# Resolutions of solar and atmospheric neutrino problems

## Solar neutrino problem

Experiments which studied beta decays of various nuclei in 1920s showed that the emitted electron has a continuous range of energies (Figure 1).

On the other hand, energy–momentum conservation predicted that the energy of the electron must be fixed and is equal to $(m_1 - m_2)c^2$, where $m_1$ is the mass of the decaying nucleus and $m_2$ is the mass of the daughter nucleus. This presented a severe crisis in physics because it seemed as if energy conservation is violated in beta decay. To preserve energy conservation, Pauli hypothesized that a neutral and very light particle is also emitted in beta decay along with the electron[1]. Based on Pauli's hypothesis, Fermi constructed his theory of beta decay which predicted electron spectra of all the known beta decays[2]. He also named Pauli's particle neutrino, meaning 'little neutral one'. The close agreement between theoretical prediction (Fermi) and experimental data led to the acceptance of the neutrino hypothesis, even though the neutrinos were not explicitly seen.

The interactions of the elementary particles are classified to be one of the three types: (a) strong, (b) electromagnetic and (c) weak. Weak interactions are mediated by very massive particles which make the overall interaction strength quite small, much smaller than the electromagnetic interaction strength at the same energy scale. Since neutrinos have only weak interactions, their interaction cross-section with matter is extremely small. An initial calculation showed it to be about $\sim 10^{-42}$ cm$^2$, which is 16 orders of magnitude smaller than cross-sections for electromagnetic processes such as Compton scattering measured at that time. Since the cross-section is so small, it was originally expected that neutrinos can never be detected. The construction of nuclear reactors during World War II provided intense neutrino sources which brought detection of neutrino interactions within the realm of possibility. Two different types of experiments were eventually constructed close to nuclear reactors. The first experiment, led by Cowan and Reines, looked for the reaction

$$\bar{\nu}_e + p \rightarrow e^+ + n,$$

where the experiment detected both the final state positron and the neutron[3]. The second experiment, led by the radiochemist Raymond Davis (Figure 2), looked for the nuclear reaction

$$^{37}\text{Cl} + \nu_e \rightarrow {}^{37}\text{Ar} + e^-. \quad (1)$$

The detection mechanism consisted of extracting the few $^{37}$Ar atoms from a large tank of chlorine. After about three years of data acquiring, the Cowan–Reines experiment observed an unambiguous signal for the reaction in eq. (1) in 1957 and provided experimental proof for the existence of the neutrino. However, the Davis experiment did not observe any signal[4]. The positive result of the Cowan–Reines experiment showed that nuclear reactors emit anti-neutrinos. Since the $^{37}$Ar producing reaction is driven by neutrinos, it was not surprising that the Davis experiment did not observe any signal. The results of these experiments also showed that the reactions of neutrinos and anti-neutrinos are quite different.

Undaunted by his setback, Davis set out to build a bigger and better detector which can detect the reaction in eq. (1). The question arises: what is an intense source of electron neutrinos? The answer: the sun. Nuclear fusion taking place in the sun emits so many neutrinos that a billion of them pass through our palms every second (Figure 3). Unfortunately, the energy of these neutrinos is too low for them to drive the reaction in eq. (1). John Bahcall, a solar physicist, provided a key input. The nuclear fusion in the sun, in addition to converting protons into helium nuclei, also produces higher mass nuclei in minute quantities. In particular, $^8$B nuclei are produced with a concentration of about $10^{-4}$, after a chain of four nuclear fusion reactions. $^8$B undergoes beta decay and produces neutrinos (not anti-neutrinos) with energy $E_\nu \leq 15$ MeV. At such energies, the reaction in eq. (1) can occur and $^{37}$Ar can be produced. Bahcall estimated the flux of neutrinos from $^8$B decay to be about $5 \times 10^6$ cm$^{-2}$ s$^{-1}$. Davis built the solar neutrino experiment in the Homestake mine, South Dakota with 600 tonnes of cleaning fluid, containing about $2 \times 10^{30}$ $^{37}$Cl atoms. The size of the detector was chosen such that one argon atom per day is produced in it. The results are quoted in terms of a unit specially defined for the experiment: solar neutrino unit (SNU), which corresponds to the production of one argon atom per $10^{36}$ target atoms per second. Bahcall's calculations predicted a rate of $8.5 \pm 0.9$ SNU for the Homestake experiment[5].

The Homestake experiment started taking data in 1964. Every month, the $^{37}$Ar atoms were collected from the detector and counted. Instead of 30 atoms expected based on Bahcall's calculations, only about 10 were actually observed[6]. The results indicated some good news and some bad news. The good news is that the idea of Davis for detecting neutrinos is fundamentally sound and the reaction in eq. (1) is indeed taking place.

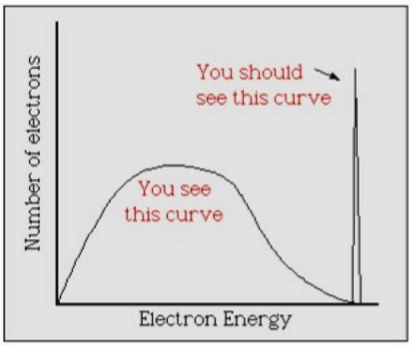


**Figure 1.** Observed and predicted spectra for beta decay [source: From a lecture prepared by Amol Dighe, Theory Group, TIFR].

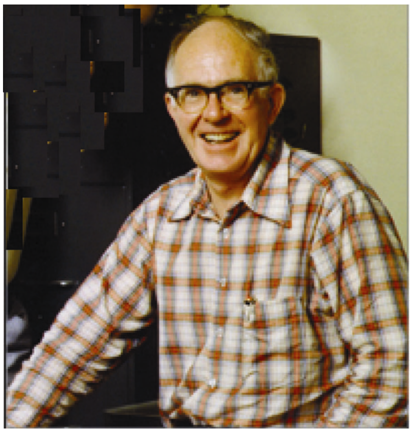


**Figure 2.** Raymond Davis [source: https://www.aip.org/history/acap/images/bios/davisr.jpg].

The bad news, of course, is that there is a factor of is three discrepancy between theoretical calculation[5] for the Homestake experiment and the experimental observation.

There is a famous saying of Niels Bohr that physics needs such discrepancies to make progress. A larger number of important discoveries were made when physicists were confronted with such situations. In this case, there are three possible resolutions to the discrepancy:

1. The theoretical calculation overestimated the production of neutrinos by a factor of three. The neutrinos driving eq. (1) come from $^8$B decay and the $^8$B nuclei in the sun are produced at the end of a chain of four reactions. Each of these reactions occurs only for those nuclei whose speeds are large enough to overcome the Coulomb repulsion between the nuclei. In the speed distribution of the particles in the sun, only a small fraction of particles has such high speeds. The net effect of this is that the concentration of $^8$B in the sun depends on the temperature of the core of the sun as $\sim T_c^{24}$. A small reduction in $T_c$ of about 5% is enough reduce the $^8$B flux by a factor of three and account for the experimental observation. Most particle physicists considered this to be the reason for the shortfall in the signal.

2. Something happened to the neutrinos as they travelled from the sun to the earth.

3. All the neutrinos reached the earth and produced 30 $^{37}$Ar atoms. However, the procedure for collecting the argon atoms is inefficient, it collected only ten atoms.

The third point was ruled out by calibrating the detector using an intense source of neutrinos. Since the radioactivity of the source is known, the rate of $^{37}$Ar production can be predicted precisely and it matched the observed collection rate. The first point was ruled out by the improvements in the modelling of the sun. Helioseismology, in particular, constrained the density profile of the sun and gave a precise estimate of the conditions in its core. A reduction in the temperature by 5% was not acceptable.

Thus physicists were forced to accept that some aspect of neutrino propagation between the sun and the earth is responsible for the shortfall of the $^{37}$Ar atoms in the Homestake experiment. In 1985, the Kamiokande experiment[7] also detected $^8$B neutrinos from the sun through elastic scattering off electrons in the detector. They observed only about 40% of the expected flux, thus providing an independent confirmation of the solar neutrino deficit.

Even before the start of the Homestake experiment, three Japanese physicists, Maki, Nakagawa and Sakata (MNS), proposed a mechanism in 1962, by which the neutrinos can be made to disappear as they propagate[8]. This is called neutrino oscillations. The neutrinos are of three types: those which occur in beta decay are electron (anti) neutrinos. In addition, there are two other types of neutrinos called muon neutrinos and tau neutrinos which interact respectively, with heavier cousins of the electron called muon and tau lepton. All of these are neutral and all are nearly massless. MNS realized that if these neutrinos have very light but non-zero masses, they can mix with one another. This mixing in turn can lead to an electron neutrino turning into a muon neutrino or a tau neutrino. The probability of oscillation is a sinusoidal function of the distance of travel (distance between the source and the detector) and hence the phenomenon is labelled neutrino oscillations.

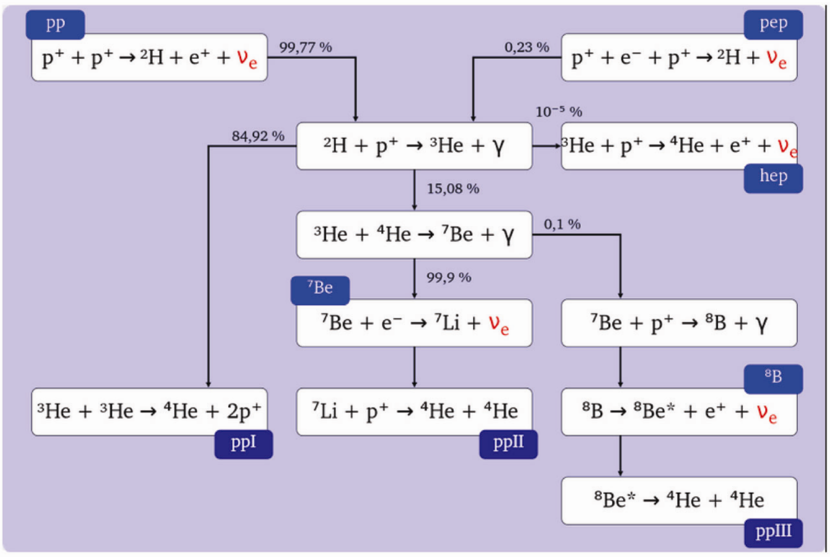


**Figure 3.** Fusion reactions taking place in the sun [source: https://en.wikipedia.org/wiki/Solar_neutrinos#/media/File:Proton_protom_cycle.svg].

Neutrino oscillations can be described mathematically in the following way. For simplicity, we assume that there are only two types (flavours) of neutrinos $\nu_e$ and $\nu_\mu$. They mix to form two mass eigenstates $\nu_1$ and $\nu_2$ with masses $m_1$ and $m_2$. The relation between these two sets of states is given by

$$\begin{bmatrix} \nu_e \\ \nu_\mu \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \nu_1 \\ \nu_2 \end{bmatrix},$$

where $\theta$ is called the mixing angle. Neutrinos are produced and detected in their flavour eigenstates (because what we see in the detector are the corresponding charged leptons, electrons or muons or their anti-particles). Between the source and the detector they travel as mass eigenstates with energies $E_i = \sqrt{p^2 + m_i^2}$. So a $\nu_e$ produced at $t = 0$ evolves in time in the following manner

$$\nu(0) = \nu_e = \cos\theta\nu_1 + \sin\theta\nu_2$$

$$\nu(t) = e^{-iE_1 t}\cos\theta\nu_1 + e^{-iE_2 t}\sin\theta\nu_2. \qquad (2)$$

We take the overlap of $\nu(t)$ with $\nu_\mu$ and square it to obtain $\nu_e \rightarrow \nu_\mu$ oscillation probability

$$P(\nu_e \rightarrow \nu_\mu) = \sin^2 2\theta \sin^2\left(1.27\frac{\Delta m_{21}^2 L}{4E}\right),$$

where $\Delta m_{21}^2 = m_2^2 - m_1^2$, $L$ is the distance of travel and $E$ is the neutrino energy.

Suppose an electron neutrino turned into a muon neutrino as it travelled from the sun to the earth. In principle, the muon neutrino can interact with $^{37}$Cl to produce a $^{37}$Ar. But it also needs to produce a muon which is 200 times more massive than the electron. The solar neutrinos simply do not have the energy to produce the muon; so the converted neutrino will simply pass through the detector, leaving no signal at all. The original MNS proposal was not widely accepted for two reasons:

1. At that time, the existence of tau lepton and tau neutrino was unknown. If the electron neutrino mixed only with the muon neutrino, then the largest suppression one could obtain is 1/2, rather than 1/3 observed in the Homestake experiment.
2. To obtain this 1/2 suppression, one needed to assume that mixing between the neutrino states is very large.

After the discovery of the tau lepton and the tau neutrino, it was possible to obtain 1/3 suppression of the electron neutrino flux from the sun, by considering the mixing of electron neutrinos with muon neutrinos and tau neutrinos. But the problem of large mixing angles still remained. And physicists were prejuidiced against large mixing of particles.

An important theoretical development in 1985 brought the solar neutrino problem to centre stage. Two Russian theorists, Mikheev and Smirnov, proposed a dynamical mechanism by which small mixing can be amplified into large mixing[9]. Particle physicists received this proposal with great enthusiasm and worked out various scenarios under which the electron neutrino shortfall observed by the Homestake and Kamiokande experiments could be explained. Data from two experiments, GALLEX[10] and SAGE[11], which could detect low-energy electron neutrinos from the sun, gave a further boost to this effort. These experiments also observed a shortfall of electron neutrinos from the sun, though the shortfall in their case is only about 1/2 rather than 1/3. This meant that, not only neutrinos are disappearing but the probability of disappearance is a function of energy of the neutrino. The analysis of these data led to two particular scenarios:

1. The mixing angle is small, which is enhanced dynamically as predicted by Mikheyev and Smirnov. As a consequence, at low energies the electron neutrino survives as such with unit probability, but at higher energies it oscillates into muon or tau neutrino with unit probability.
2. The mixing angle is large. The Mikheev–Smirnov mechanism is still operative and it induces a moderate change in the electron neutrino survival probability from 0.5 at low energies to 0.33 at high energies.

Now the question is, which of the two pictures is correct? More importantly, are we correct in assuming that the electron neutrino is oscillating into a muon neutrino or a tau neutrino?

The SNO experiment was designed to answer these questions. The leader of the experiment is Arthur B. McDonald (Figure 4) who is one of the Physics Nobel Prize awardees of 2015. The experiment consists of 1000 tonnes of heavy water in a transparent sphere which is surrounded by 9500 photomultiplier tubes. The ‘hydrogen’ in the heavy water contains deuterium (a weakly bound state of proton and neutron) as the nucleus. A neutrino interacting with a deuterium can induce one of two possible reactions

$$\nu_e + \mathrm{d} \rightarrow \mathrm{p} + \mathrm{p} + \mathrm{e}^- \quad \text{charged current (CC)}, \tag{3}$$

$$\nu_x + \mathrm{d} \rightarrow \mathrm{p} + \mathrm{n} + \nu_x \quad \text{neutral current (NC)}, \tag{4}$$

where $\nu_x$ stands for any of the three neutrino types. The energy threshold for both these reactions is a few mega electron volts, which meant that only the $^8$B neutrinos could drive these reactions. In the CC reaction, the energetic electron emits Cerenkov light as it passes through the heavy water, which is detected by the photomultiplier tubes. In the NC reaction, the neutron is absorbed by the neutron-absorbing nuclei in the detector. Post-absorption, the excited nucleus de-excites by emitting a gamma ray, which again is detected by the photomultiplier tubes. There are a number of other processes which try to mimic these signals. The SNO experimenters made heroic efforts to minimize these effects so that the signal events can be cleanly identified.

**Figure 4.** Arthur B. McDonald.

After a year of acquiring data, the results were first announced in June 2001 (ref. 12). The results of the CC reaction were in perfect agreement with those of the Homestake and Kamiokande experiments, showing that only one-third of the electron neutrinos are reaching the earth. The results of the NC reaction were in perfect agreement with the $^8$B flux calculation of Bahcall. This shows that the full complement of the electron neutrinos was produced in the $^8$B decays in the sun, but about two-thirds of them oscillated into muon/tau neutrinos during their journey out of the solar core. These converted neutrinos were incapable of driving the CC reaction but were perfectly capable of driving the NC reaction, which is why the measured NC reaction rate agreed with Bahcall’s predicted rate. In addition, the energy dependence of these rates conclusively established that the mixing angle is indeed large, forcing particle physicists to overcome their prejudice against such phenomena. For establishing the phenomenon of the oscillation of electron neutrinos from the sun into muon/tau neutrinos, the SNO experiment (Figure 5) and Arthur McDonald have been awarded the well-deserved Nobel Prize.

## Atmospheric neutrino problem

Fermi, whose theory of beta decay played a large role in the general acceptance of Pauli’s neutrino hypothesis, was known as ‘Il Papa’ (the Pope) among his students. Just as the Pope is supposed to be infallible in all matters religious, for Fermi’s students, he was infallible in all matters of physics. On one occasion, Fermi made the following pontifical statement: ‘Yesterday’s discovery is today’s calibration’, to which Richard

Feynman added, ‘It will become tomorrow’s background (nuisance)’. The case of atmospheric neutrinos is one where this statement is turned on its head. Here what was supposed to be a background turned out to be an important signal, which led to the Nobel Prize for the other awardee this year, Takaaki Kajita (Figure 6).

Cosmic rays from outer space, mostly protons, come and collide with the nuclei of atoms in the atmosphere. These collisions lead to copius production of pions which decay according to the following chain

$$\begin{aligned}\pi^{\pm} &\to \mu^{\pm} + \nu_{\mu}/\bar{\nu}_{\mu}, \mu^{\pm} \\ &\to \mathrm{e}^{\pm} + \nu_{\mathrm{e}}/\bar{\nu}_{\mathrm{e}} + \bar{\nu}_{\mu}/\nu_{\mu}. \qquad (5)\end{aligned}$$

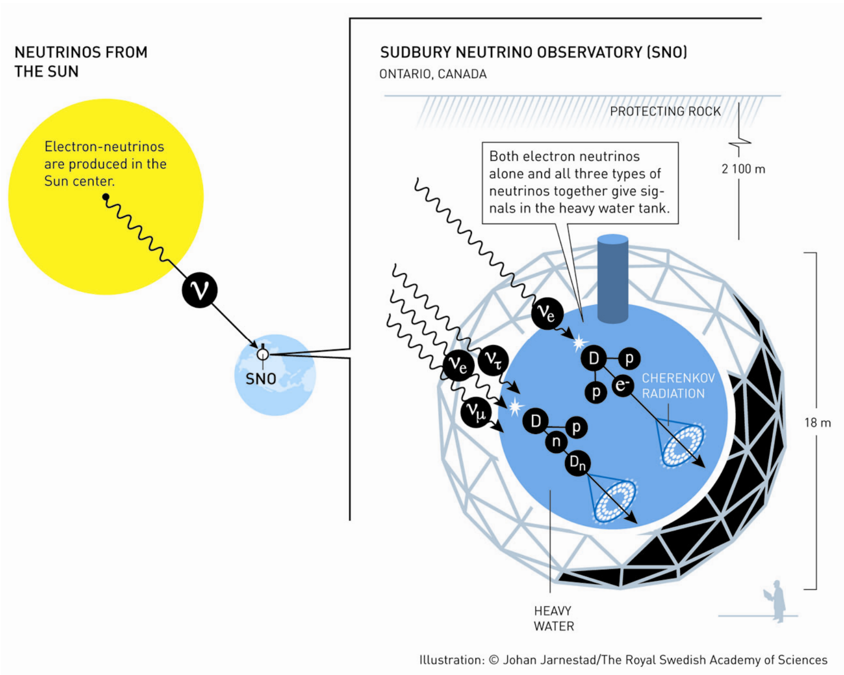
Copius numbers of muon neutrinos/anti-neutrinos and electron neutrinos/anti-neutrinos are produced due to the cosmic ray interactions in the atmosphere. The flux of these neutrinos is much smaller than that of the solar neutrinos, but these neutrinos are quite energetic with $E_{\nu} >$ 200 MeV. Because of the larger energy, their interaction cross-section is much larger and the neutrinos are energetic enough to produce a muon in the final state. Thus we can distinguish between the interaction of an electron neutrino versus a muon neutrino based on whether the final state contains an electron or a muon. However, most atmospheric neutrino detectors cannot determine the charge of the final state particles; hence they cannot distinguish between the interactions of neutrinos and anti-neutrinos. Due to this, in the case of atmospheric neutrinos, it is usual to club neutrinos and anti-neutrinos together and label them as neutrinos. By this nomenclature, the decays in eq. (5) indicate that for every electron neutrino produced, two muon neutrinos are produced in the atmosphere. This fact plays an important role in the further development.

Weinberg and Salam, based on the work of Glashow, constructed the electroweak model which unified electromagnetic and weak interactions, just as Maxwell's theory unified electric and magnetic interactions. The electroweak model is based on an important principle called the gauge principle. Later Politzer, Gross and Wilczek constructed a model for strong interactions, called quantum chromo-dynamics, which is also based on the gauge principle. Soon particle physicists became bold and using the same gauge principle constructed grand unified theories (GUTs) which unified strong, electromagnetic and weak forces. One problem with such unification is that it takes place at very high energies $E \sim 10^{16}$ GeV, which is a trillion times higher energy than the highest energy available to us at the most powerful accelerators. So the question arises: How to test these theories?

The proton is considered to be a stable particle. Experimentally there is no evidence for any decay of the proton. However, the symmetries in GUTs compel us to introduce interactions into the theory which lead to proton decay

$$\mathrm{p} \to \mathrm{e}^{+}\pi^{0}. \qquad (6)$$

Since these interactions take place at an energy scale of $10^{16}$ GeV, their effect at ordinary energies is strongly suppressed. Thus GUTs predict that the proton is not stable but it has a very long lifetime of about $10^{30}$ years. To test this hypothesis, one builds a detector with about $10^{32}$ protons and makes observations for a year to see if about 100 of them decay. To observe the decay, one needs to detect the decay products. To have $10^{32}$ protons, we need to have a detector with about a billion moles, which means a mass of about a few kilotonnes. The decay products $\mathrm{e}^{+}$ and $\pi^{0}$ will each have energy of about 500 MeV and they will be moving with relativistic speeds. Ultra-relativistic charged particles, moving through water, create Cerenkov light, which can be used to detect the positron. The $\pi^{0}$ promptly decays into two high-energy photons, each of which further splits into electron–positron pairs. These particles also emit Cerenkov light and can be detected. Thus, the detector design calls for a large tank of ultra pure water, which provides the necessary number of protons that can decay and



Electron-neutrinos are produced in the Sun center.

Both electron neutrinos alone and all three types of neutrinos together give signals in the heavy water tank.

Illustration: © Johan Jarnestad/The Royal Swedish Academy of Sciences

**Figure 5.** The SNO experiment.

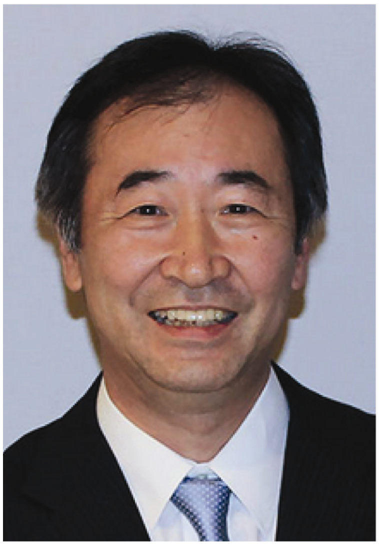


**Figure 6.** Takaaki Kajita.

also acts as a detecting medium. The water tank is surrounded by large photomultiplier tubes which detect the Cerenkov light emitted by charged particles.

Two such experiments were built: one in the US was called Irvine–Michigan–Brookhaven (IMB) collaboration and the other in Japan was called the Kamiokande collaboration. As is the case with all particle physics experiments, these experiments also needed to work out if there are random events which can mimic the proton decay signal. That meant worrying about reactions in the detector which can produce charged particles. The millions of neutrinos produced in the atmosphere pass straight through the earth and can enter the detector. Occasionally, one of these neutrinos interacts with a nucleus (either oxygen or hydrogen) and produces a charged particle. This charged particle is a muon if the neutrino is a muon neutrino and is an electron if the neutrino is an electron neutrino. Because the muon is 200 times more massive than the electron, it loses energy very slowly. The pattern of Cerenkov light emitted by the muon is quite different from that emitted by the electron. Thus it is possible to identify whether the interaction which produced the charged particle was due to a muon neutrino or an electron neutrino.

As the IMB and Kamiokande experiments started acquiring data, they accumulated a large number of such neutrino events. Eventually, they did not find any signal for proton decay and set a lower limit on proton lifetime of $>5 \times 10^{32}$ years. However, when they compared their neutrino event rates with the theoretical expectations, they found a curious result. The number of muon neutrino interactions was only about 66% of the expected number[13,14]. Here again, the uncertainty in the theoretical predictions led to the question: is the deficit really there? The experiments were able to overcome this obstacle by an ingenious method. The main source of uncertainty in the prediction of neutrino interaction rate is the uncertainty in the cosmic ray fluxes. This uncertainty affects both the muon neutrino interaction rate and the electron neutrino interaction rate in the same way. If one takes a ratio of these two rates, then the largest source of uncertainty cancels out and the ratio can be predicted quite accurately. The observed ratio was found to be about 0.7 of the predicted ratio. Thus the so-called background atmospheric neutrino events of the proton decay experiments provided an unexpected new puzzle which came to known as the atmospheric neutrino problem.

Given that observed $\mu$/e ratio is smaller than the predicted one, we are again faced with three choices:

1. $\mu$ rate is smaller and e rate is equal to the prediction,
2. $\mu$ rate is smaller and e rate is larger, and
3. $\mu$ rate is equal to the prediction and e rate is larger.

To resolve which possibility is correct, a similar experiment, but five times larger in scale, was built. This is the super-Kamiokande experiment (Figure 7) which conclusively proved that the muon neutrinos produced in the atmosphere are oscillating into tau nuetrinos.

The super-Kamiokande experiment used the same principles as Kamiokande to detect the interactions of atmospheric electron and muon neutrinos. Because of the larger size, the rate of interactions is much more. More importantly, the photomultiplier tubes in the super-Kamiokande were specially designed with exceptional light sensitivity. Because of them, the super-Kamiokande experiment was especially efficient in reconstructing the cone of Cerenkov light emitted by the electrons and muons. This better reconstruction led to a precise determination of energy and the direction of these charged particles. The larger number of events and the measurement of the energy and direction of the charged particles meant that the super-Kamiokande could study the rate of interactions as a function of energy as well as direction.

At higher energies, there is a good correlation between the direction of the neutrino and the direction of the charged particle it produces. Consider an electron or a muon going in the downward direction in the detector. The neutrino which produced it must have come from above, which means that the neutrino has travelled a distance of a few kilometres, which is the thickness of the atmosphere. Now consider an electron or a muon going in the upward direction in the detector. The corresponding neutrino must have come from below the detector, which means that it was produced on the other side of the earth and it travelled for thousands of kilometres before it entered the detector and interacted with a nucleus in it. Thus we have this picture of neutrinos with energies in the range 200 MeV–10 GeV, travelling distances varying from 10 to 13,000 km, entering the detector and producing interactions (Figure 8). Depending on their energy and distance of travel, different neutrinos oscillate with different probabilities.

With three flavours $\nu_e$, $\nu_\mu$ and $\nu_\tau$, we assume that they mix to form three mass eigenstates $\nu_1$, $\nu_2$ and $\nu_3$, with masses $m_1$, $m_2$ and $m_3$. The flavour and mass

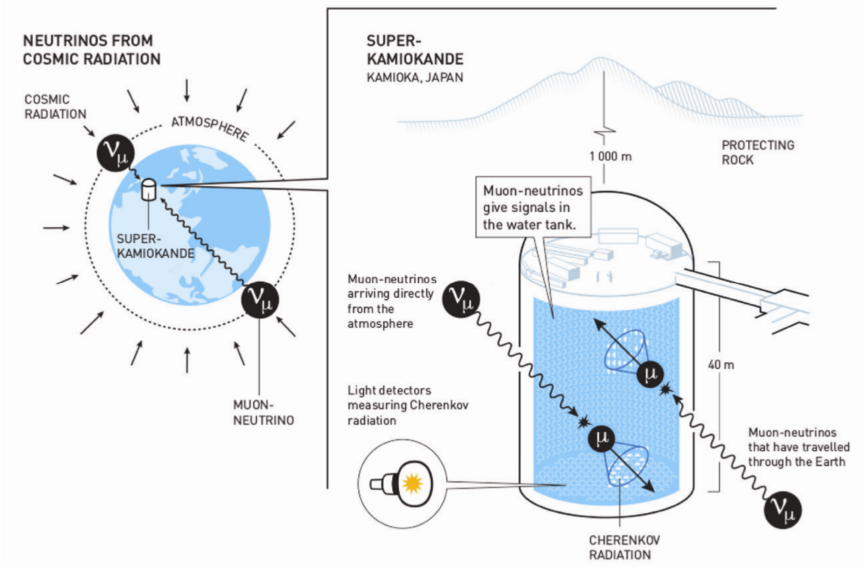


**Figure 7.** Super-Kamiokande experiment.

eigenstates are related to each other by the unitary transformation

$$\begin{bmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{bmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{bmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{bmatrix}$$

$$= R_{23}(\theta_{23}) U_{13}(\theta_{13}, \delta_{CP}) R_{12}(\theta_{12}) \begin{bmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{bmatrix}, \quad (7)$$

where we note that the $3 \times 3$ mixing matrix is parametrized by three angles and one phase. Using the same time evolution that was used in eq. (2), we can calculate the probabilities

$$P(\nu_e \to \nu_e) = 1 - \sin^2 2\theta_{13} \times \sin^2\left(1.27 \frac{\Delta m_{31}^2 L}{E}\right),$$

$$P(\nu_\mu \to \nu_\mu) = 1 - \sin^2 2\theta_{23} \times \sin^2\left(1.27 \frac{\Delta m_{31}^2 L}{E}\right), \quad (8)$$

where $\Delta m_{31}^2 = m_3^2 - m_1^2$. In deriving the above neutrino survival probabilities, we have made an approximation. With three masses, there are two independent mass-squared differences: $\Delta m_{31}^2$ and $\Delta m_{21}^2 = m_2^2 - m_1^2$. We have assumed that $\Delta m_{21}^2 \ll \Delta m_{31}^2$. Neutrino oscillations are the solution to the solar neutrino problem. Fitting the oscillation formula to the solar neutrino data gives us the constraint $\Delta m_{21}^2 \simeq 10^{-4}$ eV$^2$. Given the energy scale of atmospheric neutrinos, the oscillation probability is very small for such a value of $\Delta m_{21}^2$. Therefore, while analysing the atmospheric neutrino data, $\Delta m_{21}^2$ is neglected. In this approximation, we get eq. (8) for the survival probabilities of electron and muon neutrinos. In 1997, reactor experiment called CHOOZ provided an upper limit[15,16]

$$\sin^2 2\theta_{13} \leq 0.1.$$

Substituting it in eq. (8), we find that electron neutrino survival probability is $\geq 0.9$. That is, there is only a small probability for the oscillation of atmospheric electron neutrinos.

The super-Kamiokande experiment did a detailed study of these interactions as a function of energy and direction. It was found that electron neutrino interactions matched the expectations for all energies and directions, which is expected based on the CHOOZ constraint. On the other hand, muon neutrino interaction rate was smaller, by a factor of two, for the up-going direction and slowly increased as a function of direction and was closer to expectation for the downward direction. For interactions with $E_\nu > 1$ GeV, the event rate in the downward direction is the same as the expectation. For neutrino energies in the range 200 MeV–1 GeV, there is some suppression of event rate in the downward direction[17]. Figure 9 provides a summary of the super-Kamiokande experiment.

These features of the super-Kamiokande data can be easily understood

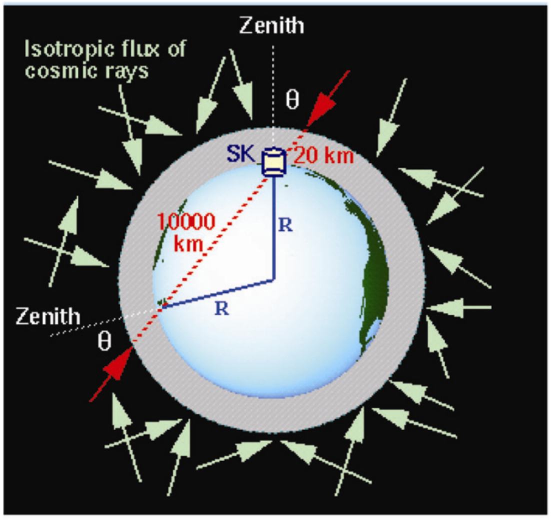


**Figure 8.** Neutrinos travelling through earth [source: http://hep.bu.edu/superk/atmnu].

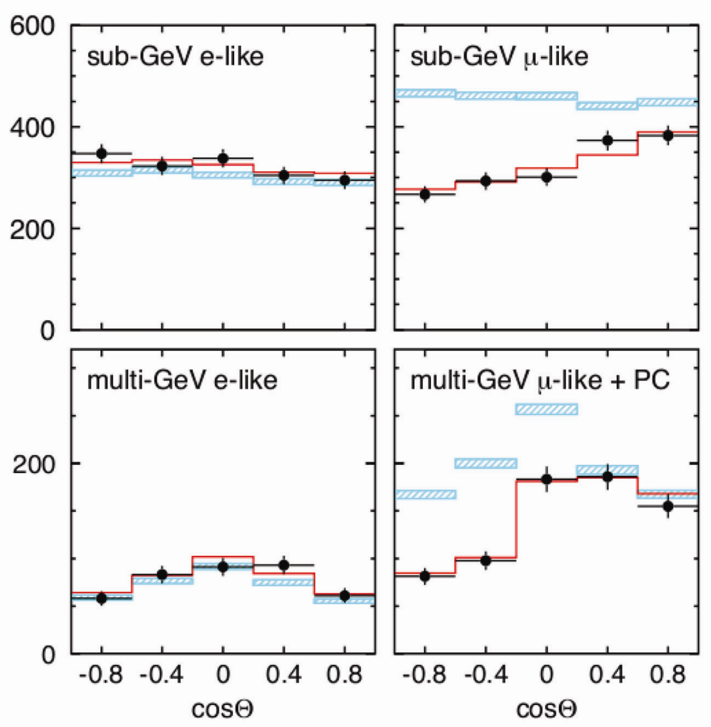


**Figure 9.** Super-Kamiokande results: the shaded bars represent the expectation in the case of no oscillations and the dark circles represent the data points. The solid line is fit assuming neutrino oscillations with $\Delta m_{31}^2 = 3 \times 10^{-3}$ eV$^2$ and $\sin^2 2\theta_{23} = 1$.

based on the expression for the muon neutrino survival probability in eq. (8). For $\Delta m_{31}^2 \simeq 3 \times 10^{-3}$ eV$^2$, we have $\Delta m_{31}^2 L/E \sim \pi/2$ for $L$ of a few thousand kilometres and $E$ of a few giga electron volts. Hence there is a large oscillation probability for upward-going neutrinos if $\sin^2 2\theta_{23}$ is large. But for smaller values of $L$ and $E > 1$ GeV, $\Delta m_{31}^2 L/E$ becomes quite small and the muon neutrino survival probability ~1. These features can be observed in the lower-right panel of Figure 9. For smaller neutrinos energies $\Delta m_{31}^2 L/E$ is of the order of a few radians for $L$ of a few thousand kilometres. Then the term $\sin^2(1.27\,\Delta m_{31}^2 L/E)$ oscillates rapidly for small changes in $L$ and $E$, and produces an average value of 1/2. Thus we expect the rate of upgoing muon events to be about half of the predicted rate for low energies. For downgoing muon events, which have $L$ of a few hundred kilometres, $\sin^2(1.27\,\Delta m_{31}^2 L/E)$ is smaller and hence their suppression factor is smaller, but is non-zero. These two features can be seen in the lower-left panel of Figure 9.

How did the super-Kamiokande experiment overcome the handicap of the large uncertainty in the prediction of cosmic ray flux, which in turn affects the prediction of the atmospheric neutrino flux? In the experiment, the cosmic ray flux was a free parameter in the predictions and the shape of the functional form of variation with direction was then obtained. This shape could be predicted with much greater accuracy and was compared to the observed shape. The predicted and observed shapes matched only under the assumption of muon neutrinos oscillating into tau neutrinos. Thus the super-Kamiokande experiment conclusively settled the atmospheric neutrino problem earning a well-deserved Nobel Prize to the leader of its atmospheric neutrino group, Takaaki Kajita.

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