ISSN 0974-5904, Volume 10, No. 03

DOI:10.21276/ijee.2017.10.0331

International Journal of Earth Sciences and Engineering

www.cafetinnova.org

June 2017, P.P. 693-696

Stability Analysis of Fluid Flow through a Flexible Pipe by Energy Balance Method for Non-Axisymmetric Disturbances

S ANBU KUMAR AND MUNENDRA KUMAR

Departmentof Civil Engineering, Delhi Technological University, Bawana Road, New Delhi - 110 042, INDIA

***Email:** anbukumars@yahoo.com*

Abstract: *In this paper, the energy balance technique is used to study the stability of flexible pipe. The pipe flow is considered as laminar flow. The outer surface of the pipe is taken as shroud. The different curves are plotted for the various flexibility parameters i.e.* $\Gamma = \sqrt{\rho_f V^2 / G}$ *and* $\mu_r = \mu_s / \mu_f$*., where* μ_s *is the viscosity of flexible material, and* μ_f *is viscosity of fluid and* ρ_f*, V and G are fluid density , centre line velocity of pipe and modulus of rigidity. It is found that the various energy terms are responsible for the stability of flexible pipe flow.*

Keywords: *energy balance technique. Flexible pipe, stability analysis*

1. Introduction

The flexible pipe flow is normally found in nature like flow of blood through veins, pharmaceutical industries etc. Kramer's [1, 2, 3, 4] found that flexible material reduce the drag on the flat surface flow. The flexible pipe flow will remain laminar for the longer period due to flexibility of surface. Reynolds [5] performed the experiment on the rigid pipe flow and observed that the center line modes are responsible for making the flow unstable. The induced disturbances make the flow unstable (turbulent).

Davey and Drazin [6] did the numerical study on the rigid pipe flow and found that flow of rigid pipe is stable at all Reynolds numbers *R* and all axial wave numbers α for all to infinitesimal axisymmetric disturbances. Garg and Rouleau [7] and Salwen and Grosch [8] also confirmed same result by numerical method. They also observed that centerline modes are more unstable as compared to wall modes.

Hamadiche and Gad-el-Hak [9] did the numerical study on the flexible tube for axisymmetric and non-axisymmetric disturbances for the normal plus tangential compliance (N+T) problem. They found that flexile pipe is unstable at all Reynolds number R i.e. low R, medium R and high R. Gajjar, Gibson and Sen [10], and particularly Gibson [11] studied the only normal compliance (N). In present paper, normal plus tangential compliance (N+T) problem and normal compliance (N) are analyzed for flexible pipe flow for 3D disturbances

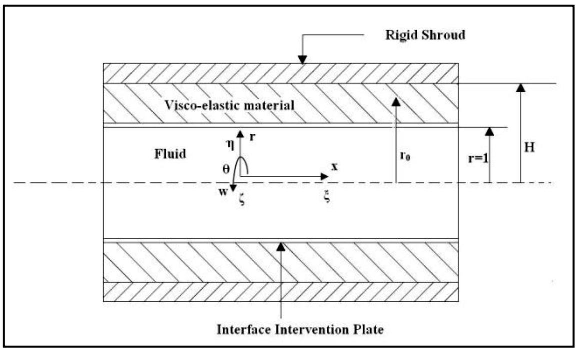

Figure 1: *Configuration of flexible pipe flow*

2. Formulation

Figure 1 shows the configuration of the flexible pipe flow. The interface of fluid and visco-elastic is considered at *r* = 1 and external surface of rigid pipe at *r* = *H*. The non-axisymmetric (3-D) disturbances are induced in the fluid flow field. The velocity components (u, *v* & *w*) for fluid-side and visco elastic material displacement components ($\hat{\eta}$, $\hat{\zeta}$ and $\hat{\xi}$) are shown in figure 1 for all three directions (x , *r* and θ). All parameters are normalized with radius *r* and centre line velocity *V*. is $R = V r_o^* / \nu$, where, R is Reynolds number, ν is the kinematic viscosity of fluid. In this paper flexible pipe flow is considered for laminar flow condition.

2.1 The fluid side energy equations

The derivations of various energy terms can be seen from any standard energy analysis paper. The energy equations of non-axisymmetric disturbances modes, for the fluid-side, are given as follows:

$$2\,\alpha\, c_i\, I_1 = I_2 + I_3 + B_4 + B_5 \qquad (1)$$

Where, I_1, I_2, I_3, B_4 *and* B_5 are as follows:

Received: January 17, 2017; Accepted: May 28, 2017; Published: June 30, 2017

International Journal of Earth Sciences and Engineering, 10(03), 693-696, 2017, DOI:10.21276/ijee.2017.10.0331

$$I_1 = \int_0^1 r \frac{\overline{(u^2)} + \overline{(v^2)} + \overline{(w^2)}}{2} dr,$$

$$I_2 = -\int_0^1 r\, \bar{u}'\, \overline{(u\, v)}\, dr,$$

$$I_3 = -\frac{1}{R}\int_0^1 r\,[\,\overline{\omega_x{}^2} + \overline{\omega_r{}^2} + \overline{\omega_\theta{}^2}\,]\, dr,$$

$$B_4 = \left[\overline{(v\tau_{rr})} + \overline{(u\tau_{xr})} + \overline{(w\tau_{r\theta})}\right]_1,$$

$$B_5 = -\frac{2}{R}\left[\overline{\left(u\frac{\partial v}{\partial x}\right)} + \overline{\left(v\frac{\partial v}{\partial r}\right)} + \overline{\left(\frac{w}{r}\frac{\partial v}{\partial \theta}\right)} - \overline{\left(\frac{w^2}{r}\right)}\right]_1$$

2.2 *The flexible side energy equations*

The flexible side energy equations can be written in terms of displacements of flexible material.

$$2\alpha c_i\, J_1 = J_2 + J_3 + D_4 + D_5 + D_6 \qquad (2)$$

Where,

$$J_1 = \int_1^2 \frac{r}{2}\left[\overline{(-i\alpha c\xi)^2} + \overline{(\eta')^2} + \overline{(-i\alpha c\zeta)^2}\;\right]dr$$

$$J_2 = -\frac{1}{\Gamma^2}\int_1^2 r\left[\begin{array}{l}\overline{\omega_{sx}(-i\alpha c\,\omega_{sx})} + \overline{\omega_{sr}(-i\alpha c\,\omega_{sr})}\\ +\overline{\omega_{s\theta}(-i\alpha c\,\omega_{s\theta})}\end{array}\right] dr$$

$$J_3 = -\frac{\mu_r}{R}\int_1^2 r\left[\overline{(-i\alpha c\,\omega_{sx})^2} + \overline{(-i\alpha c\,\omega_{sr})^2} + \overline{(-i\alpha c\,\omega_{s\theta})^2}\right]dr$$

$$D_4 = \left[\overline{(-i\alpha c\eta)\sigma_{rr}} + \overline{(-i\alpha c\xi)\sigma_{xr}} + \overline{(-i\alpha c\zeta)\sigma_{r\theta}}\;\right]_1$$

$$D_5 = \frac{2}{\Gamma^2}\left[\overline{(-i\alpha c\eta)\eta'} + \overline{(-i\alpha c\xi)(i\alpha\eta)} + \frac{\overline{(-i\alpha c\zeta)(in\eta)}}{r} - \frac{\overline{(-i\alpha c\zeta)\zeta}}{r}\right]_1$$

$$D_6 = -\frac{2\mu_r}{R}\left[\begin{array}{l}\overline{(-i\alpha c\eta)(-i\alpha c\eta')} + \overline{(-i\alpha c\xi)(\alpha^2 c\eta)} \;\; + \dfrac{\overline{(-i\alpha c\zeta)(n\alpha c\eta)}}{r} - \\ \dfrac{\overline{(-i\alpha c\zeta)^2}}{r}\end{array}\right]_1 .$$

2.3 *Boundary conditions*

The boundary conditions for the combined fluid-solid problem, for different azimuthal wave numbers *n*, are given as below.

2.3.1 Centerline of the pipe, $r = 0$:

for n = 1:

$$u(0) = 0,\;\; v + iw = 0,\;\; v'(0) = 0 \;\; \text{(or)} \;\; w'(0) = 0). \qquad (3)$$

for n > 1:

$$u(0) = 0,\quad v(0) = 0,\quad w(0) = 0. \qquad (4)$$

2.3.2 Outer rigid surface

Outer surface of the visco-elastic pipe is at $r = H$. The boundary conditions are given as follows

$$\xi(H) = 0,\quad \eta(H) = 0,\quad \zeta(H) = 0 \qquad (5)$$

2.3.3Fluid and the visco-elastic interface (r = 1)

The boundary conditions at the interface are continuity of velocities and continuity of stresses.

$$\frac{\partial\hat{\xi}}{\partial t} = \hat{u} + \hat{\eta}_w\bar{u}_w',\quad \frac{\partial\hat{\eta}}{\partial t} = \hat{v},\quad \frac{\partial\hat{\zeta}}{\partial t} = \hat{w}; \qquad \text{(6a, b, c)}$$

Particularly, eq. (6a,b,c) is the tangential no-slip boundary condition. The above interface velocities can also be written as follows:

$$-i\alpha c\xi_w = u_w + \eta_w\bar{u}_w',\; -i\alpha c\eta_w = v_w,$$
$$-i\alpha c\zeta_w = w_w. \qquad \text{(7a, b, c)}$$

The stress matching conditions at the interface are given as follows:

$$\sigma_{rr} = \tau_{rr},\;\; \sigma_{rx} = \tau_{rx},\;\; \sigma_{r\theta} = \tau_{r\theta}; \qquad \text{(8a,b,c)}$$

3. Finite Difference Technique

The finite difference technique is used to solve the differential equations for the fluid and the flexible material..

The differential equations may be written as given below:

$$[A_{IJ}][\Gamma_J] = [P_I],\, I,\, J = 1, 2, 3, 4, ..(2N+2) \qquad (9)$$

Here, N is the number of intervals in the above equation. $[A_{IJ}]$ is the coefficient matrix. The finite difference method is solved by computer programming in FORTRAN and Programs are compiled by Lahey Fijtsu complier.

4. Results

The different energy curves are plotted for the various parameters for normal (N) and normal plus tangential (N+T) problem. The energy balance method has been used to analyze fluid-solid stability problem.

4.1 The N+T problems

The stability of normal plus tangential compliance problem (N+T) for the non-axisymmetric mode has been studied here.

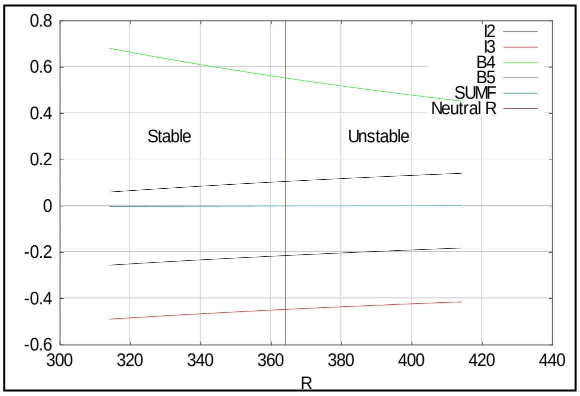

Figure 2: *The variation of different energy terms for the N+T for the fluid-side. H =2.0,* μ_r *= 0.0, Γ = 8, α = 2.076*

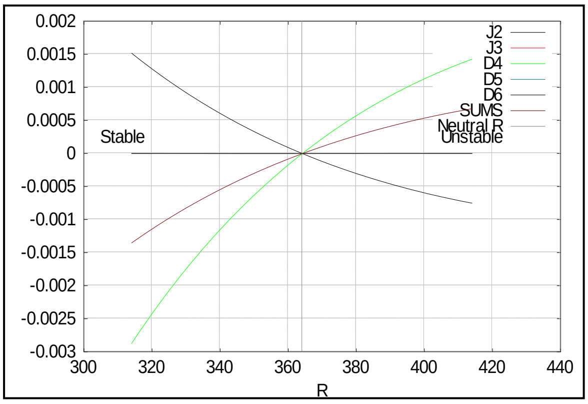

Figure 3: *The variation of different energy terms for the N+T, for the solid-side. H =2.0, μ_r = 0.0, Γ = 8, α = 2.076*

Figures 2 and 3 show the variation of the different energy terms respectively for the fluid-side and the solid-side. These curves are drawn different Reynolds number *R* versus different energy terms for the fluid-side and the solid-side both. These curves correspond to the nose region of the neutral curve for the non-axisymmetric modes. It is seen in figure 2 that production term I_2 is positive everywhere in the range, but relatively small in magnitude. The terms I_3 and B_5 are negative everywhere, both being large, with I_3 larger in size than B_5. The tractive work term B_4 is positive everywhere and very large in magnitude. So, the tern B_4 is the main source of energy in the fluid-side. Basically, therefore, B_4 is balanced by $I_3 + B_5$ or ($B_4 \rightarrow I_3 + B_5$).

Next in figure 4, it is seen that solid-side energy terms corresponding to the case above in figure 3. It is observed that the term J_2 is positive in stable region and negative in the unstable region and is large in magnitude. The terms J_3 and D_6 are zero, since μ_r =0. The tractive work term D_4 is positive in the unstable region and negative in the stable region and is the largest term. The D_5 term is positive in the stable region and negative in the unstable region and is negligibly small. Basically, D_4 is balanced by $J_2 + D_5$ or ($D_4 \rightarrow J_2 + D_5$).

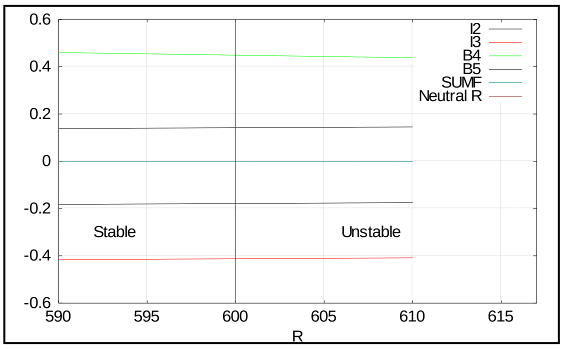

Figure 4: *The variation of different energy terms for the N+T for the fluid-side. H =2.0, μ_r = 0.1, Γ = 8, α = 2.632*

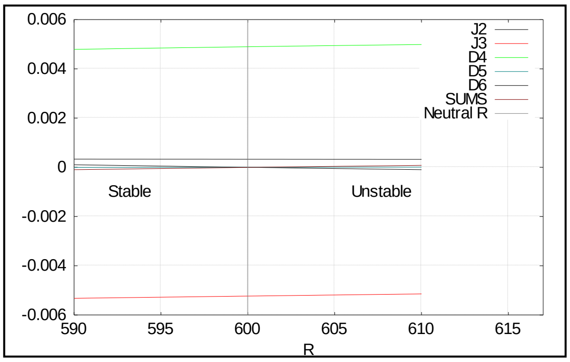

Figure 5: *The variation of different energy terms for the N+T, n = 3 mode for the solid-side. H =2.0, μ_r = 0.1, Γ = 8, α = 2.632*

Next in figures 4 and 5, the energy terms for the fluid-side and the solid-side respectively for μ_r = 0.1 were studued. The parameters are N+T, H =2.0, μ_r = 0.1, Γ = 8, α = 2.632. In figure 4 for the fluid-side, it is found that the production term I_2 is positive everywhere in the range and is not very large. The term I_3 is negative everywhere and is large in magnitude. The B_4 term is positive everywhere and large. This term B_4 is the main source of energy in the fluid-side. Also the wall normal work term B_5 is negative everywhere and is not very large. This, amongst the two source terms, B_4 is larger than I_2, and amongst the two sink terms, J_3 is larger than B_5. Here, $B_4 + I_2$ is balanced by I_3 + B5 or ($B_4 + I_2 \rightarrow I_3$ + B5).

Figure 5 shows the variation of the different energy terms for the solid-side and parameters are same as given above in the fluid-side energy distribution in figure 4. Here it is observed that J_2 and D_5 are negligible everywhere. The term J_3 is negative everywhere and large in magnitude. The work term D_4 is positive everywhere, is large, and is the main source of energy in the solid-side. The term D_6 is negative everywhere and of negligible size. Basically, D_4 is offset by J_3 or ($D_4 \rightarrow J_3$).

3.2 The N problems for the visco-elastic wall

In this sub-section, Normal compliance (N) problem for non-axisymmetric disturbances has only studied.

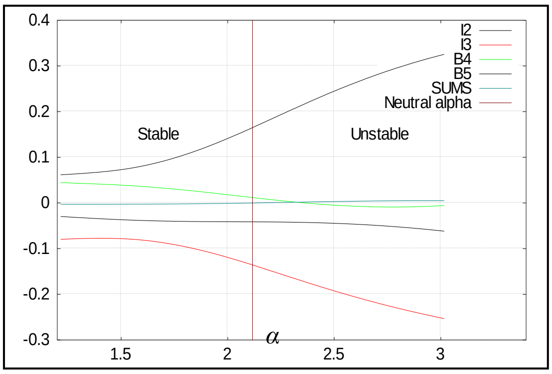

Figure 6: *The variation of different energy terms for the N, mode for the fluid-side. H =2.0, μ_r = 0.0, Γ = 8, R = 500*

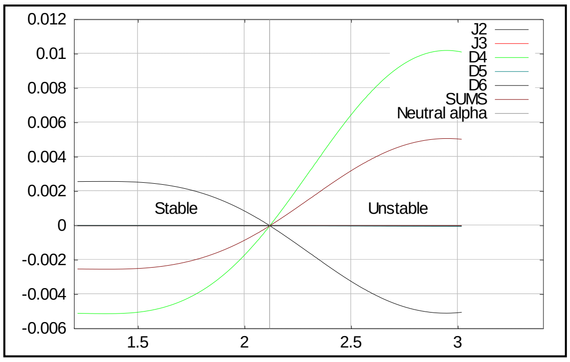

Figure 7: *The variation of different energy terms for the N, mode for the solid-side. H =2.0, μ_r = 0.0, Γ = 8, R = 500*

Normal compliance (N) problem has been studied. Figures 6 and 7 showed the variation of different energy terms, for the fluid-side, and solid-side respectively.The parameters studied are N, $\mu_r = 0.0$, $\Gamma = 8$ and R = 500. It is seen that the production term I_2 is positive everywhere and large is in magnitude. Thus I_2 is the main source of energy in the fluid-side. The dissipation term I_3 is negative everywhere and is also large. The tractive work term B_4 is positive in the stable region and negative in the unstable region and is small in magnitude as compared to I_2. The term B_5 is negative everywhere and is small in magnitude. Basically therefore I_2 is balanced by I_3.

In figure 7, the solid-side energy terms were considered corresponding to the case above in figure 6. It is observed that the term J_2 is positive in the unstable region and negative in the stable region and is reasonably large. The terms J_3 and D_6 are zero, since, $\mu_r = 0$. The attractive work term D_4 is positive in the unstable region and negative in the stable region and is large. The term D_5 is negligible. Basically therefore D_4 is offset by J_2.

4. Conclusions

From the above discussions, the following conclusions were drawn as follows:

(i) The neutral curves for N+T, modes are again similar to those for axi- symmetric modes. However the energy balance is more like axi- symmetric modes. Basically $B_4 + I_2$ is offset by $I_3 + B_5$ both for $\mu_r = 0$ and $\mu_r \neq 0$.

(ii) For the N problem, modes, the general pattern is regular, with $I_2 + B_4$ is offset by $I_3 + B_5$. Also D_4 is offset by J_2 when $\mu_r = 0$.

(iii) The present results based on the energy method provide much deeper insight into the relevant mechanism involved. It can been seen for which class of modes, I_2 is the dominant production term, and for which other class of modes B_4 term is the dominant production term. The energy method therefore provides alternative tools of analysis giving the further insight into the exchange mechanism taking place both in the fluid-side and in the solid-side.

References

[1] Kramer, M. O. Boundary layer stabilization by distributed damping. J. Aeronaut Science, 24, P.P 459 – 460,1957

[2] Kramer, M. O. Boundary layer stabilization by distributed damping. J. Aero/Space Science, 27, P.P 69,1960a

[3] Kramer, M. O. Boundary layer stabilization by distributed damping. J. Am. Soc. Naval Engineers, 74, P.P 341 – 348,1960b

[4] Kramer, M. O. Hydrodynamics of dolphin, in advance Hydro-science, ed. V. T. Chow, Academic press, New York. P.P 110 – 130,1965

[5] Reynolds, O. An experiment investigation of the circumstances which determines the motion of water in pipe, Phill. Trans. Roy. Soc. 174, P.P.953-982,1883

[6] Davey, A. and Drazin, P. G. Stability of Poiseuille flow in a pipe, J. Fluid Mech., 36,P.P 209-218,1969

[7] Garg, V. K. and Rouleau, W. T. Linear spatial problem of the pipe-Poiseuille flow, J. Fluid Mech., 54, P.P 113-127,1972

[8] Salwen, H. and Grosch, C. E. The stability of Poiseuille flow in a pipe of circular -section, J. Fluid Mech., 54, P.P. 93-112,1972

[9] Hamadiche, M. and gad-el-hak, M. (2002) Temporal Stability of flow through a visco-elastic tubes, J. Fluid Structure, 16, no. 3, P.P 331-359,2002

[10] Gajjar, J.S.B, Gibson, A. and Sen, P. K. Instability in compliant pipe and related flows, presentation at IUTAM Symposium, on flow in collapsible tubes and past other highly compliant boundaries, University of Warwick, England, 26-30 March.2001

[11] Gibson, A. Stability of internal flows bounded by compliant walls: A Kinematic approach, Ph.D. Thesis University of Manchester U.K. 2001