



Analysis of Lining Water Pressure of High-pressure Water-rich Karst Tunnel Based on Fluid-structure Interaction

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Abstract: To ensure the normal operation of the mountain tunnel with large burial depth and high water head, the water pressure of the tunnel lining should be accurately determined. The two-dimensional seepage around the tunnel meets Laplace equation, and the conformal variation theory is adopted to convert the free water surface of the underground water of the mountain tunnel and the periphery of the grouting circle into the circular ring with inner diameter of a and outer diameter of l in w plane. The universal solution of Laplace equation is obtained in the conformal variation circle area, and the analytical solution of the water pressure of the tunnel lining with steady seepage is deduced according to boundary condition and seepage continuity. Meanwhile, the analytical solution formula for the water pressure of the lining and the calculation software based on FLAC^{2d} finite difference method are adopted to calculate the analytical solution and the numerical solution of the water pressure of Ying-shan-yan tunnel lining of Chengdu-Guiyang Passenger Transport Line. The comparison between above two solutions shows that the analytical solution has higher practicability and reference value for calculating the water pressure of the high-pressure water-rich karst tunnel lining.

Keywords: High-pressure Water-Rich Karst Tunnel; Conformal Conversion; Water Pressure of Lining; Fluid-structure Interaction

1. Introduction

Along with the rapid development of transportation infrastructure construction, more and more mountain tunnels are constructed in karst area. The high-pressure water-rich karst tunnels can not only bring great difficulty to tunnel construction, but also influence tunnel operation. In the karst tunnel in operation, the tunnel lining occasionally suffers from structural cracking, deformation, damage, etc. due to the action of the karstic water[1][2][3]. In order to ensure safe and normal tunnel operation, we need to accurately determine the water pressure of the tunnel lining and the distribution characteristic thereof. Although there are many methods for researching the water pressure of tunnel lining, yet the seepage analysis method is one of the methods widely applied. Specifically, Tong Lei[4] adopted complex variables functions to convert the seepage equation for soil mass into Dirichlet problem of the circular ring area similar to the lining part in the two-dimensional Laplace equation so as to solve the seepage problem of the lined tunnel in the semi-infinite space; Ma Longxiang, et al. [5] assumed the total water head acting on the outer lining wall in the tunnel with steady seepage as a unknown constant to deduce the seepage field of the soil mass around the tunnel and calculate the water head of the outer lining wall; Xiong Hao [6] adopted the conformal conversion method to find the water-carrying capacity and the water head distribution of the lined tunnel under overlaying soil condition according to the principle of inter-layer flow equivalence. In this paper, the

conformal variation equation is adopted to convert the free water surface of the underground water of the mountain tunnel and the periphery of the grouting circle into the circular ring in w plane. The two-dimensional seepage around the tunnel meets Laplace equation according to Darcy law and mass conservation law. Laplace equation is solved according to boundary condition and seepage continuity to obtain the analytical solutions of the seepage flows of the lithosphere, the grouting layer and the lining layer of the mountain tunnel and the water pressures of the lining. Meanwhile, the numerical calculation model based on flac^{2d} finite difference method is also established to compare the analytical solution and the numerical solution, thus concluding that the analytical solution deduced in this paper for the water pressure of the karst tunnel lining has certain practicability and reference value.

2. Establishment of calculation model

2.1. Calculation model and hypothesis

The calculation model proposed in this paper is as shown in Figure 1, the inner diameter of the tunnel is r_0 , the outer diameter is r_1 , the radius of the grouting circle is r_2 , and the distance between the tunnel center to the free water surface of the underground water is h .

The following hypotheses are made for convenient calculation in the paper: 1) the surrounding rock and the lining are uniform saturated isotropous continuous medium, and the soil mass and the water cannot be compressed; 2) the groundwater recharge is sufficient and

the tunnel drainage water does not influence the existing underground water level, and the seepage occurs along the radial direction in the surrounding rock; 3) the total water heads acting on the inner and outer walls of the tunnel lining and the outer wall of the grouting circle are respectively unknown constants h_0 , h_1 and h_g ; 4) the permeability coefficients of the rock, the grouting circle and the lining are respectively k_r , k_g and k_l ; 5) the tunnel center is set as the zero potential plane.

2.2. Control equation

The two-dimensional seepage around the tunnel meets Laplace equation according to Darcy law and mass conservation law:

$$\frac{\partial \phi^2}{\partial x^2} + \frac{\partial \phi^2}{\partial y^2} = 0 \quad (1)$$

Where $\phi = Y + \frac{P}{\gamma_w}$ (2)

Φ is the total water heads, Y is the potential water head, P is the water pressure and γ_w is the water weight per unit volume.

2.3. Rock seepage field solution

In the model, in order to solve above Laplace equation, Formula (3) for the conformal transformation [7] is adopted to convert the free water surface of the underground water and the round opening on the contact surface between the rock and the grouting circle into the circular ring with the inner diameter of α and outer diameter of 1 in w plane. Specifically, the conformal transformation of the rock-soil stratum is as shown in Figure 2.

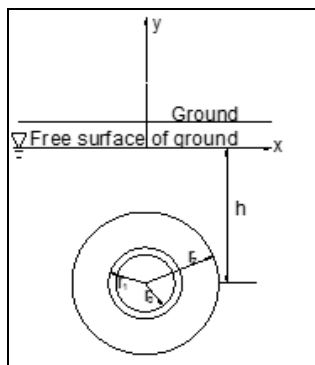


Figure 1. Mountain Tunnel Section Diagram

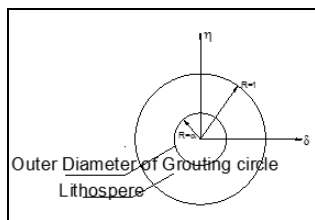


Figure 2. Conformal Transformation of Rock-soil Stratum

The conformal transformation formula is as follows:

$$z = w(\eta) = -iA \frac{1+\eta}{1-\eta} \quad (3)$$

$$A = \frac{h(1-\alpha^2)}{(1+\alpha^2)} \quad (4)$$

$$\alpha = \frac{h - \sqrt{h^2 - r_2^2}}{r_2} \quad (5)$$

In the circle area, Laplace equation can be represented as the general solution under $(\rho-\beta)$ polar coordinates in η plane:

$$\phi = C_1 + C_2 \ln \rho + \sum_{n=1}^{\infty} (C_3 \rho^n + C_4 \rho^{-n}) \cos n\beta \quad (6)$$

The polar coordinates of the lining and the grouting circle are as shown in Figure 3

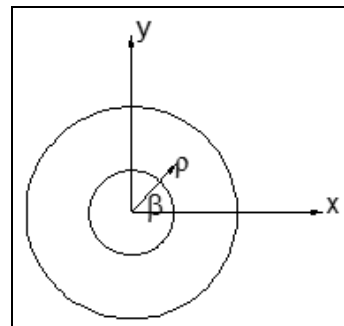


Figure 3. Polar Coordinates of Lining and Grouting Circle

2.3.1. Analysis of surrounding lithosphere seepage equation

w plane includes the following boundary conditions:

$$\phi_{s(\rho=1)} = 0 \quad (7)$$

$$\phi_{s(\rho=\alpha)} = h_g \quad (8)$$

Formulae (7) and (8) are put into general solution (6) to obtain the surrounding rock seepage field solution as follows:

$$\phi_s = \frac{h_g}{\ln \alpha} \ln \rho \quad (9)$$

According to Darcy law, the seepage flow of the surrounding stratum is obtained as follows through the integration of formula:

$$Q_r = k_r \int_0^{2\pi} \frac{\partial \phi}{\partial \rho} \rho d\theta = \frac{2\pi k_r h_g}{\ln \alpha} \quad (10)$$

2.3.2. Analysis of the equation for the seepage in the grouting circle

Since the grouting circle itself is a symmetrical circular ring, there is no need to carry out conformal transformation. Therefore, the seepage in the grouting

circle can be regarded as the axisymmetric constant seepage problem.

The continuous differential equation for the seepage in the grouting circle can be written as follows:

$$\frac{\partial \phi_g^2}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial \phi_g}{\partial \rho} = 0 \quad (11)$$

Boundary condition $\phi_g(\rho=r_2) = h_g, \phi_g(\rho=r_1) = h_l$ in the grouting circle is put into Formula (11) to obtain:

$$\phi_g = \frac{h_g - h_l}{\ln\left(\frac{r_2}{r_1}\right)} \ln\left(\frac{\rho}{r_2}\right) + h_g \quad (12)$$

The above formula is integrated according to Darcy law to obtain the seepage flow of the grouting circle.

$$Q_g = k_g \int_0^{2\pi} \frac{\partial \phi_g}{\partial \rho} \rho d\theta = \frac{2\pi k_g (h_g - h_l)}{\ln\left(\frac{r_2}{r_1}\right)} \quad (13)$$

Similarly, the seepage in the lining is as follows:

$$Q_l = k_l \int_0^{2\pi} \frac{\partial \phi_l}{\partial \rho} \rho d\theta = \frac{2\pi k_l (h_l - h_0)}{\ln\left(\frac{r_1}{r_0}\right)} \quad (14)$$

2.3.3. Inter-layer water head calculation

The following formulae are obtained according to the inter-layer seepage continuity conditions: $Q_r=Q_g$ and $Q_g=Q_l$.

$$h_g = \frac{Ak_g k_l}{Ak_g k_l - Bk_l k_r - Ck_g k_r} h \quad (15)$$

$$h_l = \frac{(Ak_g k_l - Bk_l k_r)}{Ak_g k_l - Bk_l k_r - Ck_g k_r} h \quad (16)$$

Where $A = \ln \alpha, B = \ln\left(\frac{r_2}{r_1}\right), C = \ln\left(\frac{r_1}{r_0}\right)$.

2.3.4 Water pressure of lining and seepage flow in lining

According to Formulae (1) and (16), the water pressure of the outer wall of the lining is as follows:

$$p = (h_l - Y) \gamma_w \quad (17)$$

Above Formula (16) is put into Formula (2) to obtain the water pressure of the lining at the tunnel center as follows:

$$p_0 = \frac{Ck_g k_r}{Ak_g k_l - Bk_l k_r - Ck_g k_r} h \gamma_w \quad (18)$$

Above Formula (18) is used for calculating the external water pressure of the lining at the horizontal line of the tunnel center. According to the characteristics of water pressure, the external water pressure of the tunnel lining

is linearly reduced upwards along the horizontal line of the tunnel center; the water pressure of the tunnel lining is linearly increased downwards along the tunnel center; Formula (18) can be regarded as the mean value of the water pressure of the lining at the round tunnel section, and the water pressure of the tunnel lining theoretically calculated in this paper is the mean value of the water pressure of the tunnel lining.

The precondition for above derivation is the uniform seepage of the surrounding rock and the support structure. For composite lining, such water drainage devices as transverse and longitudinal water drainage pipes are set between the initial support and the secondary lining; equally, the weep holes are additionally set for the secondary lining, and the water drainage quantity of the composite lining can be uniformly assigned to the secondary lining[8].

3. Calculation instance

3.1. Project profile

Ying-shan-yan Tunnel of Chengdu-Guiyang Passenger Transport Line, located at Daba Miaozu Village, Xingwen County, Sichuan Province and Jiucheng Town, Weixin County, Yunnan Province, crosses the provincial boundaries and the watershed of Nan-guang River and Gusong River. According to the passenger train, the tunnel is designed as the two-way tunnel for passenger transport line and the travel speed is designed as 250km/h. For this tunnel, the entrance distance is D2K254+362, the exit distance is D2K261+223, the total length is 6861m, the maximum burial depth is about 750m, and most burial depths are about 200~300m[9]. The tunnel passes through limestone, dolomite and karst breccias of middle-lower Triassic system, and the karst is strongly developed, and the tunnel has wide and gentle anticline and syncline, without any obvious fault.

3.2 Tunnel structure design

For the tunnel, the inner outline width of the rail surface lining is 1,220cm, the inner outline height of the lining from the rail surface to the vault is 868cm, and the profile area above the rail surface is 92m². The tunnel trunk mainly includes level III, IV and V surrounding rocks and is provided with composite lining. Therein, C25 concrete with the thickness of 28cm is jetted to level V surrounding rocks, the grating steel frame with the longitudinal distance of 0.8m is set for support reinforcement, and C45 reinforced concrete with the thickness of 50cm is molded for the lining. According to the principle of "Blockage Orientation and Limited Discharge", the water drainage system is reasonably designed for the water drainage of the tunnel in order to meet the requirements for structural design and use.

3.3. Tunnel parameter selection for analytical calculation and numerical calculation

1) Tunnel parameter selection for analytical calculation: according to the equivalent area method, the inner diameter of the tunnel is set as $r_0=5.5m$, the outer

diameter of the tunnel is set as 6m, the burial depth of the tunnel is set as 230m, the underground water level is 30m below the ground line, h is set as 200m, the permeability coefficient of the surrounding rock is set as $k_r = 1 \times 10^{-4} \text{ cm/s}$, m and n are respectively set as $m = k_r/k_z$ and $n = k_r/k_l$.

2) Tunnel parameter selection for numerical calculation: in this paper, shiliba syncline segment D2K260+000 is selected as the fracture surface to establish the numerical model for numerical calculation. In the model, the range of 8 times of the truck diameter, 240m in total, is transversely taken from the tunnel excavation sides to the two sides; the range of 80m is vertically upwards taken from the vault, and the range of 5 times of trunk height, 160m in total, is taken downwards from the tunnel bottom. The burial depth of the fracture surface is 230m, and the underground water level is 30m below the ground line [10-11]. The numerical calculation model is as shown

in Figure 4 and the numerical calculation parameters are as shown in Tab.1.

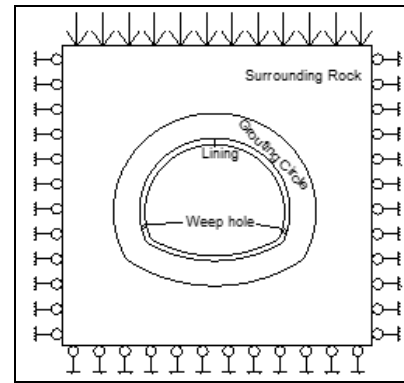


Figure 4. Numerical Calculation Model for Yingshanyan Tunnel

Table 1: Numerical Calculation Parameters for Yingshanyan Tunnel

Material	Density (kg/m^3)	Elasticity Modulus (GPa)	Poisson's Ratio	Internal Friction Angle ($^\circ$)	Cohesion Force (MPa)	Porosity	Permeability Coefficient (cm/s)
Surrounding Rock	2350	1.5	0.4	35	0.5	0.2	1×10^{-4}
Grouting Circle	2400	1.95	0.35	40	0.55	0.15	2×10^{-6}
Lining	2500	35	0.2	-	-	0.1	1×10^{-10}
Weep Hole	2500	35	0.2	-	-	0.5	5.23×10^{-2}

3.4. Calculation result comparison analysis

3.4.1. Influence of water drainage mode on water pressure of lining

In this paper, the surrounding rock and the support structure are assumed to have uniform water seepage in the theoretical formula. For composite lining, the water drainage quantity thereof can be uniformly assigned to the secondary lining, namely: the permeability coefficient of the lining is adjusted to simulate the water drainage of the tunnel. If the lining has larger permeability coefficient, then the tunnel has stronger water drainage capacity; oppositely, the tunnel has weaker water drainage capacity. According to the actual conditions of Ying-shan-yan tunnel and relevant engineering design parameters, the parameter setting is as follows: $k_l = 10^{-4} \text{ cm/s}$, $k_l = 10^{-10} \text{ cm/s}$ and $k_l = 10^{-7} \text{ cm/s}$, namely: $n=1$, $n=1000000$ and $n=1000$ are respectively set to calculate the external water pressures of the lining under three tunnel water drainage modes ---- full water drainage, full water blockage and limited water drainage, and the theoretical calculation results are as shown in Figure 5~7. Additionally, the external water pressures of the lining, numerically calculated under three tunnel water drainage modes ---- full water drainage, full water blockage and limited water drainage, are as shown in Figure 8~10.

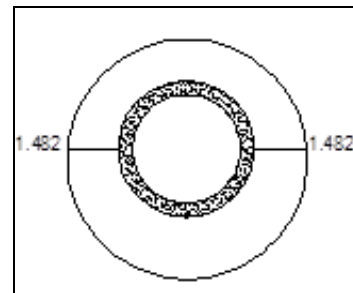


Figure 5. Water pressure of full Drainage (MPa)

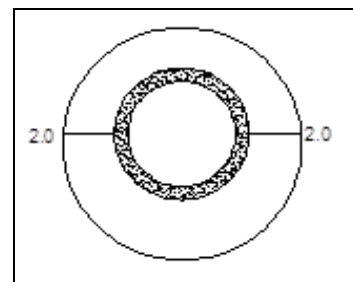


Figure 6. Water pressure of full blockage (MPa)

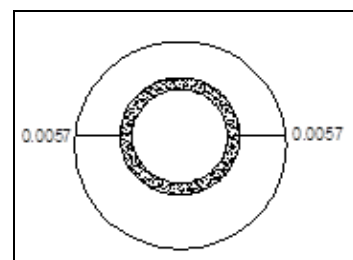


Figure 7. Water pressure of limited drainage (MPa)

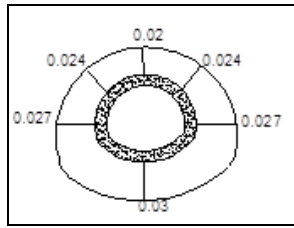


Figure 8. Water pressure of full drainage (MPa)

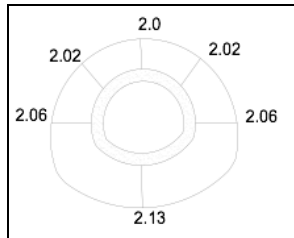


Figure 9. Water pressure of full blockage (MPa)

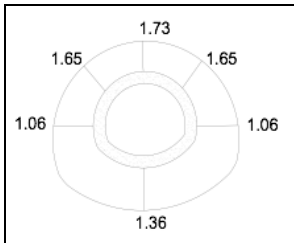


Figure 10. Water pressure of limited drainage (MPa)

According to the figures, under the full water drainage mode, the mean value of the theoretical calculation of the water pressure of lining is 0.0057 MPa, and the numerical calculation result of the water pressure of the lining is maximally 0.03MPa at the inverted arch and minimally 0.02MPa at the vault, and the water pressure of the lining is linearly reduced from the vault to the inverted arch, and the numerical calculation result is slightly greater than the theoretical calculation result. Under the full water blockage mode, the theoretical calculation result of the water pressure of the tunnel lining is consistent with the numerical calculation result, namely about 2MPa, because the water pressure of the lining cannot be discounted under full water blockage mode and the water pressure is equivalent to the hydrostatic pressure near to the tunnel. Under the limited water drainage mode, the mean value of the theoretical calculation of the water pressure of the lining is 1.482 MPa, and the numerical calculation result shows that the water pressure of the lining is smaller at the side wall and the inverted arch but is larger at the vault and the haunch; because the weep holes are set at the side wall and the inverted arch and the place near to the weep holes has large water drainage capacity, thus the water pressure of the lining is small. According to the comparison, the theoretical calculation result and the numerical calculation result both reflect the rule --- the stronger the tunnel water drainage capacity, the smaller the water pressure of the lining. The theoretical calculation result is basically consistent with the numerical calculation result.

3.4.2. Influence of grouting circle thickness on water pressure of lining

When k_l is set as $k_l = 10^{-7} \text{ cm/s}$ and the grouting circle thickness is set as 3m, 5m and 8m, the water pressures of the tunnel lining are respectively calculated under the limited water drainage mode. Specifically, the theoretical calculation results are as shown in Figure 11~13, and the numerical calculation results are as shown in Figure 14~16.

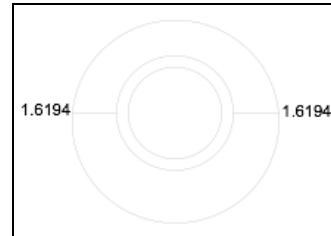


Figure 11. Water pressure of 3m thickness (MPa)

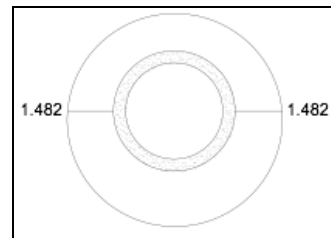


Figure 12. Water pressure of 5m thickness (MPa)

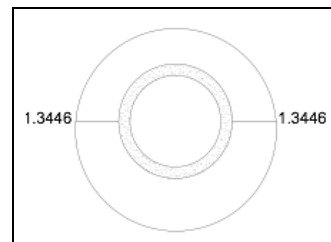


Figure 13. Water pressure of 8m thickness (MPa)

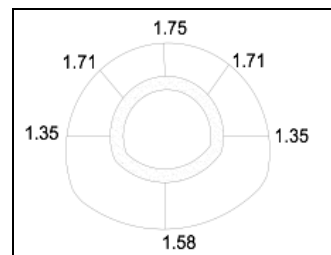


Figure 14. Water pressure of 3m thickness (MPa)

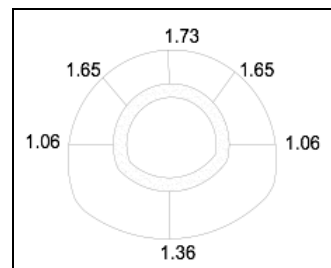


Figure 15. Water pressure of 5m thickness (MPa)



Figure 16. Water pressure of 8m thickness (MPa)

According to the figures, when the grouting circle thickness is respectively set as 3m, 5m and 8m, the mean values of the theoretical calculation of the water pressure of the lining are respectively 1.6194 MPa, 1.482 MPa and 1.3446 MPa, and the numerical calculation results of the water pressure of the lining at the vault are respectively 1.75 MPa, 1.73 MPa and 1.67 MPa, and the numerical calculation results of the water pressure of the lining at the inverted arch are respectively 1.58 MPa, 1.36 MPa and 1.12 MPa. The mean value of the numerical calculation of the water pressure of the lining at various parts is equivalent to that of the theoretical calculation. According to the comparison, the mean value of the theoretical calculation of the water pressure of the lining is basically consistent with that of the numerical calculation, and the mean value of the water pressure of the lining is gradually reduced along with the increase of the grouting circle thickness.

3.4.3. Influence of grouting circle permeability on water pressure of lining

When k_l and k_r are respectively set as $k_l = 10^{-7} \text{ cm/s}$ and $k_r = 1 \times 10^{-4} \text{ cm/s}$ and the permeability coefficient of the grouting circle is respectively set as $k_g = 3.33 \times 10^{-6} \text{ cm/s}$, $k_g = 2 \times 10^{-6} \text{ cm/s}$ and $k_g = 1 \times 10^{-7} \text{ cm/s}$, the water pressure of the tunnel lining is respectively calculated in this paper. The theoretical calculation results are as shown in Figure 17~19, and the numerical calculation results of the water pressure of the tunnel lining under the same conditions are as shown in Figure 20~22.

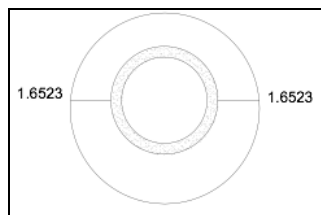


Figure 17. Water Pressure of $m=30$ (MPa)

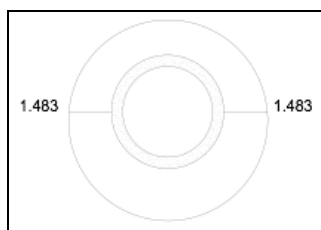


Figure 18. Water Pressure of $m=50$ (MPa)

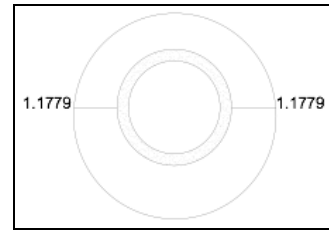


Figure 19. Water Pressure of $m=100$ (MPa)

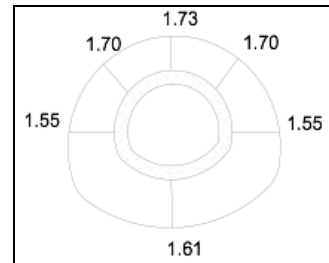


Figure 20. Water Pressure of $m=30$ (MPa)

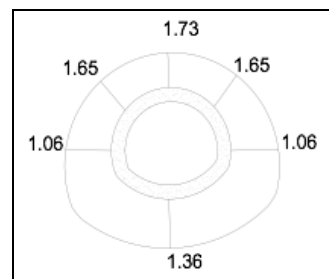


Figure 21. Water Pressure of $m=50$ (MPa)

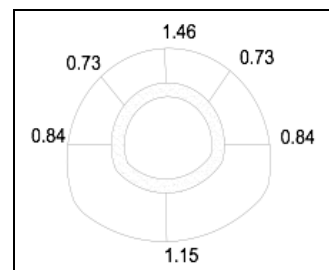


Figure 22. Water Pressure of $m=100$ (MPa)

According to the figures, when m is respectively set as $m=30$, $m=50$ and $m=100$, namely: the permeability coefficient of the grouting circle is respectively set as $k_g = 3.33 \times 10^{-6} \text{ cm/s}$, $k_g = 2 \times 10^{-6} \text{ cm/s}$ and $k_g = 1 \times 10^{-7} \text{ cm/s}$, the mean values of the theoretical calculation of the water pressure of the lining is respectively 1.6523 MPa, 1.483 MPa and 1.1779 MPa, and the numerical calculation results of the water pressure at the haunch are respectively 1.70 MPa, 1.65 MPa and 0.73 MPa, and the numerical calculation results of the water pressure of the lining at the inverted arch are respectively 1.61 MPa, 1.36 MPa and 1.15 MPa. According to comparison, the mean value of the theoretical calculation of the water pressure of the lining is basically consistent with that of the numerical calculation, and both the numerical calculation value and the theoretical calculation value are reduced along with the reduction of the permeability coefficient of the grouting circle.

4. Conclusion

- 1) The conformal variation theory is adopted in this paper. The general solution of Laplace equation is found in the conformal variation circle area to deduce the analytical solution of the water pressure of the tunnel lining with steady seepage.
- 2) For the theoretical derivation in this paper, the surrounding rock and the support structure are assumed to have uniform water seepage; for composite lining, the water drainage quantity of the composite lining between the initial support and the secondary lining can be uniformly assigned to the secondary lining.
- 3) The result of the comparison between the analytical solution and the numerical solution of the water pressure of the tunnel lining shows: the analytical solution of the distribution characteristic of the water pressure of the tunnel lining is basically consistent with the numerical solution of the same, and the analytical solution in this paper has higher reference value for calculating the water pressure of the high-pressure water-rich karst tunnel lining.

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