



On Solving the Shortest Path of 3D Terrain Based on Remote Sensing Elevation Data and Ant Colony Optimization

JUN ZHENG¹, ZHONG-JU MENG² AND QUAN-SHENG HAI³

¹Network Information Center, Baotou Teachers' College, Baotou, China

²College of Ecology and Environment, Inner Mongolia Agricultural University, Hohhot, China

³Department of Geographical Sciences, Baotou Teachers' College, Baotou, China

Email: zhengjun_025@163.com

Abstract: Solving the shortest path of 3D terrain is widely requested. However, researchers have not been able to get a good solution due to the limitations of traditional technical means. Thanks to recent development and maturation of remote sensing technology, this paper presents a method of building 3D terrain model with remote sensing elevation data and finding the shortest path with Ant Colony Optimization (ACO). At the beginning, the paper analyzes the mainstream methods of solving the shortest path, and evaluates their advantages and disadvantages when applied to solving the shortest path in 3D space. In view of the excellent performance of the ACO, it is eventually adopted as the algorithm to solve the shortest path of 3D terrain. After introducing the principle and algorithm model of traditional 2D ACO, the paper improves the algorithm by re-designing the heuristic function, pheromone update strategy, and transfer rules, thereby applying the traditional algorithm to 3D space. After that, the author downloads and resamples the remote sensing elevation data of Shipping County, Yunnan Province, extracts the elevation data from the resampled data, and established a 3D mountain environment model. Based on the model, the author successfully conducts an experiment of solving the shortest path of 3D terrain with ACO.

Keywords: remote sensing, ant colony optimization, the shortest path, 3D terrain

1. Introduction

Solving the shortest path is a common concern and the object of study by researchers from various backgrounds[1-3]. In a 2D space, path solving is one of the basic research targets in graph theory. Researchers often have to solve the shortest path when using graph theory to build math models for all sorts of practical problems, such as finding the shortest distance in logistics transportation, site selection for logistics centers, planning of communication lines, to name but a few. When it comes to 3D space models, the issue of solving the shortest path in 3D space arises. A good solution is needed for many practical issues, e.g. optimizing urban 3D pipe network, identifying the optimal route for submarines, improving the flight path of unmanned planes, selecting the routes for road pavement, and planning the line of march. As a result, it is very urgent to solve the question of finding and calculating the shortest path in 3D space.

Fortunately, thanks to the rapid development of remote sensing technology, high-resolution satellite remote sensing images and stereopairs are now available to researchers. After processing and analyzing these data, researchers can easily extract dynamic surface topography and terrain features in real time[4-5]. These feature data help to generate 3D spatial data easily and quickly. Against this backdrop, this paper proposes to solve the shortest path in 3D

space through the combination of remote sensing elevation data and artificial intelligence algorithm.

2. The Selection of Intelligent Algorithms

At present, researchers at home and abroad mainly use the following algorithms in path planning: A* search algorithm, artificial potential field method, genetic algorithm and ACO.

A* search algorithm is a heuristic search algorithm first proposed in 1968[6]. The algorithm pinpoints the best position through evaluation of the neighboring or adjacent positions of each search position, searches from the best position all the way to the target position, and finally determines the least-cost path from the starting position to the target position. This method improves search efficiency by eliminating a great number of unnecessary search paths. A* search algorithm has good performance in 2D space, but runs rather slowly in 3D space due to the drastic increase of state places.

The artificial potential field method is proposed by Khatib in 1986[7]. Its basic idea is to virtualize the motion space of a robot into a gravitational potential field so that the robot moving in the field is attracted by the gravitational field of its target position and repelled by the repulsion field of the obstacle, and control the movement by the combined effect the resultant force. Although it is simple and easy to implement, the algorithm often stops at local optimal solutions and lacks stability.

Developed at the intersection of life science and engineering science, genetic algorithm is a new algorithm that finds global optimal solution through randomized search and simulation of the natural evolution process. The algorithm is a four-step process, involving coding, selection, crossover and mutation. To use this algorithm, the user has to abstracts a specific problem to be optimized as a set of individuals, and encodes a “0”, “1” string for each individual. Next, substitute the encoded individuals into the fitness function, and select the superior individuals according to fitness level to form a new population. Crossing and mutation of the newly formed population re-create a new population. The new population is more adaptable to the environment than previous generations. Find an approximate optimal solution by decoding the individuals of the last population[8]. When genetic algorithm is adopted for 3D path planning, it works well in a simple environment, but when the environment is complex, it will be difficult to find a feasible path to satisfy the constraints.

The ACO is a bio-inspired optimization algorithm proposed by the Italian scholar Dorigo et al. in early 1990s in light of the group foraging habit of ants. When ants go out foraging, they leave a volatile substance called pheromone on the path. The more ants pass through the path, the higher the accumulation of pheromone is. During the foraging process, the ants that pass through later can perceive the strength of this substance on the path and tend to select the path with a high concentration of pheromone. The group decision-making ensures that ants can find the best path from the nest to the food. This algorithm has been widely used, and has been applied to the traveling salesman problem, graph coloring problem, quadratic assignment problem, robot path planning problem, workpiece scheduling problem, vehicle routing problem and network routing problem[9]. The ACO has many advantages over other algorithms, such as being simple and easy to implement, less dependent on empirical parameters, insensitive to initial value and parameter selection, strong robustness, fast convergence speed and not easy to fall into local optimal solution. If it is extended from 2D space to 3D space, the ACO would feature little storage space and low computational complexity. Based on the above analysis, the author decides to use the ACO to solve the shortest path of 3D terrain space.

3. Implementation of the ACO

3.1. The Biological Description of the ACO

According to bionics studies, ants have no vision. To make up for the loss of sight, they would release pheromone, a kind of secretion, during path finding. Ants tend to choose the path of high pheromone concentration. Therefore, when a certain path is relatively short, the pheromone volatilization on this

path is smaller over the same period, resulting in higher pheromone concentration. The number of ants that choose this path will increase. In turn, the more ants choose this path, the higher amount of pheromones released on this path. Over time, the vast majority of ants would move along the shortest path, also known as the optimal path. Citing Figure 1 as an example, the author would explain the path selection principle of the ACO.

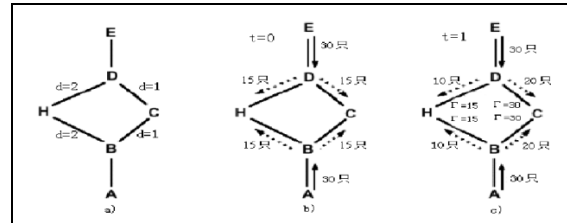


Figure 1: The Path Selection Principle of the ACO

As shown in Figure 1.a, suppose Point A is the nest, Point E is the food point, and Points C, B, D & H are the path points; assume the length of Paths BC & CD are both 1 and that of Paths BH & HD are both 2. If 30 ants go out foraging at the same time, the following situations would occur.

At $t=0$, there is no pheromone on any path, and the probability that ants choose each path is equal. As shown in Figure 1.b, 15 ants select Path BHD, and the 15 ants choose Path BCD. The ants leave pheromones on their path, which evaporate over time. Assuming that the speed of advance of each ant is 2, the release and evaporation rate of pheromones are 1 per unit distance, then at the next moment, $t=1$, the pheromone accumulated on Path DCB is 30, and the pheromone accumulated on Path DHB is 15. Thus, when 30 ants return to nest A again, 10 ants select Path DHB, and 20 choose Path DCB, as shown in Figure 1.c. As the pheromone on all paths continues to get enhanced and volatilized, eventually the ants only move on the shortest path ABCDE from nest to food, while other longer paths are gradually abandoned.

3.2. The Math Model of Traditional Ant Colony Algorithm

Established on the basis of bionics characteristics of ant foraging, the ACO is illustrated with the following math model:

Without loss of generality, let the number of ants in an ant population be M , and the probability that ants at position I at time t choose to arrive at the position j is:

$$P_{ij}^k(i, j) = \begin{cases} \frac{\tau_{ij}^\alpha(t) \eta_{ij}^\beta(t)}{\sum_{u \in \text{allowed}_k} \tau_{iu}^\alpha(t) \eta_{iu}^\beta(t)} & , j \in \text{allowed}_k \\ 0 & , j \notin \text{allowed}_k \end{cases} \quad (1)$$

Where, $\eta_{ij}^\beta(t)$ is the heuristic function displaying the heuristic information of the ants moving from position i to position j at time t . β is the parameter to adjust the influence degree of the heuristic function to the

decision. The larger the value of β , the greater the heuristic function plays in the transfer. $\tau_{ij}^{\alpha}(t)$ stands for the pheromone concentration on path $\langle i, j \rangle$ at time t , and α is the importance factor of pheromone. The larger the value of α , the greater the pheromone concentration plays in the transfer. $allowed_k$ means the position that the k -th ant can reach.

During the path-finding process from the starting point to the target point, the ants keep releasing pheromone, while the pheromone on each path gradually evaporates. Therefore, the pheromone on each path needs to be updated in real time. The pheromone update formula is:

$$\begin{cases} \tau_{ij}(t+1) = (1 - \rho)\tau_{ij}(t) + \Delta\tau_{ij} \\ \Delta\tau_{ij} = \sum_{k=1}^M \Delta\tau_{ij}^k \end{cases} \quad 0 < \rho < 1 \quad (2)$$

Where, ρ is the pheromone evaporation factor describing the degree of evaporation; $\tau_{ij}(t)$ denotes the pheromone concentration on the path $\langle i, j \rangle$ at time t ; $\Delta\tau_{ij}^k$ refers to the amount of pheromone released by the k -th ant on the path $\langle i, j \rangle$; $\Delta\tau_{ij}$ represents the total concentration of pheromones released by all ants on the path $\langle i, j \rangle$.

In the pheromone update formula, $\Delta\tau_{ij}^k$ refers to the amount of pheromone released by the k -th ant on the path $\langle i, j \rangle$. According to different pheromone release methods, the traditional ACO can be divided into three different basic models, namely: ant quantity system, ant density system and ant cycle system. The models are described as follows:

ant quantity system:
$$\Delta\tau_{ij}^k = Q/d_{ij} \quad (3)$$

Where, Q is a constant representing the pheromone concentration of the ACO and d_{ij} illustrates the length of the path $\langle i, j \rangle$ covered by the k -th ant.

ant density system:
$$\Delta\tau_{ij}^k = Q \quad (4)$$

Where, Q is a constant representing the pheromone concentration of the ACO.

ant cycle system:
$$\Delta\tau_{ij}^k = Q/L_k \quad (5)$$

Where, Q is a constant representing the pheromone concentration of the ant colony algorithm, and L_k illustrates the length of the path covered by the k -th ant.

4. Implementation of the Solution to the Shortest Path of 3D Terrain Based on Remote Sensing Elevation Data and Ant Colony Optimization

4.1. Data Preparation

In this study, the author selects the 15km long square area centering on the location at 102.5°E, 24.5°N as the research object. Situated in Shiping County, Yunnan Province, the area is dominated by mountainous terrain.

The author downloads GDEM DEM 30M digital elevation data of the research object from Geospatial Data Cloud (<http://www.gscloud.cn>). The elevation data thus acquired is shown in the following figure:

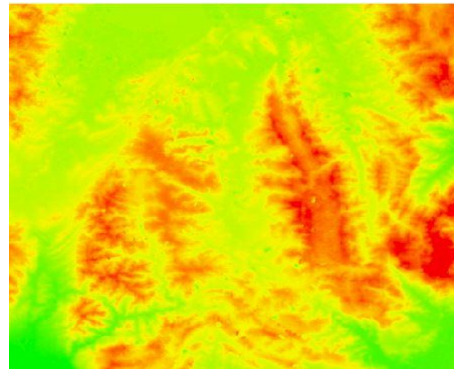


Figure 2: Elevation Data of the Research Object

Due to the excessively large size, the original remote sensing data needs to be re-sampled. In this study, cubic convolution interpolation[10] is used for resampling. The resampled results are shown in the following figure:

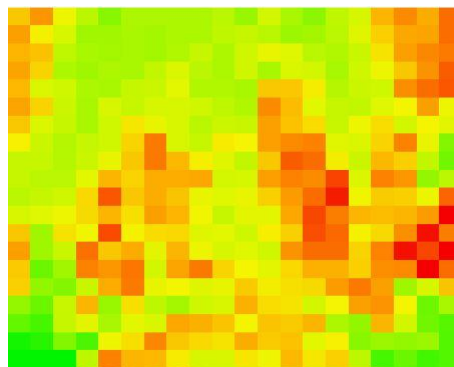


Figure 3: Resampled Results

4.2. 3D Environment Modeling Based on Remote Sensing Images

Environmental modeling aims at simulating real 3D spatial environment information in an abstract way. In this study, the geo-elevation data extracted from the above resampling process is used to simulate real mountain topography. The simulation results are shown in the following figure:

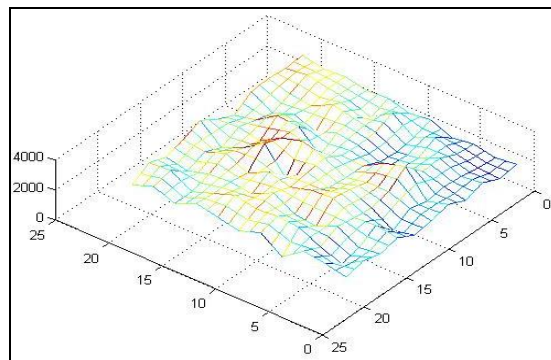


Figure 4: Mountainous Terrain Simulation Diagram

On the basis of the mountainous terrain simulation diagram, build a 3D space coordinate system with the vertex at the lower left corner as the coordinate origin O, the ascending direction of the longitude as x-axis, the ascending direction of latitude as y-axis and the ascending direction of height as z-axis. Then, map the vertex at the lower left corner of the mountainous terrain model to the coordinate origin O, and establish the path space of the simulated mountainous terrain ABCD-EFGH with the length, width and height of the mountain as the boundary. After the 3D path space is established, discretize the 3D space into grid points by dividing the space into aliquots. The method goes as follows: obtain n-1 spaces $\Pi_i (i=1,2,\dots,n-1)$ by dividing the space of Edge AB into n equal segments along axis-x. Likewise, divide the space of Edge AE into m equal segments along axis-y, and divide the space of Edge AD into k equal segments along axis-z. With the above operations, it is easy to find the intersections of these segmented planes, which are the 3D space path points to be planned. In this way, the 3D space ABCD-EFGH is discretized into a number of grid points in a 3D space. The results are shown in the following figure:

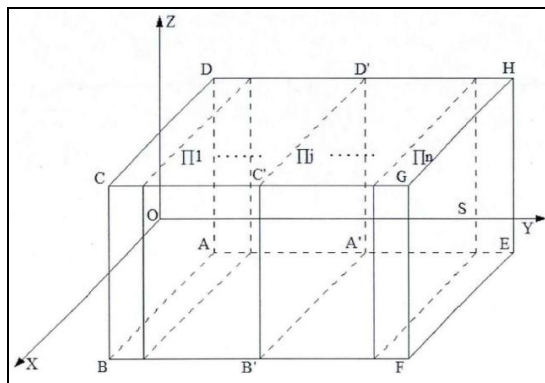


Figure 5: Grid Points in 3D Space

4.3. Solution to the Shortest Path of 3D Space with the ACO

4.3.1. Design of the Heuristic Function

The heuristic function is designed based on the principle of shortest path reachability and path distance. According to Equation 1, the heuristic function plays an important role in planning a feasible path as it affects the convergence and stability of the ACO. For any point (i, j, k), the heuristic function H(i,j,k) goes as follows:

$$H(i, j, k) = [S(i, j, k)]^{w_1} \times [D(i, j, k)]^{w_2} \times [Q(i, j, k)]^{w_2} \quad (6)$$

S (i, j, k) is the path reachability factor. If the path is reachable, its value is 1. If the path is unreachable, its value is 0. w_1 is a constant representing the influence coefficient of path reachability factor.

D (i, j, k) is the shortest distance factor, while w_2 is a constant representing the influence coefficient of the shortest factor. D (i, j, k) causes the ants to choose the

closer path. Assuming that a is the current point and b is the next point, D (i, j, k) is calculated as:

$$D(i, j, k) = 1/\sqrt{(x_a - x_b)^2 + (y_a - y_b)^2 + (z_a - z_b)^2} \quad (7)$$

Q (i, j, k) is the distance from the target location factor, while w_2 is a constant representing the influence coefficient of the distance from the target location factor. Assuming that a is the target point, b is the next point, Q (i, j, k) is calculated as:

$$Q(i, j, k) = 1/\sqrt{(x_a - x_b)^2 + (y_a - y_b)^2 + (z_a - z_b)^2} \quad (8)$$

4.3.2. The Design of Pheromone Update Strategy

The pheromone is stored in discrete grid points of the 3D model and the pheromone at each discrete point decreases with time. When an ant passes by a certain point, the pheromone would increase. According to Pheromone Update Equation 2, the pheromone update model of all discrete points on the 3D model is:

$$\tau_{ijk}(t + 1) = (1 - \rho)\tau_{ijk}(t) \quad (9)$$

Where, t refers to time, τ_{ijk} the pheromone value on the point (i,j,k), and ρ the attenuation coefficient of the pheromone.

When an ant passes by a certain point, the pheromone at the point would be updated. The ant quantity system (See Equation 3) to update the pheromone model, which goes as follows:

$$\begin{cases} \tau_{ijk}(t + 1) = (1 - \rho)\tau_{ijk}(t) + \Delta\tau_{ijk} \\ \Delta\tau_{ijk} = \sum_{m=1}^M 1/d_{ijkm} \end{cases} \quad 0 < \rho < 1 \quad (10)$$

Where, t refers to time, τ_{ijk} the pheromone value on the point (i, j, k), ρ the attenuation coefficient of the pheromone, and d_{ijkm} the length of the path covered by the m-th ant passing by the spatial point (i, j, k).

4.3.3. The Design of Transfer Rules

Assuming that p_a is a point on the plane Π_a , p_{a+1} is a point on the plane Π_{a+1} , and allowed is the set of all accessible points on the paths from p_a to p_{a+1} , then the probability that an ant chooses to go to p_{a+1} from p_a is:

$$S_{a+1} = \begin{cases} \frac{\tau_{a+1}H_{a+1}}{\sum \tau_{a+1}H_{a+1}} & a + 1 \in \text{allowed} \\ 0 & a + 1 \notin \text{allowed} \end{cases} \quad (11)$$

Arrange the probabilities of every selected points p_{a+1} in ascending order in the interval [0,1], and randomly generate a number p from the interval. Find the interval which the S_{a+1} corresponding to the number p falls into for it contains the point to be chosen for the next path.

5. Experimental Results

The program is set as follows: abstract the 3D terrain space as a 21x21x21 3D space of discrete points, set the number of ants m=50, the maximum number of cycles n=200, the path reachability factor $w_1 = 1$, the influence coefficient of the shortest distance factor $w_2 = 1$, the influence coefficient of the distance from

the target point $w_3=1$, and the pheromone attenuation coefficient $\rho=0.4$, and regard the point $[0,10,2]$ START as the starting position and the point $[0,12,1]$ END as the target position. After running the program, the author finds the shortest path of the 3D mountain terrain. The results are shown in the following figure.

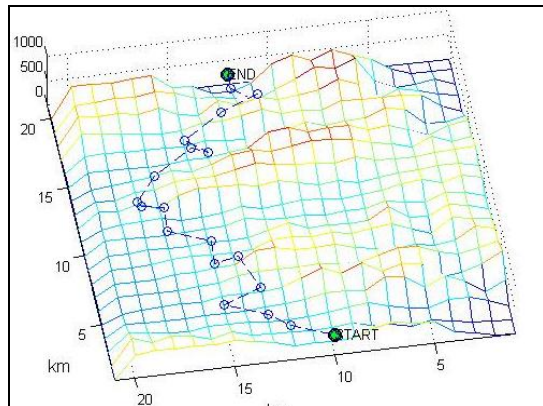


Figure 6: Search Results of the Shortest Path of 3D Mountain Terrain

Acknowledgements

This work was financially supported by “the College Science Research Project of Inner Mongolia, China “(Grant No. NJZY16238).

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