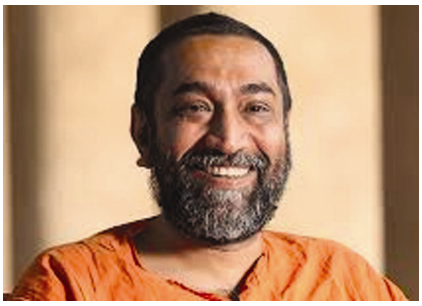
Mahan Mj



The Infosys Prize 2015 in Mathematical Sciences has been awarded to Mahan Mj, for his outstanding contributions to geometric group theory, low-dimensional topology and complex geometry. In particular, Mahan Mj has established a central conjecture in the Thurston program to study hyperbolic three-manifolds and introduced important new tools to study fundamental groups of complex manifolds. The award carries a purse of Rs 65 lakhs, and a 22-carat gold medal along with a citation.

Mahan Mj studied at St Xavier's Collegiate School, Kolkata. He then went to Indian Institute of Technology, Kanpur, initially to major in electrical engineering, but switched to mathematics. He graduated with a Master's degree in 1992. He is alumnus of University of California, Berkeley, where he completed his Ph D in mathematics with Andrew Casson as his advisor. His thesis was titled 'Maps on boundaries of hyperbolic metric spaces'. After earning a doctorate in 1997, he worked briefly at the Institute of Mathematical Sciences, Chennai. Spiritually inclined, he joined the Ramakrishna Mission in 1998, being impressed by the life and work of the Vedantic philosopher Swami Vivekananda; received his saffron robe and ordained a monk in 2008. He was a monk at the headquarters of Ramakrishna Mission at Belur Math and was an Associate Professor of Mathematics at the Ramakrishna Mission Vivekananda University at Belur Math. Currently, he is a professor of Mathematics at the Tata Institute of Fundamental Research (TIFR), Mumbai. He has published and presented his work widely and is the author of a book titled *Maps on Boundaries of Hyperbolic Metric Spaces*. He is the recipient of several fellowships and awards: Earle C. Anthony Fellowship, U. C. Berkeley in 1992–93 and the prestigious Sloan Fellowship for 1996–97. In 2011, he won the Shanti Swarup Bhatnagar Award in Mathematical Sciences.

In conversation with *Current Science*, Mahan Mj spoke on his research interests, his love for mathematics and his take on science and religion.

Please tell us about the creative side of mathematics? Is there a philosophy you have of what mathematics is or what is its place in society?

The creative aspect of mathematics is at the heart of the actual doing of it. I really don't have much to add to what greats like Poincaré have said about it. Suffice to say that this is done best when one's entire being is involved in the process. As regards its position as a human endeavour, there are two aspects. To the actual practitioner, the subjective motivation is largely that of a creative art – to create music out of reason, so to speak. To the spectator or user, it is the language of science and technology: something that gives precision and solidity to the scientific endeavour.

Help us understand the concept of manifolds and Thurston's program for which you have been awarded the Infosys Prize.

The area of geometry that I work on is called hyberbolic geometry (a kind of non-Euclidean geometry). Geometry deals with properties of lines, circles, planes, points, etc. Imagine two branches from the trunk of a tree – they will form a *Y* shape along with the trunk of the tree. If a squirrel wants to move from one branch to another (assuming it does not jump), it will have to travel down from one branch to the point where it meets the trunk and then climb up the other branch. Let's name the base of the tree as 0, the tip of one branch as 1 and let's call the tip of the second branch 2. Let's name the point at which these branches meet as *X*. Thus, the squirrel will have three paths: 1–*X*–2; 0–*X*–1; and 0–*X*–2. These are the analogues of the three sides of a triangle in the plane geometry that we studied in school. In plane Euclidean geometry, you can imagine squeezing out whatever is inside a triangle till it becomes a *Y* (of the tree). You can play the same game with a tree with more branches. So what's the difference between a Euclidean triangle and this *Y*? If you scale the *Y* by making the tips longer, they still have a meeting point. But if you continue to make the sides of a Euclidean triangle longer, the inside fat area will become fatter and fatter. Spaces which have triangles that are uniformly thin are called hyberbolic spaces.

What we have described above is one side of the story: hyperbolic geometry. Next, when you observe a tree a little closely, you will notice that the initial branches are not as long as the trunk. And the branches continue to grow shorter and shorter till you get very small, slender branches. The question that a hyberbolic geometer would ask is: Why do the branches become smaller as you go further away from the trunk? Is it an accident? Why, for instance, are leaves not as long as stems? The answer is that had they been so, you would have not been able to fit them in 3D space. What's happening here is that the hyperbolic geometry of the tree is conflicting with the flat Euclidean geometry of the three-dimensional space in which we live. If you look closely at the foliage of the tree, you will notice that the large structure of the tree is being replicated at the tip of the branches. This kind of geometry at the surface of the tree is self-replicating. Thus, it has a property that may be described as 'self similarity at all scales'. The foliage is an example of what is called a fractal. In effect, there is a constant interplay between the two-dimensional fractal geometry on the surface of the tree and the 3D hyberbolic geometry of the tree itself. This is a kind of naive visual representation of most of the work.

What attracted you to the field of topology and geometry? What is the distinction between these two fields?

People get attracted to pure mathematics typically through one of two channels in school. Either playing with numbers, or Euclidean geometry. For me it was the latter. My way of thinking is primarily visual. So something that appeals to that instinct naturally catches hold of the imagination. Then at IIT Kanpur, we had some outstanding teachers. I recall, in particular, Shobha Madan teaching us a number of courses in topology and that really sort of fuelled my love for that subject. In the summers of 1990 and 1991, I visited TIFR as a summer student (the formal visiting students research programme (VSRP) had not started in mathematics back then) and got to study algebraic topology with V. Srinivas. M. S. Raghunathan put me on to Guillemin–Pollack's differential topology. And I remember having frequent conversations with Indranil Biswas, who was then a Ph D student. This, last, has been exceptionally fruitful, both personally and mathematically. We are collaborators and friends. Berkeley, where I went for a Ph D, had an outstanding all-star cast: Bill Thurston, Andrew Casson (my advisor), Curt McMullen, John Stallings, Rob Kirby – all in and around geometry and topology. Grisha Perelman was a post-doc. So those were heady times for anyone wanting to work in the area. Geometry and topology deal with shapes: lines, planes and so on. While lengths and distances are important in geometry, they are not important in topology. So you are allowed to twist and turn and continuously deform shapes in topology, so long as you do not tear.

In one of your previous interviews you mentioned that mathematics is the 'music of reason'. Can you please elaborate?

I was quoting something I had heard. What I meant was that the language of mathematics is the most precise unambiguous language available to us: a crystallization of the faculty of reason common to all of us. But the body of mathematics that we build from this language is something more. It all holds together cogently, giving the subject its depth and abstract and sublime beauty. Just as individual notes are the language of music, but when put together in particular patterns, they give something more. The whole composition takes on greater meaning, depth and significance to the trained ear. In that sense, there are marked parallels between classical music and mathematics.

Is pure mathematics all about generating patterns and revolutionary insights or does it have transformative applications?

Generating patterns and getting insights is what any serious scientific research is about. Mathematics is no different there. What kicks in then is its relevance to a larger group of people. This is where its social relevance lies. Perhaps what distinguishes mathematics is that a serious application to other fields of science and technology takes a long time – perhaps several generations. It is only in exceptional situations that serious applications of a mathematician's work to other fields arise in his/her lifetime. Therefore, the joys of discovery and understanding have to remain, at a personal level, the principal motivation.

How would you visualize the landscape of mathematics and its real-world applications in the 21st century?

Mathematics as a subject is too vast an arena for me to make any sensible comment in this generality. On the other hand, the style of doing geometry that I belong to – an essentially visual one – with Thurston and Gromov as its primary exponents in recent times, is one that has come to stay as a school of thought. It consists in trying to 'live' within the space one is trying to understand. This today is a part of our common mathematical heritage – a part of the Thurston–Gromov legacy.

What would be your advice to students aiming to take pure mathematics as their research discipline?

Frankly, I would find it difficult to advise per se. On the other hand, to those who actually enjoy the figuring out of hidden abstract structures, I can only say that mathematical research is a deeply satisfying endeavour.

How easy or difficult is the simultaneous pursuit of the two contradicting fields, viz. science and religion?

First, I do not subscribe to any organized religion or theological dogmas. These contradict the very ethos of science and I do not believe that such dogmatism and the scientific spirit can really co-exist. So I cannot honestly comment on the question as phrased. On the other hand, science itself has a dimension that appeals to something that almost defines us as human beings. First, it is based on enquiry into the truth of things without coloration by personal prejudice. Second, its discoveries are a property of all. These can constitute a fundamentally human and universal dimension to the scientific endeavour.

In the context of Ramakrishna Mission, where I have received my training as a monk, there are these twin ideals of a personal quest for knowledge on the one hand and service to human beings on the other. So, when we are doing our research, it's the quest for knowledge. It's a personal thing. The other part is service – to be of relevance to a larger group of people. So when one teaches mathematics, it is the service part to a larger community; when one does research, it's the personal part. There are some stereotypes associated with this word called 'monk', which is what perhaps contributes to this perceived contradiction. For me personally, there isn't much of a contradiction, really.

Please take us through the literature which moulded you as a mathematician and a person that you are.

Mathematically, two sets of notes have left a permanent imprint. Thurston's 'Geometry and topology of 3-manifolds' and Gromov's 'Hyperbolic groups', and in terms of my personal life, Vivekananda's writings have influenced me a great deal.

Your thoughts on G. H. Hardy's view that mathematics is a 'young man's game'.

This is a highly over-rated comment. What remains true is the obvious: If one starts young, one has a headstart. But then what really matters is the determination to keep at it over extended periods of time.

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