Enhanced richness of notes by modulation of boundary conditions in a stringed musical instrument

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The richness of the sound emanating from a stringed musical instrument can be improved by the making it linger for longer. This has traditionally been performed in sitar and sarod by modification of boundary conditions while introducing a jawari. We try to understand the effect of the jawari by assuming periodic modulation of boundary conditions of the string in a sonometer and find that the lobe-like effect can indeed be observed as an interference between the frequency-modulated sideband of one mode with the next higher-order mode. The contributions from different modes can be added up. The size of the beat signal increases while increasing the depth of modulation.

Keywords: Beats, boundary conditions, frequency modulation, note richness, string musical instruments.

MUSICAL instruments have fascinated mankind since time immemorial. The melodies from the violin, guitar, sitar, piano, flute and many other kinds of instruments have captivated audiences transcending barriers of religion, caste, creed or nationality to bring people from all walks to the same platform. The stringed instruments[1,2] hold a special place in the list, two of which are the sitar and the sarod[3].

The richness of the sounds emanating from such stringed instruments has drawn a lot of interest into the very causes for such effects. The effects of the music can be significantly altered by making each note linger. The easiest way is to reduce the damping of the instrument. However, the sound may also be made to linger by producing beats intrinsically through modification of parts of the very instrument. Such an effect has traditionally been used in the tanpura[4], sitar[3,5,6] and sarod[7].

A sonometer is a basic device used to mimic the complex musical instrument. In such a device, the strings are tied up at the ends and plucked to oscillate at frequencies corresponding to the point of plucking, the tension in the string and the quality of the wire[8]. The addition of a jawari to this may complicate the entire frequency spectrum by introducing a unique boundary condition. An analysis of the quality of stringed musical instruments can be initiated by a study of the vibrations of the strings under such boundary conditions. In this communication, we study the effect of the jawari on the instrument and suggest a possible cause for this.

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We initiate the study of stringed instruments by making an adapted sonometer. It is a wooden frame with strings firmly attached at both ends. Plucking the string at 1/4 of the total length gives the response, as shown in Figure 1.

There are various frequencies superimposed on one another in the response. This can be observed in the corresponding Fourier transform shown in Figure 2. The spectrum shows discrete, equally spaced lines which extend from the fundamental frequency to higher ones.

The vibration of a realistic string attached at both ends is governed by the wave equation with damping given as

$$\frac{\partial^2 Y}{\partial t^2} + \gamma \cdot \frac{\partial Y}{\partial t} = c^2 \cdot \frac{\partial^2 Y}{\partial x^2}. \quad (1)$$

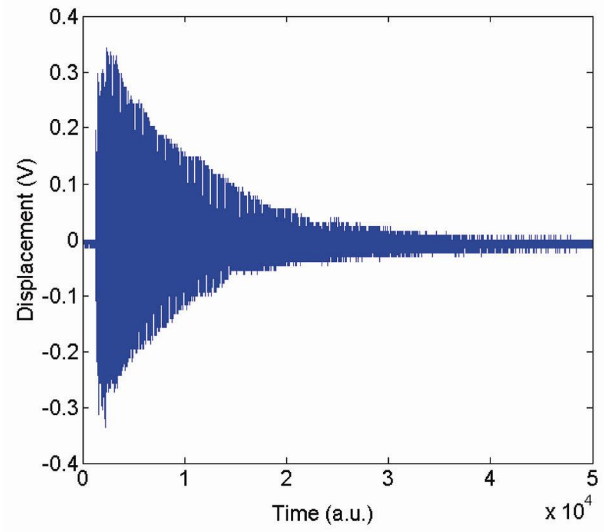

Figure 1. Time series of the sonometer signal without the jawari. The device has a string of length 1 m and attached firmly at both ends. It is plucked at ¼ of the total length and recorded using a microphone. The signal also shows a decay due to the damping.

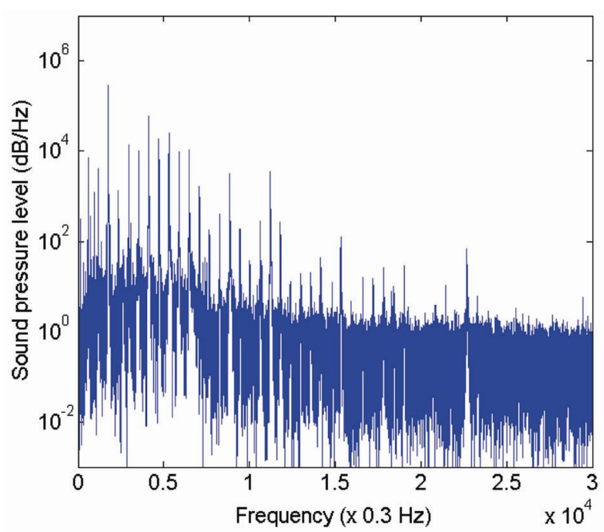

Figure 2. Fourier transform of the sonometer signal without the jawari. The discrete modes of the sonometer can be observed.

This equation can be solved by the separation of variables method upon performing $Y(x, t) = X(x) \cdot T(t)$, and plugging back into the equation to get

$$\frac{1}{T}\left(\frac{\partial^2 T}{\partial t^2}+\gamma \cdot \frac{\partial T}{\partial t}\right)=c^2 \cdot \frac{1}{X}\frac{\partial^2 X}{\partial x^2}=-k^2, \tag{2}$$

where k is some constant. The equation for X can be solved to obtain $X = A \sin(kx/c)$. When the boundary condition of $X = 0$ for $x = L$ is considered, we get the following condition for k

$$k=\frac{n\pi c}{L}. \tag{3}$$

This parameter can be put back into the equation for T to obtain

$$\frac{\partial^2 T}{\partial t^2}+\gamma \cdot \frac{\partial T}{\partial t}+\left(\frac{n\pi c}{L}\right)^2 \cdot T=0. \tag{4}$$

The corresponding solution to this equation for integral values of n ranging from 1 to 20 yields the waveform given in Figure 3. The exponential decay of the envelope is similar to that from the sonometer.

If a cylindrical piece of wood is inserted under the sonometer string so as to touch it over an extended length, the axis of the cylinder being orthogonal to the string, the response becomes different. The sound emanating from the device shows an extra lobe forming in the temporal response (Figure 4). The piece of wood is known as the jawari in some Indian stringed musical instruments like the sitar.

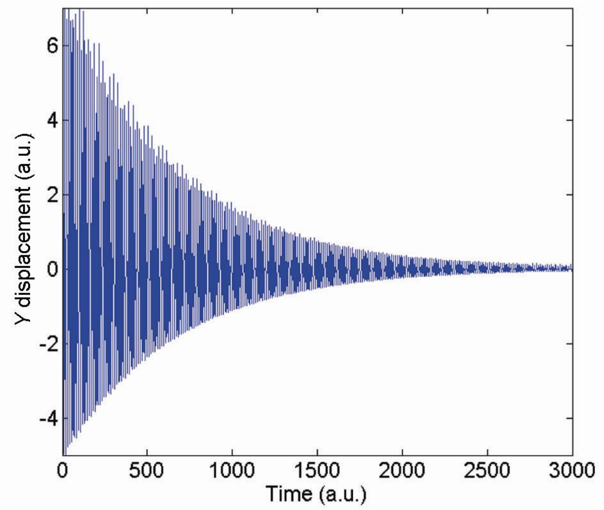

Figure 3. Theoretically computed time series upon solving eq. (2) shows the expected response of the plucked string when the jawari is not present. Here $a = 1$, $b = 0$, $L_0 = 1$, $c = 0.2/\pi$ and $\gamma = 0.003$. The exponential decay of the simulated time series can be observed.

We believe that this behaviour is the effect of the jawari on the boundary conditions of the sonometer. We modelled this by assuming that the string touches the round surface of the jawari in such a way as to imply a periodic modulation of the total length of the string. When the region of the string close to the boundary touches the surface, it effectively shortens the total length of the string. While the string is vibrating at a certain frequency, the displacement close to the boundary is also

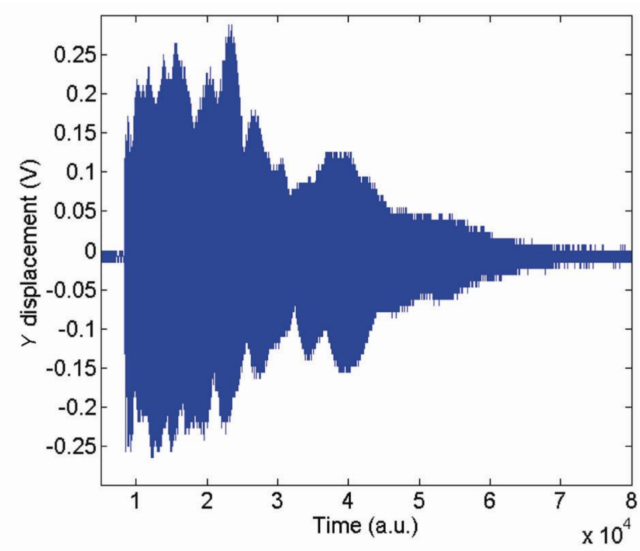

Figure 4. Time series of the Y displacement of the string as recorded from the sonometer. An extra lobe is visible in the temporal response.

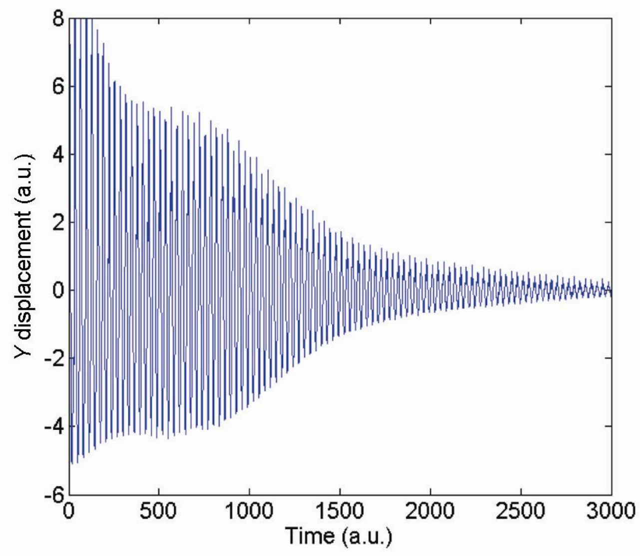

Figure 5. The result of numerically solving eq. (5). The lobe-like structure in the time series could be successfully generated by the simulation. Here L_0 was assumed to be 1, $c = 0.2/\pi$, $\gamma = 0.003$, $a = 1$ and $b = 0.012$. The modulation in the length of the sonometer string required to generate the lobe was only 1.2% and the damping was much smaller than c^2.

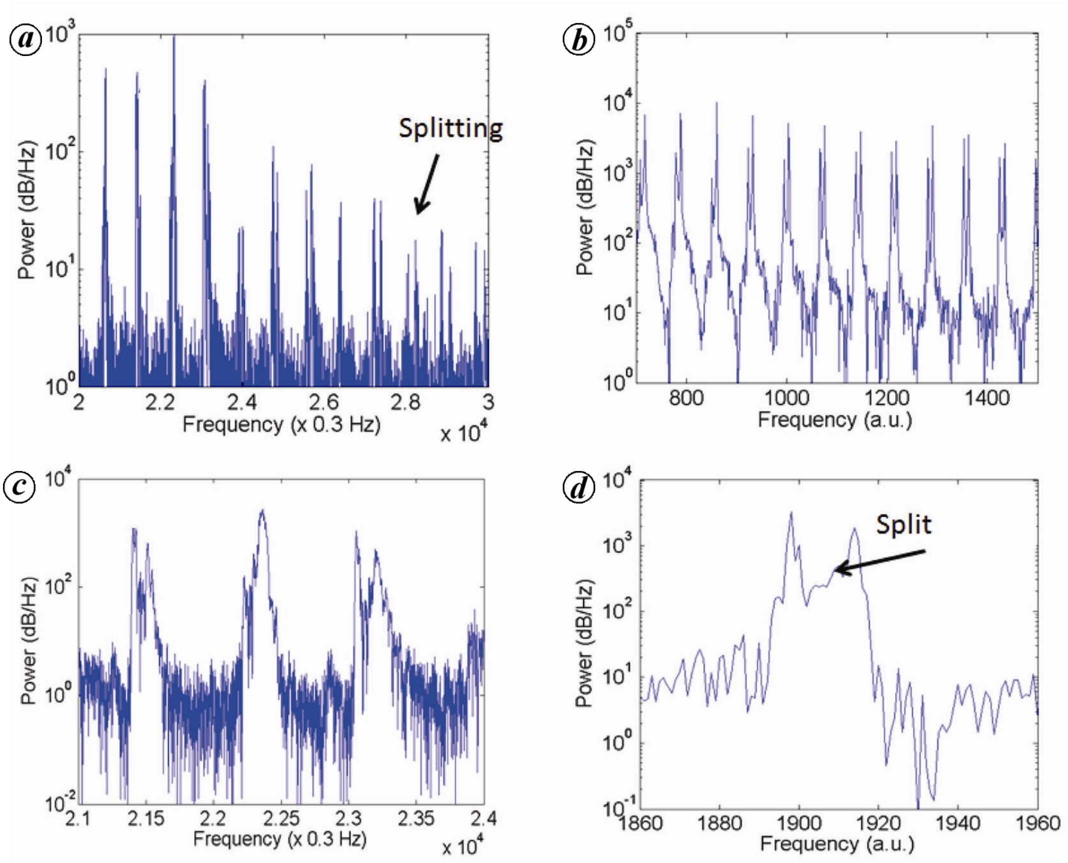

Figure 6. Spectra for (***a***) experimentally determined response of the sonometer with the jawari and (***b***) theoretically simulated result of eq. (5). ***c***, Zoomed-in view of the high harmonics in the experimentally obtained data. ***d***, Zoomed-in view of the theoretical data of a high-frequency mode. The spectra show splitting of peaks at high harmonics, which is not observed when the jawari is not used. The harmonics start are single peaks at low harmonics, but gradually split at higher ones. It is suspected that these are the FM sidebands with the next order mode.

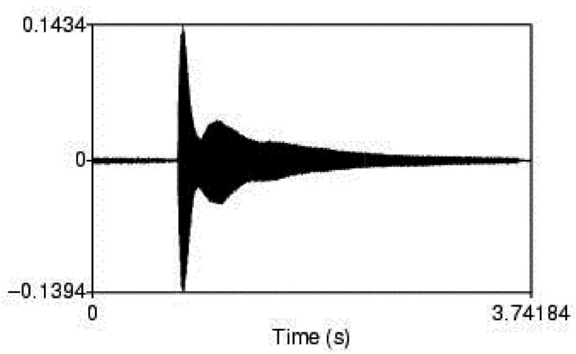

Figure 7. Time series for a real sitar. It clearly shows the extra lobe on the envelope indicating the beats which enhance the richness of the music.

being modulated at that same frequency. This was incorporated into eq. (4) using $L = L_0(a + b \times \cos(\omega t))$ to modify it as

$$\frac{\partial^2 T}{\partial t^2} + \gamma \cdot \frac{\partial T}{\partial t} + \left(\frac{n\pi c}{L_0(a + b \times \cos(\omega t))} \right)^2 \cdot T = 0. \qquad (5)$$

The ω in eq. (5) was assumed to be the same frequency as the oscillation frequency of the string, which is $n\pi c/L_0(a + b\cos(\omega t))$. The coefficient a was assumed to be 1 and b was assumed to be 0.012. Since b was small compared to a, its effect on ω after the second iteration was assumed to be negligible when L was just assumed to be L_0. The resulting equation was solved numerically for each mode and the displacements due to the individual modes added up in-phase to obtain the y displacement of the string as a function of time. The phases of the excitations were assumed to be the same since these are caused by the same excitation (Figure 5). We find that the modulation of 1.2% of the string length itself is sufficient to yield significant lobes, as is visible in Figure 5. It is assumed in the simulation that each mode contributes equally to the final displacement of the string.

By construction, an equation in the form eq. (2) can only have sharp resonances at frequencies corresponding to various values of n, when there is no external modulation of the parameters of the system. We find upon performing the Fourier transform on the experimental data that the lower harmonics are single peaks, but gradually appear to be split at high modes.

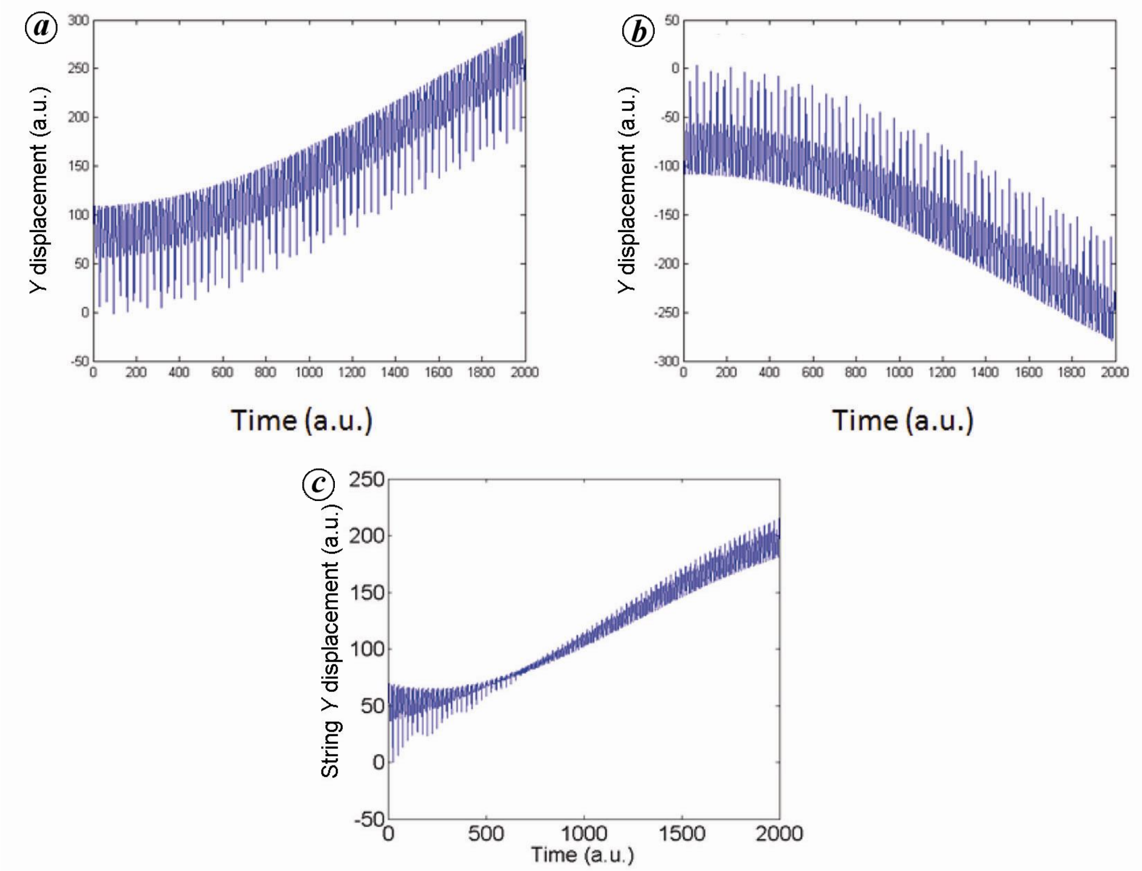

Figure 8. ***a***, Increase in the displacement of the string as a function of time with an constant positive force due to friction. ***b***, Decrease in the displacement of the string in the reverse stroke of the violin when the bow slips. ***c***, Displacement in a hybrid instrument when the string is plucked and a bow is used to apply a constant force. There are beats, yet the mean of the displacement increases.

This splitting was also observed in the Fourier transform of the theoretically generated time series as shown in Figure 6. It can be understood to be caused by an interference effect. Equation (5) corresponds to that of frequency modulation (FM) of a wave at a frequency ω. When the degree of modulation is sufficiently large, an FM sideband is generated. This sideband interferes with the next higher mode of the sonometer to form beats. When the degree of modulation indicated by the b coefficient is larger, larger is the sideband amplitude leading to larger amplitude of beats. The sideband and the higher-order mode are both excited by the same plucking excitation and hence are phase-correlated. This is the cause for the formation of the beats.

The richness of the sound emanating from stringed musical instruments increases upon using boundary conditions that mimic the effect of the jawari. This can be in the form of introducing round-shaped pieces of wood, like in the sitar. A typical waveform from a sitar is shown in Figure 7. The sound persists for longer giving a unique experience to the listener. The time series exhibits an extra lobe as observed in the case of the sonometer with the jawari.

This analysis can be extended to bowed string instruments like the violin by realizing that these primarily differ from the plucked string instruments by the mode of excitation. As has been noted by Benson[9] and also studied by Raman[10], when the bow is pulled across the string, the initial stroke has a lot of friction, while the reverse stroke involves slipping of the bow across the string. Thus eq. (4) should be modified by adding a constant force $+\alpha$ on the right-hand side, while the reverse stroke should have $-\beta$. The equation needs to be solved under these conditions to ascertain the effect of the bowed excitation. The result is shown in Figure 8 *a* and *b*. The Y displacement of the string increases as shown in Figure 8 *a* and under the asymptotic limit, becomes linear. Here is the damping of eq. (4) has been assumed negligibly small. Similarly, the reverse stroke makes the Y displacement reduce. This trend is expected[9] when the displacement of the string has been shown to exhibit saw tooth response locally as the displacement increases linearly and then reduces linearly at a different slope.

The plucked string and bow string instruments are entirely different with respect to the means of excitation as the force for the bow string instrument is constant, while the plucked string instrument has a delta function excitation. A way of combining both might be to conceive a violin with a jawari, in which case the *Y* displacement would be as shown in Figure 8 *c*. This waveform is for a device that combines sudden impulse of the plucked string with that of continuous excitation by a bow.

We have analysed the effect of a jawari on a sonometer. In the absence of the jawari, the signal resembles a damped harmonic oscillator. However, as the jawari is introduced, the boundary condition is modified to periodically modulate the length of the string. The modulation frequency of the boundary condition for a particular mode is the same as the oscillation frequency. Combining many such modes, the experimentally observed lobe of the time series could be successfully replicated. The periodic modulation of the boundary condition causes the generation of FM sidebands, which interfere with the higher-order modes in the frequency spectrum to generate the beats observed in the time series of the sonometer with the jawari.

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ACKNOWLEDGEMENTS. I thank Sanjoy Bandyopadhyay, Sangeet Research Academy, Kolkata for bringing the problem to our notice. I also thank Pratik Khastgir and Somnath Bharadwaj for their help in setting up the sonometer and providing inputs for the calculation; Soumitro Banerjee for his crucial advice regarding the simulation; and the Indian Institute of Science Education and Research, Kolkata for providing the Senior Research Fellowship.

Received 13 April 2014; revised accepted 25 May 2014

Variation of black carbon concentration associated with rain events at a tropical urban location

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Mass concentration of black carbon (BC) near the surface (within the planetary atmospheric boundary layer) was measured using a seven-channel aethalometer at Kolkata, a metropolitan city in the Indian tropical region, during the period from June 2012 to May 2013. The diurnal variation of BC concentration shows a prominent increase in the morning and evening hours, an usual feature seen over continents. However, an anomalous feature of the BC variation is observed subsequent to rain events. On normal days, the BC mass concentration during noon and early afternoon hours remains around 8000 ng/m^3 at Kolkata. However, after the occurrence of isolated thundershowers, interestingly, the BC concentration increases (rather than decreasing due to washout) and at times reaches above 20,000 ng/m^3 during noon and early afternoon hours. This increase is found to be associated with the formation of local temperature inversion within the atmospheric boundary layer during and after the occurrence of rain, which would suppress or inhibit vertical mixing and dispersion in contrast to non-rainy days. Results are presented to indicate the above-mentioned behaviour of BC concentration.

Keywords: Black carbon concentration, diurnal variation, rain effect, temperature inversion, tropical urban location.

ATMOSPHERIC black carbon (BC) is an important aerosol species in climate change studies and has significant impacts on human health. It strongly absorbs the solar radiation over a wide spectral band[1–3] contributing to atmospheric warming. BC is emitted during incomplete combustion[4,5]. There is a rapid increase in fuel demand with increase in the daily energy needs for domestic, industrial and transport sectors. This change in fuel utilization has caused a historical change in BC emission in the past decades[6]. The presence of significant amount of BC in the clouds may 'burn off' the clouds[7,8]. However, it has a cooling effect at the surface, which affects the temperature profile in the troposphere and consequently the microphysical structure of the cloud as well as the rainfall mechanism[9,10]. Usually BC particles are hydrophobic and are less scavenged by precipitation. BC particles may be

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