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Improvement of Fractal Dimension to Valuation of Eutrophication based on Correlation Analysis

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Abstract: In this paper, Correlation analysis is used to analyze Fractal dimension to valuation of eutrophication. The results are showed that correlation dimension of five index with the corresponding index are the same direction. In the fact, correlation dimension of four indexes with the corresponding index are the same direction and correlation dimension of one index with the corresponding index is the different direction. The diversification of the different direction will cause an adverse change in the evaluation results and is the opposite of the same direction. So we provide a method for pre-processing of data that is normalization method and conducted a correlation analysis and verification of accuracy. The results are showed that correlation dimension of five indexes with the corresponding index are the same direction, and that the correlation between correlation dimension and the corresponding index is very high, and that the accuracy of evaluation results is very high.

Keywords: *Eutrophication, Correlation Dimension, Correlation Analysis, Normalization Method, Fractal*

1. Introduction

As we all know: because of the rapid economic and social development, various types of environmental problems will become increasingly prominent, and become the constraints of economic and social development. To carry out water quality assessment is based on certain standards and methods to give a qualitative and quantitative description, and indicated the water quality of a region and the development of water quality change law, and find out the main problems in water quality evaluation area. And it can provide theoretical support for the control of water pollution and water function zoning, water environment, water environment planning management.

At this stage, mathematical methods to carry out water quality evaluation are relatively common, such as single factor evaluation method and comprehensive index evaluation method [1,2], of which only a comprehensive index evaluation method has reached more than 10 kinds, mainly include: fuzzy comprehensive evaluation method [3], grey evaluation [4], neural network evaluation [5,6], genetic algorithm [7-9], nonlinear evaluation [10].

2. Fractal dimension calculation and its correlation analysis

2.1. Fractal

Fractal theory is an active branch of nonlinear scientific research, and it is the fractal geometry as a mathematical basis, and it research and reveals the hidden complex natural and social phenomena of regularity, level and scale invariance. Mander Pabo, founder of fractal pointed out that the fractal consists of three elements: shape, opportunity, dimension [10-14]. Dimension is the basic parameter to describe the fractal quantitatively. It is the invariant under scale transformation, which is scale invariance.

2.2. Fractal Algorithm

There are many methods to calculate the fractal dimension, such as the box dimension, the similar dimension, the information dimension and the correlation dimension. Using the correlation fractal dimension algorithm [15-26] to evaluate the water quality mainly has the following steps:

(1)Select the appropriate evaluation criteria according to the water quality indicators;

(2)Constructing and determining the correlation dimension of the matrix using the evaluation criteria as a matrix;

(3)Take the correlation dimension of the evaluation standard as the evaluation weight value, and carry out the comprehensive evaluation of water quality indicators, and determine the water quality evaluation grade of each monitoring point.

1) Evaluation Standard

The evaluation criteria are mainly determined by combining the samples of water quality indexes with the evaluation objects. When eutrophication evaluation is carried out, eutrophication evaluation standard can be adopted. When carrying out surface water quality evaluation, it can be determined by referring to the standard value of surface water. We can use the following matrix to perform operations:

$$X = \begin{vmatrix} X_{1,1} & X_{1,2} & \cdots & X_{1,n} \\ X_{2,1} & X_{2,2} & \cdots & X_{2,n} \\ \cdots & \cdots & \cdots & \cdots \\ X_{m,1} & X_{m,2} & \cdots & X_{m,n} \end{vmatrix} \quad (1)$$

Where: m is the number of evaluation indicators, n is the number of evaluation criteria grading.

2) Correlation Dimension

(1) 1 to 7 dimensional phase space is established, and each phase space is as follows:

$$X_1 = \begin{vmatrix} X_{i,1} \\ X_{i,2} \\ \cdots \\ X_{i,n} \end{vmatrix}; X_2 = \begin{vmatrix} X_{i,1} & X_{i,2} \\ X_{i,2} & X_{i,3} \\ \cdots & \cdots \\ X_{i,n-1} & X_{i,n} \end{vmatrix}; \ldots;$$

$$X_7 = \begin{vmatrix} X_{i,1} & X_{i,2} & \cdots & X_{i,7} \\ X_{i,2} & X_{i,3} & \cdots & X_{i,8} \\ \cdots & \cdots & \cdots & \cdots \\ X_{i,n-6} & X_{i,n-5} & \cdots & X_{i,n} \end{vmatrix} \quad (2)$$

Where: i=1,2,…,m.

(2) The distance $r_{p,q}(s)$ between two points in the 1 to 7-dimensional phase space is calculated, and the average distance Δx_s is calculated as follows:

$$r_{p,q}(s) = \sqrt{\sum_{l=1}^{s} (x_{p,l} - x_{q,l})^2} \quad (3)$$

$$\Delta x_s = \sum_{p=1}^{n-s+1} \sum_{q=1}^{n-s-1} \frac{r_{p,q}(s)}{(n-s+1)^2} \quad (4)$$

Where: p,q=1,2,…,n-s+1;s=1,2,…,w(w is the phase space dimension).

(3) The probability $C_k(s)$ of the distance between two points in the 1 to 7 dimensional phase space is calculated respectively, and the calculation is based on the $r_{s,k}$:

$$C_k(s) = \frac{1}{(n-s+1)^2} \sum_p \sum_q H(r_{s,k} - r_{p,q}(s)) \quad (5)$$

Where: k is the phase space of the double dimension; $r_{s,k} = \frac{k}{10} \Delta x_s, (k = 1,2,\cdots,14)$;

$$H(r_{s,k} - r_{p,q}(s) = \begin{cases} 0, \text{当} r_{s,k} < r_{p,q} \\ 1, \text{当} r_{s,k} \geq r_{p,q} \end{cases}$$

(4) The dimension of each dimension:

$$D_s = \lim_{r_{s,k} \to 0} \frac{\ln(C_k(s))}{\ln r_{s,k}} \quad (6)$$

(3) Water Quality Assessment

By using the fractal dimension value obtained by the formula 6 as the corresponding weight value of the index and the evaluation standard value, we can get the corresponding evaluation criteria, and the specific calculation is as follows:

$$\begin{vmatrix} D_1 \times x_{1,1} + D_2 \times x_{1,2} + \cdots + D_7 \times x_{1,7} \\ D_1 \times x_{2,1} + D_2 \times x_{2,2} + \cdots + D_7 \times x_{2,7} \\ \cdots \\ D_1 \times x_{m,1} + D_2 \times x_{m,2} + \cdots + D_7 \times x_{m,7} \end{vmatrix} \quad (7)$$

In formula 7, each line is the standard value of the corresponding level, and the corresponding water quality evaluation level can be obtained by comparing the index value of each monitoring point with the corresponding phase space dimension.

2.3. Correlation Analysis:

After a model is established, its rationality and applicability should be analyzed, and the rationality of the model is the most commonly used analysis method. The correlation analysis is whether there is a certain dependency relation between research and the specific phenomenon, there is the phenomenon of dependency relation to investigate the direction and degree of correlation, and is a statistical method to study the relationship between the random variables. The correlation analysis mainly includes three aspects: the related direction, the related form, the related degree. Among them, the related direction is the increase or decrease of the increase or decrease of the variables, the related form is linear or nonlinear; the related degree is closely related to the degree of change.

Here, the correlation calculation process can be found: the related direction associated with each indicator, need to be discussed in detail; the related form is non-linear; the related degree can be discussed with the correlation coefficient, the existing calculation does not need to discuss.

In order to facilitate the comparative analysis, this paper uses the results of relevant literature [16-19] to carry out the relevant analysis; the specific dimensions are shown as table.1:

The correlation dimension of each index and the related direction table shows that: Chla, TP, TN, COD, SD should be related to the evaluation results, that is, the value of the 5 indexes increased and the evaluation results should be increased. However, the literature [14] Table 2 shows: if the SD value increases, the evaluation of the results should be good (the value should be low), it is SD and evaluation results should show a reverse correlation. The literature [21] sampling point SD J12 value is changed in order to observe the changes in the evaluation results, to make the results obvious SD value of each increase of 2 units, the specific calculation are shown as table.2.

From the evaluation results in the table above, it can be seen that the evaluation results of the SD value increase with the increase of the SD value, it is the evaluation results show that the water quality of the evaluation object should be deteriorated. But in reality, the increase of SD value is a good performance of water quality, so the evaluation effect produced by SD algorithm is inconsistent with the real situation; the reason is the direction correlation of the indicator has a problem.

Table 1: *Correlation Dimensions and the Related Directions*

Project	Chla	TP	TN	COD	SD
Correlation Dimensions	0.328555	1.165475	1.000183	1.592868	1.1833827
Related Directions	the same	the same	the same	the same	the same

Table 2: *The Analysis of the Related Direction of SD*

Project	*Chla(mg/m^3)*	*TP(mg/m^3)*	*TN(mg/m^3)*	*COD(mg/m^3)*	*SD(m)*	*Result*
1	*65.49*	*181*	*18380*	*110.2*	*0.37*	*18791.80349*
1-1	*65.49*	*181*	*18380*	*110.2*	*2.37*	*18794.17025*
1-2	*65.49*	*181*	*18380*	*110.2*	*4.37*	*18796.53702*
1-3	*65.49*	*181*	*18380*	*110.2*	*6.37*	*18798.90378*

3. Algorithm Improvement and Application

From the above analysis, we can find out that the main problem of the correlation algorithm in the related literature is that the participation in the algorithm is not completely consistent with the direction dependence of the data, so the improved algorithm must take into account the direction correlation. In order to obtain the correlation dimension and the direction of the evaluation results, we must deal with the relevant data before it is calculated.

In data processing, normalization method is the most common method of data preprocessing. Therefore, the data pretreatment on each index of the algorithm is carried out using normalization method.

In this paper, the data are preprocessed according to the following rules [15, 21]:

1) If an indicator of formula 1 increases its water quality deterioration, we use $X_i = \frac{x_i - x_{\min}}{x_{\max} - x_{\min}}$ to normalize, where: $x_{\min}$ is the minimum value of the index, $x_{\max}$ is the maximum of the index;

2) If an indicator of equation 1 increases its water quality becomes better, we use of $X_i = \frac{x_i - x_{\min}}{x_{\max} - x_{\min}}$ to normalize, of which: $x_{\min}$ is the minimum index, $x_{\max}$ is the maximum of the index.

The above criteria were used to pre-treat the eutrophication evaluation standard. The results are are shown as table.3:

Table 3: *SNV Achievement of Eutrophication Assessment*

Evaluation Standard	**Chla**	**TP**	**TN**	**COD**	**SD**
Oligotropher	*0*	*0*	*0*	*0*	*0*
Mesotropher	*0.006289*	*0.0041841*	*0.003350084*	*0.00404858*	*0.51546*
Eurptropher	*0.018868*	*0.0376569*	*0.045226131*	*0.06882591*	*0.87629*
Light Eurptropher	*0.056604*	*0.0794979*	*0.078726968*	*0.14979757*	*0.92784*
Middle Eurptropher	*0.402516*	*0.3305439*	*0.32998325*	*0.39271255*	*0.98969*
Hyper Eurptropher	*1*	*1*	*1*	*1*	*1*

After we used the eutrophication evaluation criteria to carry out the relevant calculation, the correlation dimension of each variable is 0.423883646, 0.335033817, 0.2596606, 0.149606893 and 0.505624144

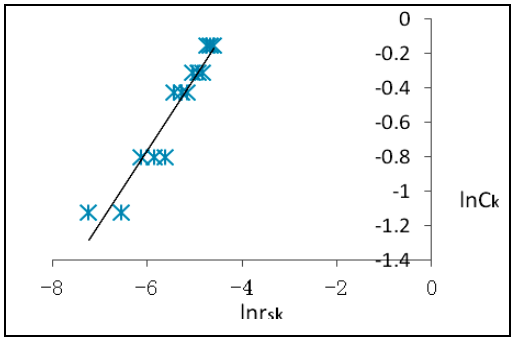

Figure 1: Line fitting of Chla

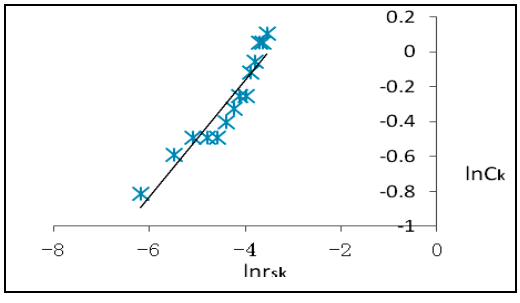

Figure 2: Line fitting of TP

4. Analysis and Discussion

In order to fully analyze and discuss the reasonableness and the scientific nature of the algorithm, the following analysis and discussion are carried out according to the relevance of the algorithm and the rationality of the results.

4.1. Correlation Analysis

Relevance analysis should be focused on three aspects: the related direction, the related form, the related degree.

1) The related Form

From the calculation process we can find out: C_k and r_{sk} is the linear correlation after double logarithmic, as shown below:

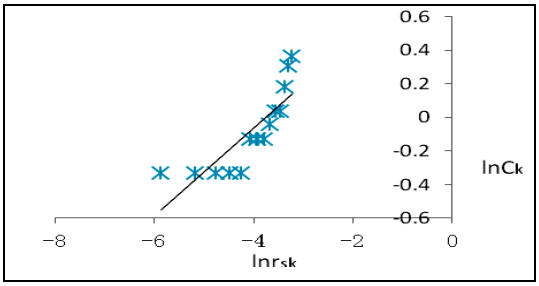

Figure 3: Line fitting of TN

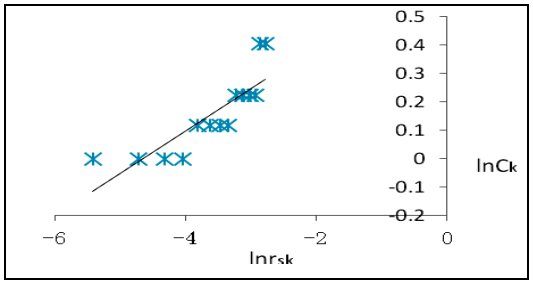

Figure 4: Line fitting of COD

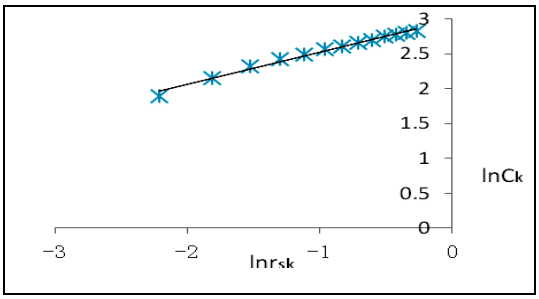

Figure 5: Line fitting of SD

The correlation coefficients of Chla, TP, TN, COD and SD were 0.960541854, 0.9445778399, 0.830097712, 0.8424406 and 0.991552798. The correlation degree of Chla, TP, TN, COD and SD was very good.

3) The Related Direction

In order to facilitate the comparative analysis, we used the parameter of the relevant literature [21] to carry out the relevant data analysis. The correlation dimensions are shown as table.4.

The results of the correlation analysis is shown that the normalized values of Chla, TP, TN, COD, and SD should be related to the evaluation results, it is the values of the five indexes should be increased. The relationship between Chla (mg/m^3), TP (mg/m^3), TN (mg/m^3), COD(mg/m^3), SD (m) was analyzed using a set of data [21] and they was as the main control project and used the interval of 20 units, 20 units, 2000 units, 20 units, 2 units, the specific calculation results are shown as table.5.

From the evaluation results in the table 5, it can be seen that the normalized values of any of the master projects (Chla, TP, TN, COD, SD) are increased and the evaluation results are inevitably increased. And the dimensionality of Chla, TP, TN, COD is in the same direction, which is consistent with the related direction of the correlation dimension obtained in Table 4. The correlation analysis is shown that the algorithm can meet the requirements of the related direction, the related form, the related degree, and can meet the requirements of calculation and analysis in the correlation analysis.

2) The Related Degree

Table 4: *Correlation Dimensions and Related Directions*

Project	*Chla*	*TP*	*TN*	*COD*	*SD*
Correlation Dimension	*0.423883646*	*0.335033817*	*0.2596606*	*0.149606893*	*0.505624144*
The related direction	The same	the same	the same	the same	the same

Table 5*: The analysis of the related direction*

	Chla		*TP*		*TN*		*COD*		*SD*		
Project	*Original value*	Normalized value	*Original value*	Normalized value	*Original value*	Normalized value	*Original value*	Normalized value	*Original value*	Normalized value	Evaluation results
	85.49	0.4056	181	0.2987	18380	3.0737	110.2	4.4494	0.37	0.9928	2.2911
Chla	105.49	0.5314	181	0.2987	18380	3.0737	110.2	4.4494	0.37	0.9928	2.3444
	125.49	0.6572	181	0.2987	18380	3.0737	110.2	4.4494	0.37	0.9928	2.3977
	65.49	0.0515	201	0.3322	18380	3.0737	110.2	4.4494	0.37	0.9928	2.249
TP	65.49	0.0515	221	0.3657	18380	3.0737	110.2	4.4494	0.37	0.9928	2.2602
	65.49	0.0515	241	0.3992	18380	3.0737	110.2	4.4494	0.37	0.9928	2.2714
	65.49	0.0515	181	0.2987	20380	3.4087	110.2	4.4494	0.37	0.9928	2.3248
TN	65.49	0.0515	181	0.2987	22380	3.7437	110.2	4.4494	0.37	0.9928	2.4117
	65.49	0.0515	181	0.2987	24380	4.0787	110.2	4.4494	0.37	0.9928	2.4987
	65.49	0.0515	181	0.2987	18380	3.0737	120.2	4.8543	0.37	0.9928	2.2983
COD	65.49	0.0515	181	0.2987	18380	3.0737	140.2	5.6639	0.37	0.9928	2.4195
	65.49	0.0515	181	0.2987	18380	3.0737	160.2	6.4737	0.37	0.9928	2.5406
	65.49	0.0515	181	0.2987	18380	3.0737	110.2	4.4494	2.37	0.7866	2.1335
SD	65.49	0.0515	181	0.2987	18380	3.0737	110.2	4.4494	4.37	0.5805	2.0293
	65.49	0.0515	181	0.2987	18380	3.0737	110.2	4.4494	6.37	0.3742	1.925

4.2. Accuracy Analysis

The evaluation criteria of eutrophication were as follows: (Oligotropher, Mesotropher, Eurptropher, Light Eurptropher, Middle Eurptropher and Hyper Eurptropher)=(0,0.527553644,0.957138026,1.085019633, 1.544501171, 2.526365808).

In this paper, the original data in [21] is used to carry out the calculation and the results are used as the basis for the accuracy analysis. The results are shown as table.6. It can be seen from the table 6 that the eutrophication evaluation was carried out using the correlation dimension after the normalization method, and only one of the evaluation results was inconsistent with the results of the comprehensive nutritional status index method, and the accuracy of the evaluation results was high. The main reasons for the inconsistency between the results of the 20th evaluation and the comprehensive nutritional status index method may be that the comprehensive nutritional status index evaluation is not completely determined by the relevant calculation results, and the range value of its evaluation is adjusted on the basis of calculation. It is possible that the correlation coefficient of the correlation dimension does not reach the value of 1, which is due to the partial error range.

Table 6: *The Analysis of Consistency*

Project	Chla	TP	TN	COD	SD	Correlation Dimension		Comprehensive Index		Consistency
						results	Rating	results	Rating	
1	65.49	181	18380	110.2	0.37	2.2378	Hyper Eurptropher	85.9	Hyper Eurptropher	Y
2	47.5	179	13750	108.4	0.52	1.9686	Hyper Eurptropher	82.8	Hyper Eurptropher	Y
3	150.65	366	21570	113.3	0.25	2.7323	Hyper Eurptropher	92.5	Hyper Eurptropher	Y
4	106.43	453	24610	117.6	0.23	2.8225	Hyper Eurptropher	93.0	Hyper Eurptropher	Y
5	159.62	333	17660	96.9	0.19	2.4714	Hyper Eurptropher	92.0	Hyper Eurptropher	Y
6	188.62	325	20990	108.6	0.26	2.7563	Hyper Eurptropher	92.4	Hyper Eurptropher	Y
7	97.26	189	10857	102.6	0.43	1.9505	Hyper Eurptropher	84.7	Hyper Eurptropher	Y
8	84.65	303	20990	98.2	0.33	2.4001	Hyper Eurptropher	88.5	Hyper Eurptropher	Y
9	170.52	338	27940	116.1	0.28	3.0620	Hyper Eurptropher	93.1	Hyper Eurptropher	Y
10	7.857	85.5	1810	76	0.83	1.0787	Eurptropher	65.8	Eurptropher	Y
11	69.39	250	26060	93.5	0.32	2.5223	Hyper Eurptropher	87.9	Hyper Eurptropher	Y
12	13.381	84.5	723	85.8	1.46	1.0721	Eurptropher	63.1	Eurptropher	Y
13	17.87	54.6	1737	83.2	0.93	1.1233	Eurptropher	66.7	Eurptropher	Y
14	1.413	80.1	2895	83.1	0.76	1.1523	Eurptropher	62.8	Eurptropher	Y
15	6.142	79.6	1737	84.9	0.9	1.1179	Eurptropher	65.0	Eurptropher	Y
16	5.876	74.8	869	80.8	1	1.0467	Eurptropher	62.0	Eurptropher	Y
17	59.6	49.7	1810	88.8	1.45	1.2418	Eurptropher	68.8	Eurptropher	Y
18	5.845	64.6	1448	81.7	1.76	1.0319	Eurptropher	61.1	Eurptropher	Y
19	3.338	83.2	2171	73.4	0.91	1.0612	Eurptropher	63.3	Eurptropher	Y
20	1.75	75.3	1810	91.3	1.57	1.1108	Eurptropher	59.7	Middle Eurptropher	N
21	6.075	72.6	2171	89.2	1.07	1.1499	Eurptropher	65.0	Eurptropher	Y
22	6.551	84.1	1810	95.8	1.75	1.1464	Eurptropher	63.7	Eurptropher	Y

From the above analysis we can find out: we can use normalization method to carry out the correlation dimension of the relevant evaluation, and the result of the evaluation has high accuracy and reliability.

5. Conclusion

The correlation dimension of fractal algorithm has a wide range of applicability, which is why the algorithm is used more and more widely. However, in the process of water environment or eutrophication evaluation, the correlation index has some imperfection in the direction correlation. In this paper, it is proposed that the evaluation indexes should be treated as mathematic normalization before the relevant calculation is carried out. Through the correlation analysis and verifying, it is show that the correlation dimension of normalization has a high correlation and the consistency of the direction of relevance. The improved algorithm can carry out the relevant aspects of analysis and research after we used the existing data of the relevant literature to verify the improved algorithm with high accuracy.

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