. In some like 1959, 1968 and 1977 they represent *all* the talks and not just the plenaries

Year	GRn	Place	Class.	Quant.	Cosm.	Ap.	GW	Exp.	Others
1955	GR0	Berne	17	5	7	0	1	2	–
1957	GR1	Chapel Hill	9	7	4	0	1	1	2
1959	GR2	Royaument	31	5	1	0	8	–	–
1962	GR3	Jablonna	26	6	5	0	1	2	1
1965	GR4	London	17	1	1	5	1	–	–
1968	GR5	Tbilisi	85	18	13	–	–	8	9
1971	GR6	Copenhagen	1 + 7	1 + 4	2 + 2	2 + 1	2 + 1	2 + 1	–
1974	GR7	Tel Aviv	4	2	1	2	1	2	2
1977	GR8	Waterloo	244	71	22	28	28	8	–
1980	GR9	Jena	7	3	–	–	3	1	4
1983	GR10	Padova	5 + 4	3 + 2	4 + 2	0 + 1	3 + 2	1 + 2	1
1986	GR11	Stockholm	6 + 7	3 + 2	1 + 3	1 + 0	1 + 2	1 + 2	=
1989	GR12	Colorado	4 + 7	3 + 5	3 + 3	2 + 1	2 + 2	2 + 2	2
1992	GR13	Cordoba	6 + 6	5 + 3	2 + 3	2 + 1	2 + 2	1 + 1	–
1995	GR14	Florence	6 + 5	3 + 3	2 + 2	2 + 1	4 + 1	–	–
1997	GR15	Pune	6 + 5	3 + 4	2 + 4	2 + 1	3 + 5	1 + 0	3
2001	GR16	Durban	6 + 5	3 + 4	2 + 4	1 + 13 + 5	0 + 1	1	
2004	GR17	Dublin	3 + 5	5 + 3	3 + 0	2 + 3	3 + 6	0 + 1	2
2007	GR18 Amaldi7	Sydney	3 + 5	3 + 4	2 + 2	1 + 2	3 + 6	1 + 1	–
2010	GR19	Mexico	3 + 3	3 + 4	2 + 2	4 + 2	3 + 5	0 + 1	–
2013	GR20 Amaldi10	Prague	5 + 6	3 + 2	2 + 23 + 1	5 + 8	1 + 1	2	

acceleration in the comparable mass case[60]. In the comparable mass case, the GW flux is known to 3.5PN order beyond the leading Einstein quadrupole formula corresponding to 6PN dissipative terms in the acceleration[61]. In the test particle limit the GW flux for the Schwarzschild case is known to 22PN (ref. 62) and for the Kerr case to 11PN (ref. 63). The PN approximations break down eventually around the LSO but resummation techniques like Pade approximants can be used to extend their domain of applicability[64]. The Effective-One-Body (EOB) approach is a particular non-perturbative resummation of PN-expanded EOM to extend validity of PN results beyond the last stable orbit (LSO), and up to the merger and ringdown[42]. Today we know that GW must exist in any relativistic theory of gravitation like GR but the properties on GW in different theories can be different. Thus, when GW detections become routine it has the potential to become an essential tool for astrophysics, precision cosmology and eventually fundamental physics[65].

A tour de force in the effort to compute GW from CCB was due to Pretorius[66] who produced the first simulation with large number of orbits through merger using modified harmonic coordinates, compactification of numerical domain at spatial infinity, singularity excision and damping of constraints. Post his amazing breakthrough in NR, one has reliable waveforms for the late inspiral and merger parts of the binary evolution which can be used for constructing templates including merger and ringdown. Other groups using other methods like Baumgarte, Shapiro, Shibata, Nakamura (BSSN) equations and puncture methods have followed. The waveforms are calibrated and interpreted by PN inspiral results. They showed late inspirals, plunge and mergers are tamer than expected: there are no spin flips at merger or signatures of nonlinear dynamics at plunge. Unforeseen was also the substantial kick of the final BH from coalescence for spinning black holes with aligned spins[67]. There is work on building bridges between analytical and numerical methods via quasi-local horizons and definitions of mass, angular momentum, multipole moments associated with them and balance laws. The aim is to construct invariant tools to extract physics from numerical simulations in fully nonlinear and dynamical regimes[68]. The state of the art in numerical relativity (NR) simulation[69] has progressed tremendously over the decade since then. From short simulations of about 20 orbits; the latest one based on Spectral Einstein Code (SpEC) is 25 times longer. We have the first NR simulation of compact binary (mass ratio 7; total mass 45.5 $M_{\odot}$, 125 orbits) whose gravitational waveform is long enough to cover the entire frequency band of Advanced LIGO. There is good consistency of various approaches and a possibility to compare them to models based on PN, EOB and phenomenological models for inspiral, merger and ringdown (IMR).

In the Chapel hill meeting of 1957 considered GR1, classical GR was referred to as *unquantized gravitation* probably reflecting the hope that quantum gravitation was just around the corner. Since then, classical GR has revealed unforeseen facets and gone far beyond the initial traditional mathematics connections to symbiotic exchanges with new emerging areas like geometric analysis,

Table 2. Interesting topics in classical GR in the GRn meetings

1955:	GR0: Equations of motion (EOM)
1957:	GR1: Initial value problem
1959:	GR2: Equations of motion, conservation laws
1962:	GR3: Petrov classification, characteristic initial value problem
1965:	GR4: Relativistic collapse, gravitational radiation, lensing
1968:	GR5: Singularity, Newman Penrose, global structure of Kerr, separability of wave equations in Kerr
1971:	GR6: Global structure, gravitational collapse, gravitational wave (GW) detection, cosmological singularities, parametrized post Newtonian framework
1974:	GR7: Bondi Metzner Sachs, Processes around BH
1977:	GR8: Algebraic computing, NR of colliding BH, BH perturbations, singularities, solution generation techniques, rotational instabilities, binary pulsar, EOM, cosmic censorship
1980:	GR9: Algebraic computation, initial value problem, BH, singularities, global issues, positive energy theorem, twistors, exact solutions
1983:	GR10: Perturbations of BH, cosmic censorship, asymptotics, EOM of compact bodies and GW, positive energy
1986:	GR11: BH uniqueness, twistors, NR, positive energy
1989:	GR12: Global properties of EE, rotating NS, NR, colliding waves
1992:	GR13: NR, stability of Minkowski, GW
1995:	GR14: NR in cosmology and BH, stability, cosmic censorship
1997:	GR15: Critical phenomena, asymptotics, NR
2001:	GR16: Inequalities, PN generation of GW, Constraint Eqns, NR
2004:	GR17: NR, mathematical GR, gravitational self-force
2007:	R18: Cosmic censorship, isolated horizons, stellar dynamics
2010:	GR19: Stability of Kerr, BH uniqueness, NR
2013:	GR20: Instability of ADS, geometric inequalities, exact solutions in higher dimensions, NR, GW by effective field theory.

numerical computations, high energy physics and gravitational wave astronomy. Classical GR has become an integral part of the core toolkit every physicist must be equipped with to investigate and comprehend the universe we live in.

In Table 1, is displayed the distribution of topics among classical GR, quantum GR, cosmology, astrophysics, gravitational waves, gravity experiments and other topics in the triennial GRn conferences since its inception in 1955. In Table 2, before concluding, I also enumerate the important topics in classical GR covered in these conferences to give a rough idea of how things evolved.

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