PERSONAL NEWS

John Forbes Nash Jr (1928–2015)

John Forbes Nash, aged 86, and his wife Alicia, travelling home in a taxicab from airport on 23 May 2015, died in a road accident in New Jersey, bringing to an abrupt end an unusual life story one would not believe but for the fact that it is true. Ironically, he was returning from Norway where he had received the prestigious Abel Prize, an overdue recognition for his contributions to pure mathematics that came a couple of decades after the recognition of the influence he has had on economic thinking, in the form of a Nobel Prize in Economics in 1994.

Nash's life story is now well known, thanks to Sylvia Naser's book *A Beautiful Mind*[1] and the eponymous movie starring Russell Crowe. Presumably the title alludes to what Lloyd Shapley, a fellow graduate student and game theorist and eventually also a Nobel Laureate, said of Nash: 'What redeemed him was a clear, logical, beautiful mind', a statement that also gives away how his peers perceived him. A short and lovely account of Nash's life is his own autobiographical essay written at the time of his Nobel Prize, reproduced in the collection in ref. 2. The opening paragraph of this essay quoted below already suggests that this man is different:

> 'My beginning as a legally recognized individual occurred on June 13, 1928, in Bluefield, West Virginia, in the Bluefield Sanatarium, a hospital that no longer exists. Of course I can't consciously remember anything from the first two or three years of my life from birth. (And, also, one suspects, psychologically, that the earliest memories have become "memories of memories" and are comparable to traditional folk tales passed on by tellers and listeners from generation to generation.) But facts are available when direct memory fails for many circumstances.'

To recap a few highlights of his life, his father was an electrical engineer and mother a former school teacher. After school he joined Carnegie Institute of Technology (now Carnegie Mellon University) to do chemical engineering, but soon realized that his true calling was mathematics. He did so well that he was given a Master's degree along with Bachelor's and went to Princeton, preferring it over Harvard because of its geographical proximity to home and the extra interest it showed in him. His admission had been secured by a one-line letter of recommendation from Carnegie: 'This man is a genius.' After his seminal thesis work, he got a position in MIT where he met his future wife, a physics student from El Salvador. He was all set for an extremely illustrious career, when schizophrenia struck him and he dropped out into oblivion. After a series of treatments and legal separation from his wife, he ended up in Princeton, wandering along Fine Hall which houses the

mathematics department and scribbling obscure formulae on blackboards. This earned him the nickname of 'Phantom of Fine Hall'. Cared for by his friends and former wife who took him in as a boarder, he gradually underwent a miraculous cure from his malady. The rest is history. He and his then ex-wife remarried and were together till the end, even in death.

Nash's early contributions to the then young field of game theory are legendary and are perhaps the only part of his work that non-experts have an inkling of. His first contribution was an axiomatic solution of the bargaining problem, the germ of which took root in his mind when he did his only elective in economics at Carnegie. This was a course in international trade during his final year. He refined the ideas in his early graduate student days at Princeton during a game theory seminar course run by Albert Tucker, the 'T' of 'KKT conditions' in optimization. Tucker later became his thesis adviser. According to his fellow graduate student and close friend Harold Kuhn, the way the article is written has tell-tale signs that it is by a teenager, with goods bargained being bat, ball, pen and so on. Nash later refined these results further to include a negotiation model.

His *coup de grâce* in game theory, however, came with his Ph D thesis, all of twenty-seven pages, where he introduced the equilibrium notion for non-cooperative games that we now know as Nash equilibrium. It has become the very foundation of non-cooperative game theory, having implications not only in economics, but in a variety of diverse disciplines such as evolutionary biology and engineering. Simple and natural as it may seem in hindsight, it has to be kept in mind that there had been other stalwarts before him who had not been able to pin down the right framework for thinking about games much beyond the so called 'two player zero sum' games where one player's loss is the other's gain. In fact the Nobel Prize committee's press release on the occasion of his Nobel Prize underscores the role he played in clearly demarcating the differences between cooperative and non-cooperative games in terms of enforceable versus non-enforceable agreements, on par with his specific contributions to the concept of equilibrium.

Kuhn describes his proof of the existence of a Nash equilibrium, based on Brouwer fixed point theorem, as clumsy but totally ingenious. He gave a short and sleek proof later using the Kakutani fixed point theorem, based on a tip from David Gale, which is standard textbook fare now. His thesis also has a discussion of conceptual issues such as two different interpretations of Nash equilibrium, which were not included in the *Annals of Mathematics* publication that carried the mathematical part. One is what he calls the rationalistic explanation, i.e. Nash equilibrium as the inevitable consequence of tenets of rational behaviour, assuming that the players are completely rational and have complete knowledge of the game in a precise sense. This has overtones of earlier works of Cournot, Von Neumann–Morgenstern, etc. The second explanation, which he called 'mass action', is, however, wholly original. It views the so-called mixed strategies, in which players randomize between their choices, in terms of a large statistical population of players. In another conceptual leap, in his *Annals* paper he talks of possible dynamic non-cooperative mechanisms leading to cooperative behaviour. In fact one of his subsequent papers does talk of a negotiation mechanism leading to a cooperative equilibrium. These ideas are amazingly prescient. They are the core of what came to be known as the Nash program which has dominated much of game theory research. One issue with Nash equilibrium is that it is often not unique, which creates problems for the rationalistic explanation. This led to several static refinements of the concept due to, among others, Harsanyi and Selten who shared the Nobel Prize with him. Much of the work on dynamic models of learning in game theory has been also motivated by the need to pin down a selection principle that will flag one or a few of the Nash equilibria as being 'natural'. Some of the more recent work, such as the biologically motivated work of Martin Nowak on evolution of altruism, is even closer to the original Nash program in spirit[3].

Yet another, rather little known but highly prescient work of his, is in experimental games[4], a methodology that has become quite standard in game theory now. Finally, there is the game of hex, which Nash invented independently, while still a graduate student, a few years after the Danish mathematician Piet Hein had invented it.

Why did his Nobel Prize come so late? In addition to his history of mental illness creating a mental block for the prize committee, there is also the genuine issue of game theory being delegated to the 'purely academic corner' of economics for many years. Whereas it was a whopping success in evolutionary biology through works of John Maynard Smith and others[5], the concrete applications in economics were slow to register. This came about thanks to disciplines such as mechanism design, matching markets, etc., which have built their edifices upon the foundations laid down by game theory. These have delivered in concrete ways, such as spectrum auctions, kidney exchanges, and so on, leading to subsequent Nobel prizes for the key players. The newest kid on the block is algorithmic game theory, a thriving subdiscipline of theoretical computer science driven by the exigencies of internet commerce and network economics.

While still basking in the glory of his achievements in game theory, Nash developed an urge to contribute to pure mathematics. His first major contribution was regarding a connection between differential manifolds and real algebraic varieties, a work, according to Kuhn, that was Nash's personal favourite and in his opinion, the most perfect. This was followed by one of his two colossal contributions cited for his Abel Prize. Apparently on a challenge from a colleague, he proved the existence of a smooth isometric (i.e. distance preserving) embedding of a Riemannian manifold into a Euclidean space of appropriate dimension. 'Smooth' here can mean continuously differentiable, k times continuously differentiable or real analytic, all three variants are due to Nash. The first, later improved by Kuiper, is of a different flavour than others and is considered more intriguing. As always, Nash's techniques were quite novel and some of them became items of independent interest. One of them is an extension of inverse function theorem that came to be known as Nash–Moser inverse function theorem. Another is the concept of 'Nash blowing up' developed by Hironaka and others subsequently.

The second major contribution was to an open problem in linear partial differential equations which he was introduced to by Louis Nirenberg, now his co-recipient of the Abel prize. This concerns Hölder continuity estimates for a class of elliptic and parabolic equations under extremely general conditions on the coefficients. Nash had to share the glory with the Italian mathematician De Giorgi who independently solved the problem. With later improvements by Moser, it is known as the De Giorgi–Nash–Moser estimate, a cornerstone of modern theory of partial differential equations.

Nash visited Mumbai in January 2003 when the Taj Mahal Hotel in Mumbai hosted a major international conference on game theory as a part of its centenary celebrations, a fortunate consequence of the fortuitous fact that the then Taj manager Ravi Dubey had a close family relationship with the Stony Brook economist–game theorist Pradeep Dubey, who was the prime mover behind the event. Nash expressed interest in visiting the Tata Institute of Fundamental Research and Pradeep Dubey asked me to host him. I picked up Nash from the Taj and took him to TIFR. He showed a childlike curiosity for everything he saw around and kept asking questions. In the brief waiting period till his formal seminar, he regaled me with some amazing computer skills he had picked from Princeton undergraduates. He spoke on 'Ideal Money and Asymptotically Ideal Money', expounding his thoughts on role of money in society and international economics. (See ref. 6 for related material.) He spoke well, though digressing occasionally, but seemed to keep track of where he should be by moving a pencil along typed slides that he was using. His clarity of thought and more importantly, the penchant for thinking about deeper issues was there for all to see. In the reception that followed, he interacted with students, faculty and guests, fielding graciously the occasional personal question from one of them. It was an unforgettable experience for me, being as it was a glimpse into an unforgettable life that made unforgettable contributions to science. Nash's demise signals the end of an era in game theory.

1. Naser, S., *A Beautiful Mind*, Simon and Schuster, New York, 1998.
2. Kuhn, H. W. and Naser, S. (eds), *The Essential Nash*, Princeton University Press, Princeton, NJ, 2002.
3. Nowak, M. A. and Highfield, R., *Super-Cooperators*, Simon and Schuster, New York, 2011.
4. Kalisch, G. K., Milnor, J. W., Nash Jr, J. F. and Nering, E. D., In *Decision Processes* (eds Thrall, R. M., Coombs, C. H. and Davis, R. L.), John Wiley, New York, 1954, pp. 301–327.
5. Smith, J. M., *Theory of Evolution and Games*, Cambridge University Press, Cambridge, UK, 1982.
6. Nash Jr, J. F., *Southern Econ. J.*, 2002, **69**(1), 4–11.

VIVEK S. BORKAR

Department of Electrical Engineering, Indian Institute of Technology Bombay, Powai, Mumbai 400 076, India
e-mail: borkar.vs@gmail.com