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New Solution of Elastic and Plastic Mechanics of Surrounding Rock Considering Strain Softening

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Abstract: By introducing rock mass strength softening modulus, the elastic-plastic mechanical analysis model of round roadway surrounding rock with consideration of strain softening condition under static pressure and mining pressure was set up. Then, the stress and new displacement explanation of elastic-plastic three area of roadway surrounding rock under static pressure and dynamic pressure were deduced. The new explanation example shows that strain softening modulus has remarkable impact on surrounding rock "three areas" scope, which can objectively reflect the features of roadway surrounding rock strength decrease along with strain increase under static pressure and dynamic pressure. This is the mechanical mechanism of large deformation for this kind of roadway. If reasonable rock physical and mechanics parameters are selected, then more reasonable analytic solutions can be obtained in theory. By improving residual strength status of fracture zone, the fracture zone radius can be effectively controlled. It is the mechanical mechanism of the roadway surrounding rock stability control, which can provide effective gist for the roadway arrangement and supporting design construction under static pressure and dynamic pressure.

Keywords: *Surrounding Rock, Elastic and Plastic Mechanics, Control of Surrounding Rock.*

1. Introduction

Numerous domestic and foreign scholars have done a lot of theoretical studies about analyzing elastic-plastic mechanics of stress and displacement “three areas” over circular tunnel [1-12]. The most widely used formulas are fever and modified fever formula (Kastner Formula). Both these two are ideal elastic-plastic body based on Mohr-Coulomb criterion [13-20]. The deficiency is that the strength peak remains unchanged and doesn’t depend on the degree of rock strain-softening behaviors of rock material can’t be ignored. The studies about strain-softening model are mainly focus on numerical simulating model and numerical solution, but hardly any one analyzed or talked the analytical solution, therefore, this paper analyzed the surrounding rock of circular roadway considering strain softening in an elastic-plastic way.

2. Model of Elastic-Plastic Mechanic Analysis Considering Strain Softening

Based on elastic and plastic mechanical analysis of predecessors, the author obtained a new solution by establishing an elastic and plastic mechanic of surrounding rock of circular roadway considering strain softening.

Suppose there is a long circular tunnel which is in a uniform stress field whose stress is P_0, the radius is r_0, and supporting face is P_i (see Fig 1)[21-33].

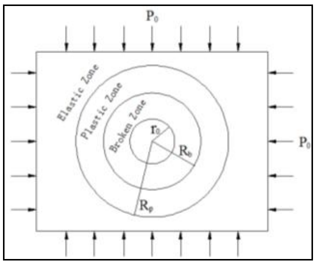

Figure 1: *Mechanical model of circular roadway*

The rock mass can be regarded as an ideal elastic-plastiaty without body force, and the material is homogeneous and isotropic. So it can be simplified as an equilibrium problem of thick-walled cylinder in a plane strain state, the fundamental equations are as follows[34-42]:

1) Equilibrium equation

$$\frac{d\sigma_r}{dr}+\frac{\sigma_r-\sigma_\theta}{r}=0 \tag{1}$$

2) Geometric equation

$$\varepsilon_r = \frac{du}{dr}, \varepsilon_\theta = \frac{u}{r} \tag{2}$$

3) Constitutive relation in elastic region

$$\varepsilon_r = \frac{1-\mu^2}{E}(\sigma_r - \frac{\mu}{1-\mu}\sigma_\theta)$$
$$\varepsilon_\theta = \frac{1-\mu^2}{E}(\sigma_\theta - \frac{\mu}{1-\mu}\sigma_r) \tag{3}$$

Considering the post-peak strain softening of geomaterials, the model of stress-strain with three segments figure in classical rock mechanics is cited(as shown in Figure 2), where the tangential and radial strain curve is shown in Fig 3.

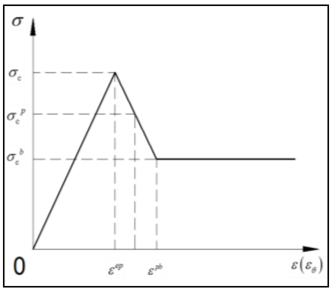

Figure 2: *Curve model of rock stress-strain with three-segment*

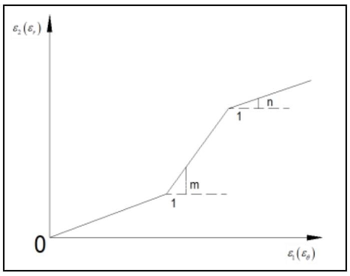

Figure 3: *Deformation characteristics of ideal elastic-plastic strain softening materials*

The rock strength softening modulus in this paper is defined as

$$Q = \frac{\sigma_c - \sigma_c^b}{\varepsilon_\theta^{pb} - \varepsilon_\theta^{ep}} \tag{4}$$

Where ${\sigma_c}^b$ -refers to the residual compressive strength of rock mass;

σ_c -refers to ultimate compressive strength;

${\varepsilon_\theta}^{pb}$ -refers to critical tangential strain of the rock mass changed from the stage of plastic softening to plastic residual;

${\varepsilon_\theta}^{ep}$ -refers to the critical tangential strain of rock mass from the stage of elastic deformation to plastic softening;

Regarding ${\sigma_c}^p$ -the plastic softening strength of failure surrounding rock mass at the periphery of a tunnel as residual compression strength, and the plastic softening ${\sigma_c}^p$ will be a function of plastic strains ${\varepsilon_\theta}^p$, the changing rule is similar to the whole stress-stain curve of rock mass.

Elastic zone, plastic softening zone and plastic residual zone (crack zone) is divided in the range of rock compression-controlled failure, which corresponds to Mohr-Coulomb yield criterion:

$$f = \sigma_\theta - K\sigma_r + \sigma_0 \tag{5}$$

Where σ_r and σ_θ refers to the radial stress and tangential stress at some point in a polar coordinate; $K = \frac{1+\sin(\varphi)}{1-\sin(\varphi)}$, φ is internal friction angle; in the beginning of yielding, $\sigma_0 = \sigma_c$; in the plastic softening region, $\sigma_0 = {\sigma_c}^p$ and in the plastic residual region, $\sigma_0 = {\sigma_c}^b$.

According to axisymmetrical theory, the maximum principal strain of rock under load will $\varepsilon_3 = \varepsilon_r$, the minimum principal strain will $\varepsilon_1 = \varepsilon_\theta$. From Fig 3, the radial strain and tangential strain in plastic zone of surrounding rock can be regarded as follows:

$$\varepsilon_r^p = (\varepsilon_r^e)_{r=R_p} + \Delta\varepsilon_r^p \tag{6}$$
$$\varepsilon_\theta^p = (\varepsilon_\theta^e)_{r=R_p} + \Delta\varepsilon_\theta^p \tag{7}$$

$\Delta{\varepsilon_r}^p$ and $\Delta{\varepsilon_\theta}^p$ meet the equation:

$$\Delta{\varepsilon_r}^p = -m\Delta{\varepsilon_\theta}^p \tag{8}$$

Where m is expansion coefficient of plastic fracture zone, generally take $m = \frac{1+\sin(\varphi)}{1-\sin(\varphi)}$.

Similarly, the theoretical solutions of radial strain and tangential strain in rupture zone can be obtained:

$$\varepsilon_r^b = (\varepsilon_r^p)_{r=R_b} + \Delta\varepsilon_r^b \tag{9}$$
$$\varepsilon_\theta^b = (\varepsilon_\theta^p)_{r=R_b} + \Delta\varepsilon_\theta^b \tag{10}$$

$\Delta{\varepsilon_r}^p$ and $\Delta{\varepsilon_r}^p$ meet the following relation:

$$\Delta\varepsilon_r^b = -n\Delta\varepsilon_\theta^b \tag{11}$$

Where n is expansion coefficient of rupture zone, generally take 1.3-1.5.

So new analytical solution of strains and displacements of "three area" of surrounding rock can be obtained:

1) Elastic zone

Following the lame formula, so

$$\sigma_r^e = p_0(1 - \frac{R_p^2}{r^2}) + \sigma_R\frac{R_P^2}{r^2}$$

$$\sigma_\theta{}^e = p_0\left(1+\frac{R_p{}^2}{r^2}\right)-\sigma_R\frac{R_p{}^2}{r^2} \qquad (12)$$

Where σ_R is the radial stress on elastic plastic interface which corresponds to Mohr-Coulomb criterion, take equations (12) into (5) and gain

$$\sigma_R = \frac{2P_0-\sigma_C}{K+1} \qquad (13)$$

Take equations (12) into (3)

$$\varepsilon_r^e = \frac{1+\mu}{E}[(1-2\mu)p_0+\frac{R_p^2}{r^2}(\sigma_R-p_0)]$$
$$\varepsilon_\theta{}^e = \frac{1+\mu}{E}\left[(1-2\mu)p_0-\frac{R_p{}^2}{r^2}(\sigma_R-p_0)\right] \qquad (14)$$

From the second equation of equations (2) we can see:

$$u^e = r\varepsilon_\theta^e = \frac{1+\mu}{E}[(1-2\mu)p_0r+\frac{R_p^2}{r}(\sigma_R-p_0)] \qquad (15)$$

2) Plastic zone

Take equation (7) and (8) into (6):

$$\varepsilon_r^p + m\varepsilon_\theta^p = (\varepsilon_r^e)_{r=R_p}+m(\varepsilon_\theta^e)_{r=R_p} \qquad (16)$$

Take equations(2) and (14) into above:

$$\frac{du^p}{dr}+m\frac{u^p}{r}=\frac{1+\mu}{E}[-2\mu P_0+\sigma_R+m(2p_0(1-\mu)-\sigma_R)] \qquad (17)$$

whose general solution is

$$u^p = C_1r^{-m}+\frac{r}{1+m}\frac{1+\mu}{E}[-2\mu P_0+\sigma_R+m(2p_0(1-\mu)-\sigma_R)] \qquad (18)$$

According to boundary conditions, when $r=R_p$, there will $u^p=u^e$, and gain

$$C_1 = -\frac{1+\mu}{E}\frac{2(\sigma_R-p_0)}{1+m}R_p^{m+1} \qquad (19)$$

So

$$u^p = A\frac{R_p^{m+1}}{r^m}+Br \qquad (20)$$

Where

$$A=-\frac{1+\mu}{E}\frac{2(\sigma_r-P_0)}{1+m} \qquad (21)$$
$$B=\frac{1+\mu}{E(1+m)}[-2\mu P_0+\sigma_R+m(2p_0(1-\mu)-\sigma_R)] \qquad (22)$$

Mohr-Coulomb criterion can also be used in the plastic zone, take equation (2-70) into equilibrium equation (1), and obtain the stress in plastic zone:

$$r\frac{d\sigma_r^p}{dr}+(1-K)\sigma_r^p=\sigma_c^p \qquad (23)$$

whose general solution is

$$\sigma_r^p = r^{K-1}\int r^{-K}\sigma_c^p dr \qquad (24)$$

Take equation (20) into equation (2) and gain:

$$\varepsilon_r^p = -mA(\frac{R^p}{r})^{m+1}+B$$
$$\varepsilon_\theta{}^p = A\left(\frac{R_p}{r}\right)^{m+1}+B \qquad (25)$$

Because the plastic softening strength $\sigma_c{}^p$ is function of plastic strain $\varepsilon_\theta{}^p$, so

$$\sigma_c^p = \sigma_c - Q(\varepsilon_\theta^p-\varepsilon_\theta^{ep}) \qquad (26)$$

Take equation (14) and (15) into it, and obtain

$$\sigma_c^p = -QA(\frac{R^P}{r})^{m+1}+C \qquad (27)$$

Where

$$C=-QB+\frac{Q(1+\mu)}{E}(2(1-\mu)p_0-\sigma_R)+\sigma_c \qquad (28)$$

Take equation (27) into (24) and integrate it

$$\sigma_r^p = \frac{QA}{mK}(\frac{R^p}{r})^{m+1}-\frac{C}{K-1}+C_2r^{K-1} \qquad (29)$$

According to the continuous rule of stress on the border of the elastic-plastic area, if $r=R_p$ then will be $\sigma_r^p=\sigma_r^e$, we can see

$$C_2 = (\frac{C}{K-1}-\frac{QA}{m+K}-\frac{\sigma_C-2p_0}{K+1})R_p^{1-k} \qquad (30)$$

3) Crack zone

Similarly, take equation (10) and (11) into (31), and gain

$$\varepsilon_r^b + n\varepsilon_\theta^b = (\varepsilon_r^p)_{r=R_b}+n(\varepsilon_\theta^p)_{r=R_b} \qquad (31)$$

Take equation (2) and (25) into (31)

$$\frac{du_b}{dr}+n\frac{u^b}{r}=(n-m)A(\frac{R_P}{R_b})^{m+1}+(n+1)B \qquad (32)$$

The general solution is

$$u^b = C_3r^{-n}+Ar\frac{n-m}{1+n}(\frac{R_P}{R_b})^{m+1}+rB \qquad (33)$$

From the boundary conditions of plastic zone and rupture zone, when $r=R_b$, there will be $u^p=u^b$, and attain

$$C_3 = A\frac{1+m}{1+n}(\frac{R_P}{R_b})^{m+1}R_b^{n+1} \qquad (34)$$

Applying Mohr-Coulomb criterion in rupture zone, take it into equilibrium equation and obtain an equation which is similar to equation (33), acquire the general stress solution:

$$\sigma_r^b = r^{K-1}\int r^{-K}\sigma_c^b dr \qquad (35)$$

Considering $\sigma_c{}^b$ is constant, so integrate equation (35) and gain

$$\sigma_r^b = -\frac{\sigma_c^b}{K-1} + C4r^{K-1} \qquad (36)$$

Because the stresses in plastic zone and crack zone are continuous, so when $r = R_b$, there will be $\sigma_r{}^p = \sigma_r{}^b$, and gain

$$C_4 = \frac{QA}{m+K}(\frac{R^P}{R^b})^{m+1} R_b^{1-K} + DR_P^{1-K} + \frac{\sigma_c^b - C}{K-1} R_b^{1-K} \qquad (37)$$

Where

$$D = (\frac{C}{K-1} - \frac{QA}{m+K} - \frac{\sigma_C - 2p_0}{K+1}) \qquad (38)$$

4) The radius R_p in plastic zone and the radius R_b in rupture zone

The stress boundary conditions of surrounding rock: when $r = r_0$, there will be $\sigma_r{}^b = q_i$, and obtain

$$-\frac{\sigma_c^b}{K-1} + [\frac{QA}{m+K}(\frac{R_p}{R_b})^{m+1} R_b^{1-k} + DR_p^{1-k} + \frac{\sigma_c^b - C}{K-1} R_b^{1-K}] r_0^{k-1} = q_i \qquad (39)$$

Besides,

$$\varepsilon_\theta^{pb} = (\varepsilon_\theta^b)_{r=R_b}, \varepsilon_\theta^{ep} = (\varepsilon_\theta^p)_{r=R_P} \qquad (40)$$

Take them into equation (4), and acquired

$$(\frac{R_P}{R_b})^{m+1} = \frac{\sigma_c - \sigma_c^b + QA}{QA} \qquad (41)$$

From equation (39) and (41) we can gain

$$R_b = r_0\{[\frac{\sigma_c - \sigma_c^b + QA}{m+K} + D(\frac{\sigma_C - \sigma_C^b + QA}{QA})^{\frac{1-K}{m+1}} + \frac{\sigma_c^b - c}{K-1}]\frac{K-1}{\sigma_c^b + q_i(K-1)}\}^{\frac{1}{K-1}} \qquad (42)$$

$$R_p = R_b(\frac{\sigma_c - \sigma_c^b + QA}{QA})^{\frac{1}{m+1}} \qquad (43)$$

3.The Elastic-plastic analysis under conditions of high and low stresses

Neglect the initial displacement of primary rock in the roadway excavation stage. For the primary rock stress $\sigma_r = \sigma_\theta = p_0$, and take it into equation(2)and (3)

$$u^0 = \frac{1+\mu}{E}(1-2\mu)p_0 r \qquad (44)$$

Let equation (15), (20) and (33) minus (44) respectively and acquire

$$u^e = \frac{1+\mu}{E}\frac{R_p^2}{r}(p_0 - \sigma_R) \qquad (45)$$

$$u^p = A\frac{R_p^{m+1}}{r^m} + \frac{1+\mu}{E(1+m)}(m-1)(p_0 - \sigma_R)r \qquad (46)$$

$$u^b = A\frac{1+m}{1+n}(\frac{R^p}{R^b})\frac{R_b^{n+1}}{r^n} + Ar\frac{n-m}{1+n}(\frac{R_p}{R_b})^{m+1} + \frac{1+\mu}{E(1+m)}(m-1)(p_0 - \sigma_R)r \qquad (47)$$

On the conditions of high stress, suppose the stress of primary rock are respectively p_1 and p_2, so there will be $R_{p1} < R_{p2}$, $R_{b1} < R_{b2}$.

When the stress of rock equals the p_1, the deformations of surrounding rock will be

$$u_1{}^{e'} = \frac{1+\mu}{E}\frac{R_{p1}{}^2}{r}(p_1 - \sigma_{R1}) \qquad (48)$$

$$u_1{}^{p'} = A_1\frac{R_{p1}{}^{m+1}}{r^m} + \frac{1+\mu}{E(1+m)}(m-1)(p_1 - \sigma_{R1})r \qquad (49)$$

$$u_1{}^{b'} = A_1\frac{1+m}{1+n}\left(\frac{R_{p1}}{R_{b1}}\right)^{m+1}\frac{R_{b1}{}^{n+1}}{r^n} + A_1 r\frac{n-m}{1+n}\left(\frac{R_{p1}}{R_{b1}}\right)^{m+1} + \frac{1+\mu}{E(1+m)}(m-1)(p_1 - \sigma_{R1})r \qquad (50)$$

Where

$$\sigma_{R1} = \frac{2p_1 - \sigma_c}{K+1} \qquad (51)$$

$$A_1 = -\frac{1+\mu}{E}\frac{2(\sigma_{r1} - p_1)}{1+m} \qquad (52)$$

$$B_1 = \frac{1+\mu}{E(1+m)}[-2\mu p_1 + \sigma_{R1} + m(2p_1(1-\mu) - \sigma_{R1})] \qquad (53)$$

$$C_1 = -QB_1 + \frac{Q(1+\mu)}{E}(2(1-\mu)P_1 - \sigma_{r1}) + \sigma_C \qquad (54)$$

$$D_1 = (\frac{C_1}{K-1} - \frac{QA_1}{m+K} - \frac{\sigma_C - 2p_1}{K+1}) \qquad (55)$$

When the primary rock stress equals the p_2, the deformations of surrounding rock is

$$u_2{}^{e'} = \frac{1+\mu}{E}\frac{R_{p2}{}^2}{r}(p_2 - \sigma_{R2}) \qquad (56)$$

$$u_2{}^{p'} = A_2\frac{R_{p2}{}^{m+1}}{r_m} + \frac{1+\mu}{E(1+m)}(m-1)(p_2 - \sigma_{R2})r \qquad (57)$$

$$u_1{}^{b'} = A_1\frac{1+m}{1+n}\left(\frac{R_{p1}}{R_{b1}}\right)^{m+1}\frac{R_{b1}{}^{n+1}}{r^n} + A_1 r\frac{n-m}{1+n}\left(\frac{R_{p1}}{R_{b1}}\right)^{m+1} + \frac{1+\mu}{E(1+m)}(m-1)(p_1 - \sigma_{R1})r \qquad (58)$$

Where

$$\sigma_{R2} = \frac{2p_2 - \sigma_c}{K+1} \qquad (59)$$

$$A_2 = -\frac{1+\mu}{E}\frac{2(\sigma_{R2} - p_2)}{1+m} \qquad (60)$$

$$B_2 = \frac{1+\mu}{E(1+m)}[-2\mu p_2 + \sigma_{R2} + m(2p_2(1-\mu) - \sigma_{R2})] \qquad (61)$$

$$C_2 = -QB_2 + \frac{Q(1+\mu)}{E}(2(1-\mu)P_2 - \sigma_{r2}) + \sigma_C \qquad (62)$$

$$D_2 = (\frac{C_2}{K-1} - \frac{QA_2}{m+K} - \frac{\sigma_C - 2p_2}{K+1}) \qquad (63)$$

When $R_0 < r < R_{b1}$, the stress of rupture zone increased from p1 to p2, the displacement increment is $u_2{}^{b'} - u_1{}^{p'}$, that is

$$\begin{aligned}\Delta u = {} & A_2\frac{1+m}{1+n}(\frac{R_{p2}}{R_{b2}})^{m+1} + A_2 r\frac{n-m}{1+n}(\frac{R_{p2}}{R_{b2}})^{m+1} + \frac{1+\mu}{E(1+m)}(m-1)(p_2 - \sigma_{R2})r \\ & -A_1\frac{1+m}{1+n}(\frac{R_{p1}}{R_{b1}})^{m+1} - A_1 r\frac{n-m}{1+n}(\frac{R_{p1}}{R1})^{m+1} - \frac{1+\mu}{E(1+m)}(m-1)(p_1 - \sigma_{R1})r\end{aligned} \qquad (64)$$

When $R_{b1} < r < R_{b2}$, the plastic zone of surrounding rock changed into rupture zone, the stress increased from p1 to p2 ,the displacement is $u_2{}^{p'} - u_1{}^{p'}$, that is

$$\Delta u = A_2 \frac{1+m}{1+n}\left(\frac{R_{p2}}{R_{b2}}\right)^{m+1} + A_2 r \frac{n-m}{1+n}\left(\frac{R_{p2}}{R_{b2}}\right)^{m+1} + \frac{1+\mu}{E(1+m)}(m-1)(p_2-\sigma_{R2})r$$
$$-A_1 \frac{R_{p1}{}^{m+1}}{r^m} - \frac{1+\mu}{E(1+m)}(m-1)(p_1-\sigma_{R1})r \quad (65)$$

When $R_{b2} < r < R_{p1}$, the stress in plastic area of surrounding rock increased from p_1 to p_2, the displacement increment is $u_2{}^{p'} - u_1{}^{e'}$, that is

$$\Delta u = A_2 \frac{R_{p2}^{m+1}}{r^m} + \frac{1+\mu}{E(1+m)}(m-1)(p_2-\sigma_{R2})r - A_1 \frac{R_{p1}^{m+1}}{r^m} + \frac{1+\mu}{E(1+m)}(m-1)(p_1-\sigma_{R1})r \quad (66)$$

When $R_{P1} \le r \le R_{P2}$, the elastic zone changed into plastic zone ,the stress increased from p_1 to p_2, the displacement is $u_2^p - u_1^e$, that is

$$\Delta u = A_2 \frac{R_{p2}^{m+1}}{r^m} + \frac{1+\mu}{E(1+m)}(m-1)(p_2-\sigma_{R2})r - \frac{1+\mu}{E}\frac{R_{p1}^2}{r}(p_1-\sigma_{R1}) \quad (67)$$

When $r > R_{p2}$, the stress of elastic zone increased from p1 to p2 ,the displacement increment is $u_2{}^{e'} - u_1{}^{e'}$, that is

$$\Delta u = \frac{1+\mu}{E}\frac{R_{p2}{}^2}{r}(p_2-\sigma_{R2}) - \frac{1+\mu}{E}\frac{R_{p1}{}^2}{r}(p_1-\sigma_{R1}) \quad (68)$$

4. Analysis of example

Applying the software MATLAB7.1 to edit file M, and to analysis the elastic-plastic model considering strain softening. The mechanical parameters are as follows: $r_0 = 3$m, $p_1 = 10$MPa, $p_2 = 20$MPa, $q_i = 0$MPa, $E = 5$GPa, $\sigma_c = 5$MPa, $\sigma_c{}^b = 0.5$MPa, $\varphi = 30°$, $Q = 1$GPa, $\mu = 0.25, n = 2$. Take the low lever stress p1 , and high lever stress p2.

The displacement of surrounding rock under high and low stresses are shown in Fig 4.

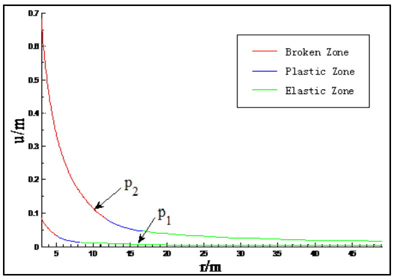

Fig 4: *Displacement distribution and displacement scope of crack zone, plastic zone and elastic zone*

Table 1: *Displacement distribution and displacement scope of crack zone, plastic zone and elastic Zone*

Conditions		Crack zone	
		Distribution range	**Displacement range**
1	low stress	3-5.17m	0.0325- 0.0871m
	high stress	3-11.62m	0.0853-0.6849m
2	**Conditions**	**Plastic zone**	
		Distribution range	**Displacement range**
	low stress	5.17-8.34m	0.0131-0.0325m
	high stress	11.62-16.63m	0.0468-0.0853m
3	**Conditions**	**Elastic zone**	
		Distribution range	**Displacement range**
	low stress	≥8.34m	0.0022-0.0131m
	high stress	≥16.63m	0.0156-0.0468m

From Fig 4 and Table 1, we can know the range of rupture zone under high stress reached 11.62m, 3.34 times of that under low stress. The range of plastic zone under high stress reached 16.63m, increased a lot, 2.99 times bigger than that under low stress, and the surface displacement of roadway under high stress reached 68.49mm, which increased so much that is 8.86 times than that under low stress.

1) By defining rock strength softening modules $Q = \frac{\sigma_c - \sigma_c^b}{\varepsilon_\theta^{pb} - \varepsilon_\theta^{ep}}$, to establish a elastic-plastic analytical model of the “three areas” of surrounding rock of circular tunnel considering strain softening, obtain new fundamental equations of “three areas elastic-plastic model” which is based on strain-softening effect of rock mass.
2) Using the common and concise Mohr-Coulomb criterion and non-relevant flow rule of boundary of “three areas”, attained new analysis solution of surrounding rock stress and displacement of “three areas” in this mechanical model.
3) After the analysis of example, attained the change rule about the displacement distribution and high and low stress conditions; with the increasing of confining pressure, the distribution range of crack zone and plastic zone increased remarkable; the surface displacement increased nonlinearly and largely. It with provide a certain significance for mechanism of deformation and failure and controlling engineering practice of surrounding rock of roadway.

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