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A Tensile Plastic Damage Constitutive Model of Concrete Based on Energy

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Abstract: Based on the theory of damage mechanics, continuum mechanics and irreversible thermodynamics, considering the coupling relation of plastic stain and damage, the certain function relation is supposed between the plastic free energy and the elastic free energy, so the damage energy release rate is defined. According to the Weibull distribution curve, the relationship between the damage variable and the energy release rate is established, thus the concrete tensile damage evolution equation is deduced. Finally, based on the plastic characteristics of concrete under the action of tensile load, the empirical formula of plastic deformation is obtained. Therefore, the tensile plastic damage constitutive model of concrete is established. The uniaxial tension test results and engineering examples are used for verification of the effectiveness and applicability of this model. Analysis results show that the tensile plasticity constitutive model can more truly reflect the concrete tensile damage evolution process.

Keywords: *Concrete material, The tensile damage, The constitutive model, The plastic free energy*

1. Introduction

For large and complex structure, the traditional linear structure analysis methods, which cannot obtain the deformation and failure characteristics of the structure, cannot guarantee the design reliability of complex structures, so we often require the analysis of nonlinear structure. For the seismic nonlinear analysis of concrete structures, the researchers had proposed a cross section fiber model with considerable accuracy. Therefore, the bottleneck of improving the accuracy of structural nonlinear analysis appears in the aspect of material model at present, and it is the key to accurately simulate the constitutive characteristics of concrete materials[1-3].

Under the seismic load, the damage of concrete dam is mainly caused by the tensile fracture of the dam body, and less by the compressive load to crush [4-5]. Because the tensile strength of concrete is far less than its compressive strength, and in the dam design, the safety coefficient of the compressive strength is relatively large, so many scholars mainly focus on the evolution of damage of concrete under tension.

In the early stage, many researchers believed that the tensile failure of concrete was not required to consider the effect of damage, and it was completely brittle failure[6-8].At the beginning of 1880s, Peterson[8] proposed the stress-strain curve, considering the damage and plastic deformation of concrete under the action of tensile stress. After more than 20 years of development, many scholars proposed various damage constitutive models based on different materials and different damage mechanism, mainly divided into meso damage constitutive model and macro damage constitutive model[10-11].The calculation of meso damage constitutive model is too large, and the problem description is more complex. It is also difficult to describe a basic unit, especially for the analysis of concrete, which is the multiphase medium material. And the experimental microscopic observation is more difficult. But the macro damage constitutive model cannot describe the micro mechanism conveniently and fine. At the same time, its establishment needs depend on the tests strongly. There are insurmountable difficulties in test and theoretical analysis for space complex stress components, so the further application and development are restricted[12-13].

Therefore, based on the damage mechanics, continuum mechanics theory and irreversible thermodynamics theory, damage variables are introduced through the plastic free energy, and damage variables are treated as internal variables. At the same time the damage variable and thermodynamic variables are used to describe the thermodynamic irreversible processes, and then the constitutive relation curve of concrete is derived, in which the influence of plasticity and damage coupling is considered, the advantages of damage mechanics and plastic mechanics are played respectively, and the damage, plastic, tension and compression softening and unilateral effects of concrete material are further described, etc.

2. Tensile Damage Constitutive Model

Assume that material is isotropy, based on the theory of thermodynamics, in the process of damage constitutive model derived ,deformation, stress and so on variable were introduced to free energy function. The free energy can be expressed as

$$\psi=\psi(\delta^e,\delta^p,\theta,V_k) \quad (1)$$

δ^e is elastic deformation. δ^p is plastic deformation. θ is temperature. V_k is internal variable.

Through the derivation of thermodynamic theory, material constitutive equation which based on free energy can be deduced, as formula (2).

$$\sigma-\rho\frac{\partial\psi}{\partial\delta^e}=0 \quad (2)$$

In general, free energy is divided into the plastic part and the elastic part, under the condition of isothermal adiabatic, the damage variables D stands for internal variable V_k , and so formula (1) can be rewritten as

$$\psi(\delta^e,\delta^p,D)=\psi^e(\delta^e,D)+\psi^p(\delta^p,D) \quad (3)$$

$\psi^e(\delta^e,D)$ is elastic free energy. $\psi^p(\delta^p,D)$ is plastic free energy.

If $C(D)$ is concrete damage after unloading stiffness, concrete material of elastic free energy $\psi^e(\delta^e,D)$ can be expressed as

$$\psi^e(\delta^e,D)=\int_0^{\delta^e}\sigma d\delta^e=\frac{1}{2\rho}C(D)\left(\delta^e\right)^2 \quad (4)$$

Because the damage caused the stiffness degradation of concrete material, and the damage is irreversible, $C(D)$ is the damage variable of non-increasing function.

The initial concrete material without damage, so the initial elastic free energy is:

$$\psi_0^e(\delta^e)=\frac{1}{2\rho}C_0\left(\delta^e\right)^2 \quad (5)$$

C_0 is concrete initial stiffness tensor.

Due to the damage of concrete is represented by the stiffness degradation, so the damage of concrete can be expressed in a damage variable to the elastic free energy $\psi^e(\delta^e,D)$.

$$\psi^e(\delta^e,D)=\frac{1}{2\rho}(1-D)C_0\left(\delta^e\right)^2=(1-D)\psi_0^e(\delta^e) \quad (6)$$

The following formula can be obtained from the formula (2) and the formula (6).

$$\sigma=\rho\frac{\partial\psi^e(\varepsilon^e,D)}{\partial\varepsilon^e} \quad (7)$$

In the process of uniaxial tension, the relationship between the tensile deformation of concrete should be satisfied:

$$\delta_t=\delta_t^e+\delta_t^p \quad (8)$$

δ_t is concrete tensile deformation. δ_t^e is concrete tensile elastic deformation. δ_t^p is concrete tensile plastic deformation.

Due to the damage would have a certain influence on the plastic properties of concrete, introduce the uniaxial tensile damage variable D_t into formula (3), and so the scalars form of elastic free energy is got:

$$\psi^e(\delta_t^e,D_t)=(1-D_t)\psi_0^e(\delta_t^e)=\frac{1}{2\rho}(1-D_t)K_0(\delta_t^e)^2 \quad (9)$$

K_0 is the initial slope of concrete under uniaxial tension stress and deformation.

Putting the formula (9) and formula (8) into the formula (7) can get the concrete damage constitutive relation under uniaxial tension cases:

$$\sigma_t=(1-D_t)K_0(\delta_t-\delta_t^p) \quad (10)$$

3. Damage evolution equation

In this paper, based on the plastic mechanics theory of yield function is clear, concrete materials in the process of yield will produce plastic deformation and damage at the same time, therefore, in order to get the damage evolution law of concrete material, first need to determine the plastic free energy expression,

$$\psi^p(\delta^p,D)=\int_0^{\delta^p}\sigma:d\delta^p \quad (11)$$

From formula (11) knows that the plastic free energy is increasing function of plastic deformation. For known materials, the total free energy is determined, so the deformation variables can be used to describe relation function between the elastic free energy and the plastic the free energy, assume that the expression is as follows:

$$\psi^p(\delta^p,D)=F(\delta^e,\delta^p)\psi^e(\delta^e,D) \quad (12)$$

Assume that after peak stress, concrete start to occur plastic deformation, peak deformation is $\delta_{pk}=\delta_{t0}^e$. After the peak deformation, plastic deformation will gradually increase, the plastic free energy also gradually increase, so the internal variable which based on deformation can be assumed that:

$$F(\delta_t^e,\delta_t^p)=k_t\frac{\delta_t^p}{\delta_t^e} \quad (13)$$

k_t is material parameters, can be determined by the uniaxial tension test data of the concrete.

The expression of the energy release rate of the tensile damage is as follow:

$$Y_t = -\rho\frac{\partial\psi}{\partial D_t} = -\rho\left[1+F(\delta_t^e,\delta_t^p)\right]\frac{\partial\psi_t(\delta_t^e,D_t)}{\partial D_t} \quad (14)$$

The formula (13) and formula (9) input formula (14) can be obtained the following formula.

$$Y_t = \frac{1}{2}\left(1+k_t\frac{\delta_t^p}{\delta_t^e}\right)K_0(\delta_t^e)^2 \quad (15)$$

In the process of tension, due to the plastic deformation of concrete elastic stage is zero, so the damage energy release rate of concrete at the limit of elasticity is only related to the elastic deformation.

$$Y_{t0} = \frac{K_0(\delta_{t0}^e)^2}{2} \quad (16)$$

Y_{t0} is initial energy release rate of tensile concrete. δ_{t0}^e is limit elastic deformation of concrete.

At present, when studying the damage of concrete and rock mass, the probability distribution of Weibull is commonly used to represent the relationship between deformation and damage variables. So the damage energy release rate and damage variable of the material can also be expressed by the Weibull distribution. The formula is as follows:

$$D_t = 1-\exp\left(-\frac{(Y_t - Y_{t0})^{a_t}}{b_t}\right) \quad (17)$$

a_t and b_t are material parameters, can be determined by the uniaxial tension test data of the concrete.

Putting the formula (15) into the formula (17) can get the concrete damage evolution relation.

$$D_t = 1-\exp\left(-\frac{\left(\frac{1}{2}\left(1+k_t\frac{\delta_t^p}{\delta_t^e}\right)K_0(\delta_t^e)^2 - Y_{t0}\right)^{a_t}}{b_t}\right) \quad (18)$$

4. Plastic deformation of concrete

At present, usually using the empirical formula of plastic deformation is index and quadratic polynomial form. The common point of empirical formula is that the total deformation is more than plastic deformation, and the plastic deformation is increased with the increase of the stress[14-16]. The equations for expressing the total deformation and the plastic deformation by deformation are as follows:

$$\begin{cases} \delta_t^p = 0 & \delta_t < \delta_{t0} \\ \delta_t^p = e(\delta_t - \delta_{t0}) & \delta_t \ge \delta_{t0} \end{cases} \quad (19)$$

δ_{t0} is initial damage deformation of concrete under tensile loading. e is material parameter, which generally takes 0.85.

5. Verification of constitutive equation of tension damage

The damage constitutive equation is divided into two stages: elasticity and softening.

(a) Elastic stage

In this stage, plastic deformation does not occur, so damage does not occur.

$$\begin{cases} D_t = 0 \\ \delta_t^p = 0 & 0 \le \delta_t \le \delta_{t0} \\ \sigma_t = K_0\delta_t \end{cases} \quad (20)$$

(b) Softening stage

In this stage, the plastic deformation occurs, and the damage is accumulated. Stress state, plastic deformation and damage variable are shown in the formula (21). Damage constitutive equations should be satisfied with $\delta_t = \delta_{t0}$ and $\sigma_t = f_{t0}$. The following parameters can be determined by the tensile test of concrete, $k_t = 4.1$, $a_t = 0.72$, $b_t = 0.496$.

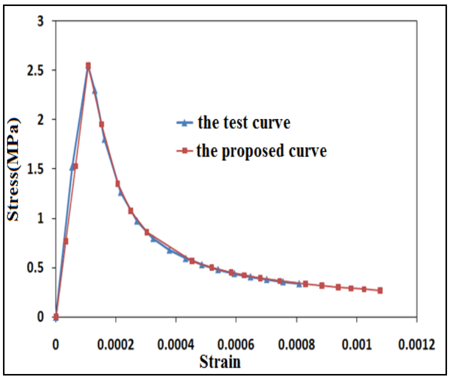
$$\begin{cases} D_t = 1-\exp\left(-\frac{\left(\frac{1}{2}\left(1+k_t\frac{\delta_t^p}{\delta_t^e}\right)K_0(\delta_t^e)^2 - Y_{t0}\right)^{a_t}}{b_t}\right) & \delta_t \ge \delta_{t0} \\ \delta_t^p = 0.85(\delta_t - \delta_{t0}) \\ \sigma_t = (1-D_t)K_0(\delta_t - \delta_t^p) \end{cases} \quad (21)$$

In order to verify the applicability of the constitutive equation of concrete damage, the test results are compared with that of Shang Renjie[18]. Test results of concrete under uniaxial tension are shown in table 1.

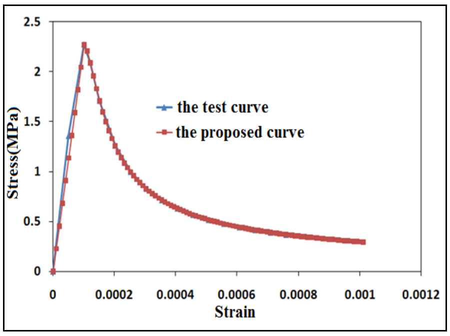
***Table 1:** The concrete uniaxial tension test results*

Specimen	Specimen length (mm)	Peak strain (10^{-3})
A[18]	100	0.20
B[18]	100	0.16
Specimen	**Peak deformation (mm)**	**Peak stress (MPa)**
A[18]	0.020	2.55
B[18]	0.016	2.27

It is showed by figure 1 and figure 2 that the proposed constitutive equation of concrete is basically consistent with the experimental results of Shang Renjie. In the case of tension, after the concrete enters the softening stage, the damage occurs and rapidly expands. With the plastic deformation increases gradually, the slope of the damage curve gradually becomes zero, and the damage index increases at this stage is not obvious.

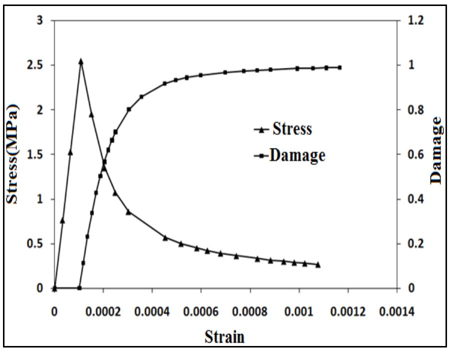


(a) Specimen A

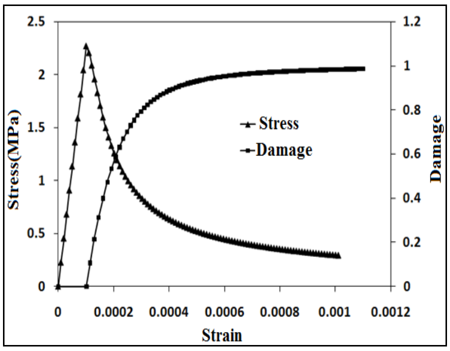


(b) Specimen B

***Figure 1**: The stress-strain curves of specimen under uniaxial tension*



(a)Specimen A



(b) Specimen B

***Figure 2:** The damage curves of specimen under uniaxial tension*

6. Engineering example verification

6.1. General situation of Koyna gravity dam

In December 11, 1967, the India Koyna gravity dam was subjected to the 6.5 magnitude earthquake, which caused many horizontal cracks in the dam, mainly focused on the slope of 629m elevation. As shown in Figure 3.

Koyna dam height is 103m, dam length is 850m. When the earthquake occurred the water level in the front of the dam is 91.75 m. The focal depth is 27km, and the epicenter distance is 13km. The epicentral intensity is Ⅷ-Ⅸ degrees. When the design reference period is 100 years, and the probability of exceeding the probability is 2%, the peak ground acceleration (PGA) is 3.99m/s^2. When the vertical earthquake and horizontal earthquake are entered in the model, the vertical acceleration peak value is the 2/3 of the horizontal acceleration peak value. As shown in Figure 4.

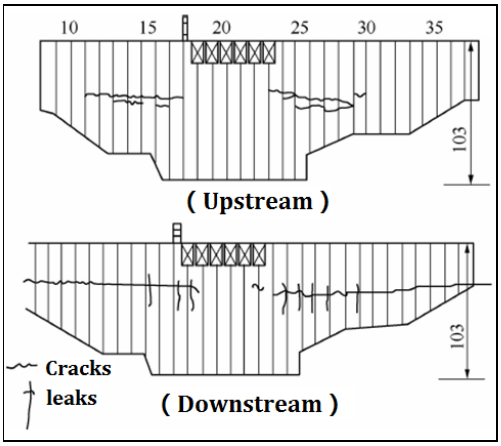


***Figure 3:** Seismic damage of Koyna dam (size unit: m)*

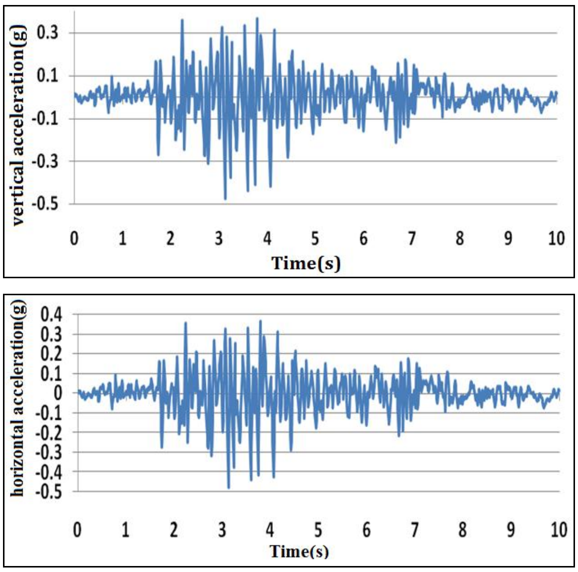


***Figure 4:** The seismic acceleration*

When the model is calculated and analyzed, the water level of the dam is normal. The Hydrodynamic pressure of reservoir water is simulated by Westergaard additive mass method, which ignores the compressibility of reservoir water. The damp is as follows: $C = \alpha M + \beta K$. Because the damping force is changed with the closing and opening of the crack, so α =0, β =0.00323. The dam material parameters are as follows,the elastic modulus is 3.1×10^4MPa, the density is 2643kg/m³, the poisson ratio is 0.15, the tensile strength is 1.90MPa, the compressive strength is 24.1MPa, the fracture energy is 200N/m, the compressive yield stress is 13MPa.

In this paper, the problem of concrete tensile damage is analyzed, and the concrete damage evolution curve, which is determined by the formula (18), is compared with the test curve of Lee and Fenvas[17]. As shown in figure 5 and figure 6. The results show that the two curves are basically consistent; therefore, the effectiveness of the proposed model is verified again. The cracking deformation is determined by the following formula.

$$\delta_t^{ck} = \delta_t^p + \frac{D_t}{1-D_t}\frac{\sigma_t L}{E_0} \tag{22}$$

L is the length of a specimen, usually the specimen length is 1m. δ_t^{ck} is the cracking strain.

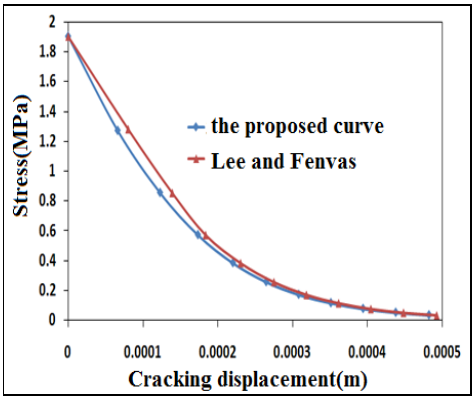


Figure 5: *The concrete stress-displacement curve*

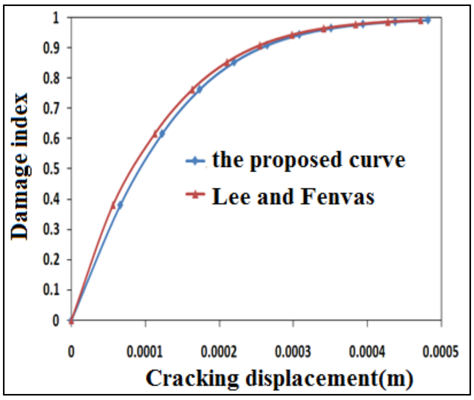


Figure 6: *The concrete damage- displacement curve*

6.2. Seismic damage analysis of Koyna gravity dam

In the process of numerical simulation, the interaction between the structure and the foundation is neglected, so the Koyna gravity dam foundation is a rigid foundation. Finite element model of Koyna gravity dam is shown in Figure 7.

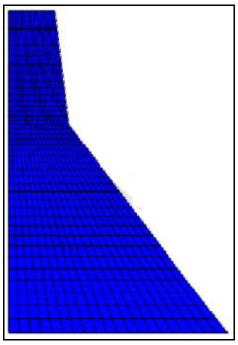


Figure 7: *Finite element model of Koyna gravity dam*

Because the damping coefficient of the dynamic analysis needs to be determined according to the frequency of the model, the modal analysis of the Koyna gravity dam is carried out firstly, the first four natural frequencies are respectively 18.86Rad/s, 49.97 Rad/s, 68.16 Rad/s, and 98.27 Rad/s. The first four vibration modal as shown in figure 8.

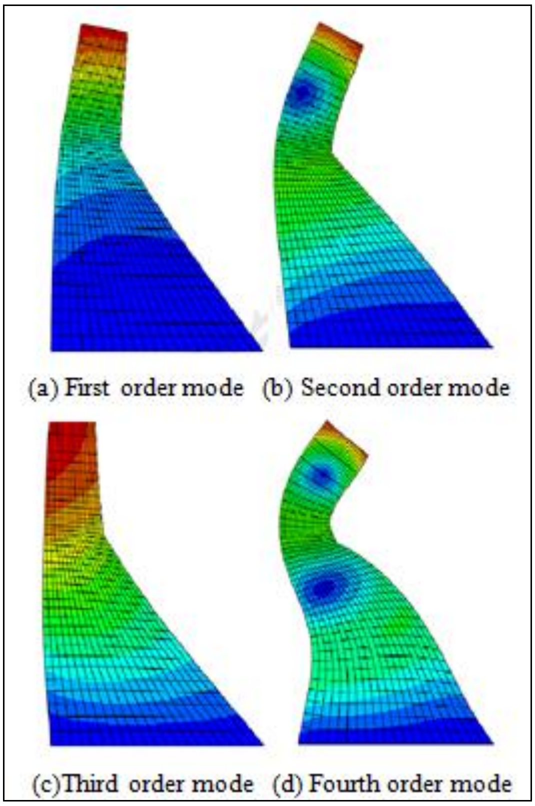


Figure 8: *The first four vibration modal of the Koyna gravity dam*

Through dynamic time history analysis of finite element software, the curves of vertical and horizontal relative displacement time history of Koyna gravity dam crest are got, as shown in figure 9. We can see from figure 9, the proposed model simulation results are basically the same with Lee Fenvas calculation results of the dam crest displacement.

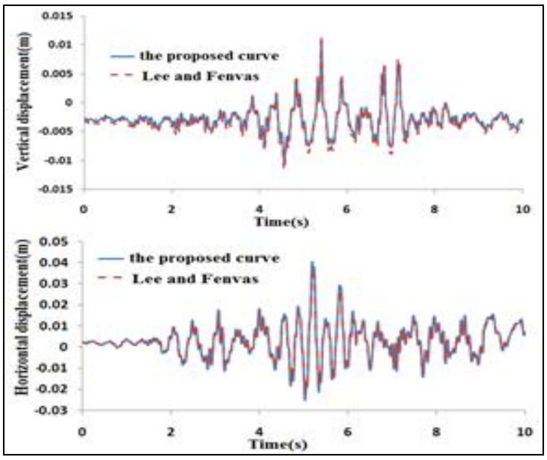


Figure 9: *The curves of vertical and horizontal relative displacement time history of Koyna gravity dam crest*

Because we cannot determine the concrete damage region, the expansion process and the extent of damage, we give the expansion process of Koyna dam damage in Figure10 and Figure 11, from which we can see concrete tension damage area and the size of the damage extent. Under the seismic load, Koyna dam tensile damage mainly occurred in the dam heel and the upper part of body. Along with the increasing duration of earthquake, the damage extent also increases gradually, and ultimately cracks in the upper part of the dam are combined together. The analysis results are basically consistent with the actual earthquake situation of dam, see Figure 3, and the calculation results of literature [17] are roughly the same.

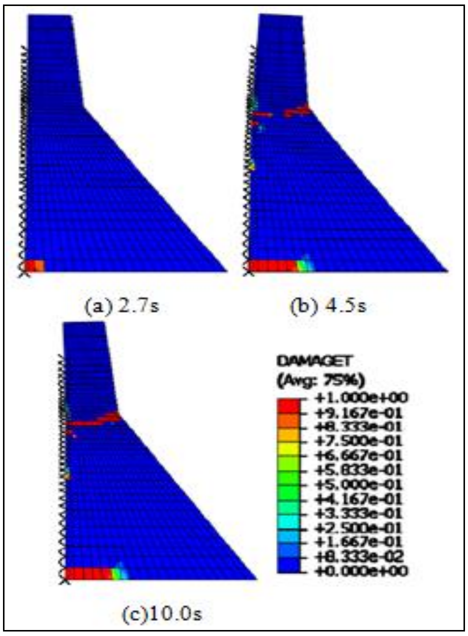


Figure 10: *The tensile damage evolution process of the proposed model*

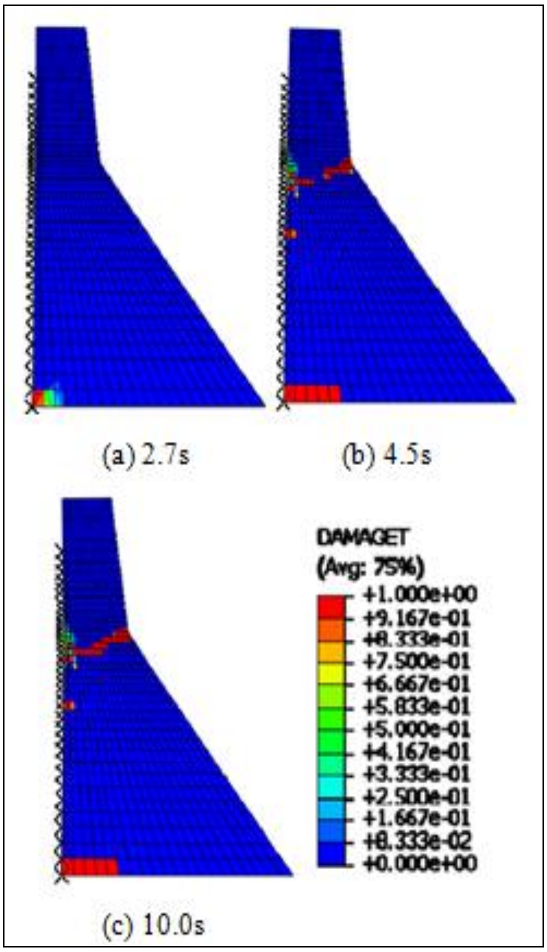


Figure 11: *The tensile damage evolution process of the literature [17]*

Under the seismic load, the Koyna dam first has the tensile damage, so the tensile damage and stiffness degradation of concrete are basically the same, as shown in Figure 12 and Figure 13.We can see from Figure 12, along with the duration of seismic load, tensile damage of Koyna dam increases gradually, but the concrete stiffness degradation occurs repeatedly, the reason is that the stiffness recovery effect of concrete structure from tension to compression process is considered in the plastic damage model. Dam heel and the downstream slope have tensile damages; the upper part of the dam also has tensile damages. Under the seismic load, the vertical seismic load makes the cracks open and close again. After the end of the earthquake process, tensile damage zone is mainly located in the dam heel, the upper part of the upstream dam and the downstream dam slope, as shown in Figure 12, most of the cracks in the dam body are in closed state, and most of the concrete stiffness recovers. The analytic results and the results in the literature [17] are roughly the same, as in Figure 13, so the concrete tensile plastic damage constitutive model in this paper can accurately describe the nonlinear characteristics of concrete under seismic load, and can be more accurate to describe the damage of concrete.

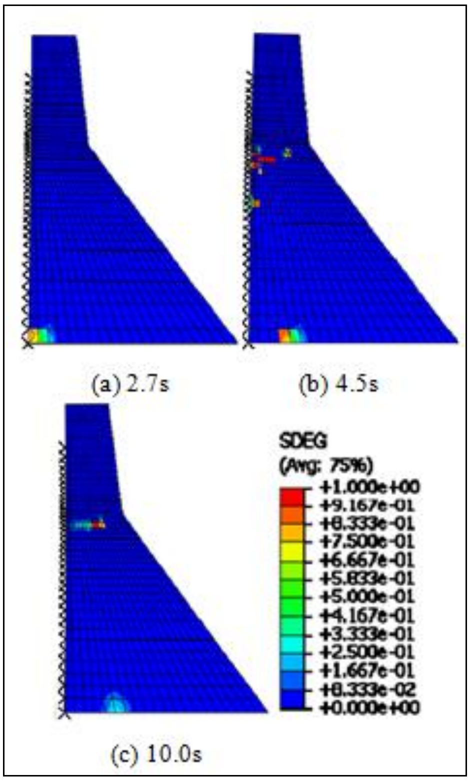


Figure 12: *The stiffness degradation process of the proposed mode*

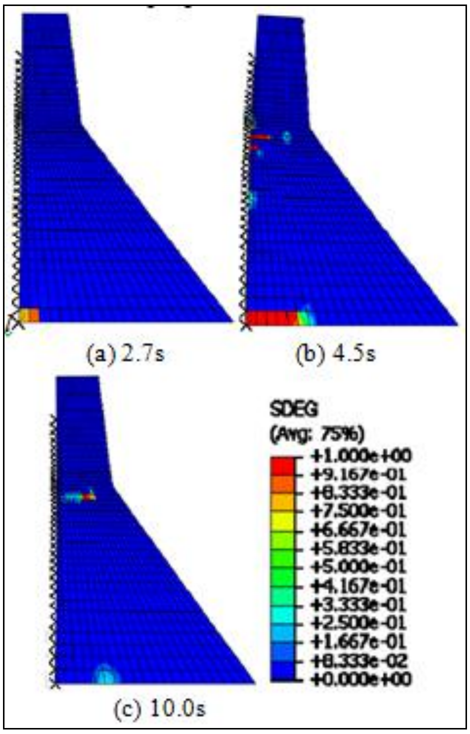


Figure 13: *The stiffness degradation process of the literature [17]*

7. Conclusions

The following conclusions can be obtained through the research of this paper.

(1) Based on the theory of damage mechanics, continuum mechanics and irreversible thermodynamics, considering the coupling relation of plastic stain and damage, the certain function relation is supposed between the plastic free energy and the elastic free energy, so the damage energy release rate is defined. According to the Weibull distribution curve, the relationship between the damage variable and the energy release rate is established, thus the concrete tensile damage evolution equation is deduced. Finally, based on the plastic characteristics of concrete under the action of tensile load, the empirical formula of plastic deformation is obtained. Therefore, the tensile plastic damage constitutive model of concrete is established.

(2) The uniaxial tension test results and engineering examples are used for verification of the effectiveness and applicability of this model. Analysis results show that the tensile plasticity constitutive model can more truly reflect the concrete tensile damage evolution process.

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