



Finite Element Analysis of HYPAR Shell Footings with Variation in Edge Beam Dimensions and Embedment Ratio

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Abstract: *This investigational study involves analysis of settlement characteristics of HYPAR shell footings with variation in the edge beam sizes using a finite element code – PLAXIS. HYPAR shell footings have been known to bring about economy in regions with high material to cost ratio when compared to flat slab footings. This study strengthens the case of HYPAR footings by confirming its superiority in the aspect of lower settlement characteristics. The effects of changing the embedment ratio on the aspect of settlement, load bearing and soil-shell interphase stresses have also been studied.*

Keywords: *HYPAR shell footings, finite element analysis, PLAXIS, edge beam, embedment ratio, soil plug width*

1. Introduction

Shells are structures that use minimum of material to maximum structural advantage. They derive their strength from 'form' rather than 'mass'. Shells when used as elements in the foundation are decisively more economical where labour is cheap but cost of materials are high [1][2]. They are effective in transmitting the loads to the soil with greater efficiency and economy that a raft of similar dimensions [3].

Among the different shells available, column footings consisting of hyperbolic paraboloidal shell quadrants joined together by a system of edge and ridge beams are commonly known as HYPAR shell footings. They were first used by renowned architect Felix Candela in Mexico [4]. This shell was used successfully in majority of his works. The HYPAR shell was further developed to support column loads in many parts of the world. Soon, the HYPAR shell form was suited for high rise buildings and used for elevated water tank structures on poor soil [5].

The hyperbolic paraboloid shell is a doubly curved anticlastic shell (Figure 1), which has translation as well as ruled surfaces [3]. It, consists of 3 major elements: shell quadrants, edge beams and ridge beams. The shell is provided with reinforcement either diagonal to the edge beam or parallel to it [7][8]; while the edge beam and ridge beam are given steel bars parallel to its length and held by stirrups.

The design of the HYPAR shell footing is carried out as per the Indian Standard 9456 [9]. The code follows a simplified membrane theory and has guidelines for detailing of the footing. The design of the footing is based on the need to maintain equilibrium between membrane stresses. The design dimensions for the edge and ridge beams tend to be conservative in most of the steps, and are often based on size of the column

and area required for concrete in compression or steel in tension.

Previous experimental and theoretical investigations into structural behaviour for shell structure, such as the membrane stresses, bending moments, shear and deflections. Theoretical analysis was done using finite difference technique and finite elements analyses [10]. In few studies, the distribution of the soil contact pressure on shell footing was also examined. The results indicated a non-uniform contact pressure distribution along the soil-shell interface. However, the currently used membrane theory assumes uniform soil contact pressure distribution [9]. All studies concluded that there was economy in the construction materials and that structural performance of the shell footing was efficient. Most of the previous studies deal with either triangular or conical shell footings, studying of the geotechnical performance HYPAR footings using FEM remains low.

Earlier studies have established that distribution of soil pressure beneath the shell remains non-uniform, yet for practical purposes of design we shall be using the simplified membrane theory, which assumes uniform distribution of soil pressure along the soil-shell interphase.

Both laboratory and FE analysis by Hannah and Abdel-Rahman [13] show that load bearing capacity of the triangular shell, with an equivalent cross sectional area, is higher than the flat slab type footing. Shell footing ensures better enclosibility of the soil inside the cavity of the footing by preventing the soil from flowing outward which is particularly helpful when the soil is poor.

The present study involves analysing the settlement characteristics of HYPAR shell footings with various configurations of the edge beam. The settlement of a typical flat slab footing designed to take up similar load in the same soil has been studied. The effects of

altering the embedment of the footing in soil on the aspect of settlement are also studied for certain configurations. The values of stress at key points have been plotted for the select configurations; this will help determine areas of structural importance.

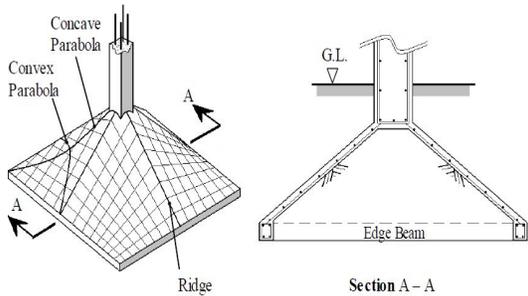


Figure 1. Typical HYPAR footing [14]

2. Design Details of the Hypar Shell Footings:

Table 1: Design dimensions

Column	500 x 500 mm	
External dimension of footing	2.4 x 2.4 m	
Adopted rise	1 in 2	
Rise	0.6 m	
Shell thickness	140 mm	
Edge beam dimensions	beam 250x300 mm	
Ridge beam dimensions	500x100mm rectangular portion below 100x500 mm triangle portion attached to bottom of shell	

Edge Beam Types:

- Edge beam type 1 – Normal - Dimension 1
- Edge beam type 2 – Dimension 1– Inverted
- Edge beam type 3 – Dimension 2–Doubled width
- Edge beam type 4 – Double edge beam–Increased Depth

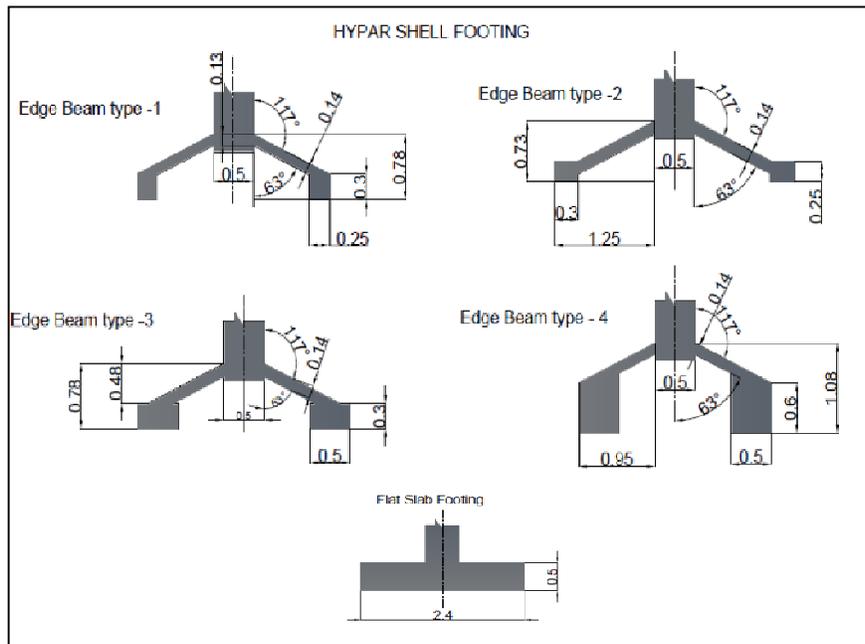


Figure 2. Designed HYPAR footings and flat slab footing

An equivalent flat slab footing was used to compare load bearing capacities for both types of footings.

3. Finite Element Modelling:

It has been concluded in studies by Abdel-Rahman [15] in 1996 that behaviour of shell footings can be predicted using FEM, experiments that compared laboratory tests and outputs of FE analysis showed good agreement.

It is common practise to divide the footing into two halves and analyze only one half owing to the symmetry of mesh for plane strain condition about the centreline the footing , this study involved modelling

of the entire footing as a whole plate and assigning properties based on EA and EI values of the designed elements to the plates. The material properties of the footings are indicated in Table 1.

The properties of the soil medium dense considered are enlisted in Table 2. In the soil model Standard Fixities were applied to the mass of dimensions 30m x 15m (Figure 3), restricting vertical and horizontal movement. Repeated refinement and densification of the mesh was done around the soil-footing interphase to get better analysis from the resulting smaller elements (Figure 4). The 15 noded meshes was generated under plane strain model with undrained

soil behaving with Mohr-Coulomb properties and the footing behaving as an elastic element. The effect of water table was neglected.

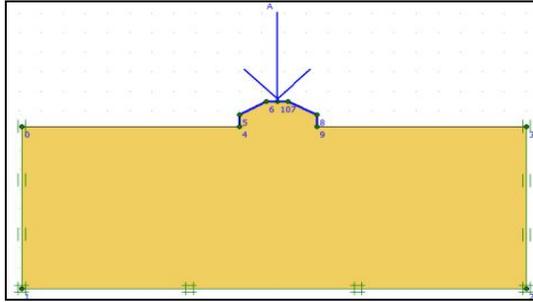


Figure 3. Modelling of the shell as a plate element

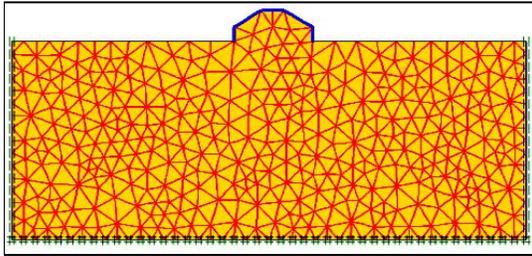


Figure 4. Generated mesh for a shell footing

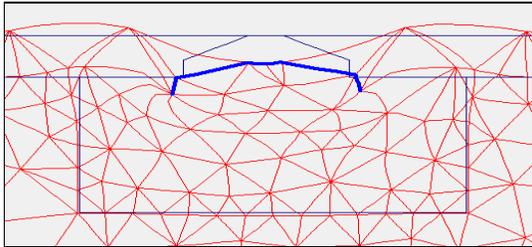


Figure 5. Deformed mesh for a fully embedded type footing

Table 2. Material Properties of Footing

Property	Units	Edge Beam Type-1	Edge Beam Type-2	Edge Beam Type-3	Edge Beam Type-4	Flat Slab Type
Cross Section Area (A)	m ²	0.723	0.801	0.902	1.199	1.535
Moment of Inertia (I)	mm ⁴	290.263 x 10 ⁹	460.453 x 10 ⁹	479.894 x 10 ⁹	686.266 x 10 ⁹	582.985 x 10 ⁹
Modulus of Elasticity (E _i)	kN/m ²	2.7 x 10 ⁷				
Poisson Ratio (ν)	-	0.3	0.3	0.3	0.3	0.3
Material Type	-	Elastic	Elastic	Elastic	Elastic	Elastic
Elastic Flexural Rigidity (EI)	kNm ²	7.812 x 10 ⁶	12.432 x 10 ⁶	12.957 x 10 ⁶	18.529 x 10 ⁶	15.7 x 10 ⁶
Axial Stiffness (EA)	kN	19.521 x 10 ⁶	21.627 x 10 ⁶	24.354 x 10 ⁶	32.373 x 10 ⁶	41.44 x 10 ⁶

The four types of HYPAR Shell footings were loaded with a point load and analyzed until failure of the soil body beneath the shell cavity. Analysis was carried out initially with Embedment Ratio (E.R)= 1. Further analysis of Edge Beam Type-1 with E.R =0.5 and 0 (Figure 6) each were carried out. The Embedment

Ratio refers to the ratio of depth of whole footing divided by the depth of footing below ground level.

Table 3: Soil Properties-Sand in Mohr-Coulomb Model

Property	Value	Units
Unsaturated Unit Weight (γ _{us})	18	kN/m ³
Saturated Unit Weight (γ _s)	20	kN/m ³
Permeability Coefficient(k _x , k _y)	1	m/day
Young's Modulus (E)	4 x 10 ⁴	kN/m ²
Poisson Ratio (ν)	0.3	-
Coefficient of Cohesion (c)	0.001	kN/m ²
Friction Angle (φ)	35	Degree
Dilatancy Angle (ψ)	2.0	Degree

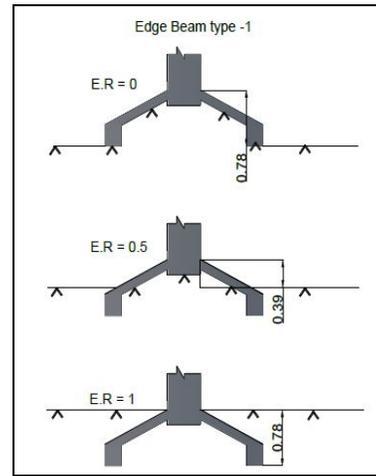


Figure 6. Variation in Embedment Ratio for Edge Beam Type-1

4. Results and Discussion

4.1. Effect of Edge Beam Configurations on Load-Settlement Characteristics

The effect of modifying the edge beam has been shown to reduce the soil pressure and increase bearing capacity with increasing width of the edge beam in triangular shell footings [17]. The current study aims to evaluate such changes in geotechnical behavior of HYPAR Shell footings with change in edge beam configuration. The four types of edge beams were analyzed and the load-settlement graph has been plotted (Figure 7).

It is observed from Figure 7 that, a wider edge beam (Type-2) is capable of better load-settlement properties than a traditional edge beam (Type-1). With increasing depth of the edge beam the load-settlement properties showed no improvement for the same value of embedment ratio (ER=1)

It was observed that higher moment of inertia of the footing section caused low initial settlement of the footing, as observed in the case of Edge Beam-Type 4. This footing showed a steep initial curve which later

flattened out with loading above 30kN. No relation can be drawn between moment of inertia and the load-settlement characteristics of the footing.

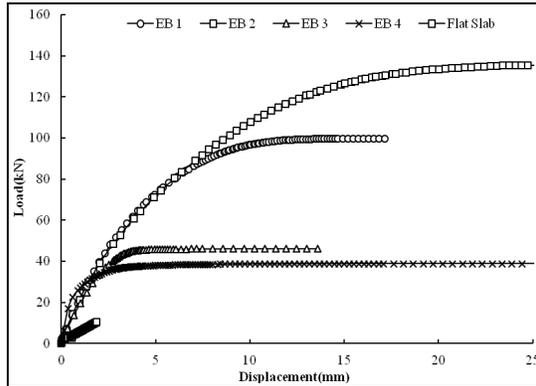


Figure 7. Load-Settlement Graphs for all footings

The aspect of Soil plug width, which does not change with embedment ratio, has been compared in Figure 8. The settlement at 15mm has been used as a criterion due to early failure of soil in each of the edge beam types. It can be inferred that increased width of enclosed soil contributes to better load-settlement properties. Edge Beam Type-2 has the maximum width of soil plug owing to maximum load at a settlement of 15mm. Edge Beam Type-4 has a deeper edge beam with very low enclosure of soil and comparatively lower load capacity at 15mm settlement. It is likely that the increased toe width of Type-3 provided additional gain to the settlement characteristics rather than the load bearing capacity of the footing. With increase in width of soil plug, the load carrying capacity increases linearly ($R^2 = 0.944$).

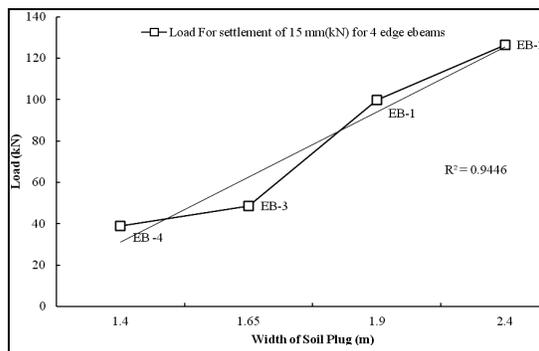


Figure 8. Effect of Soil Plug Width on Settlement

4.2. Effect of Varying Embedment Ratio on Load-Settlement and Stress Characteristics:

It was observed (Figure 9) that for Edge Beam Type-1, the load-settlement characteristics improved with increasing embedment ratio for loads up to 100 kN. For loading greater than 100 kN, embedment ratio of 0.5 was found to give better load-settlement properties. Fully embedded HYPAR Shells have shown lower stresses in the soil-shell interphase than partially embedded footings (Figure 10). For any

given embedment it was observed that stresses were maximum around the edge beam soil interphase. The stresses were least at the soil interphase directly below the point of application of load.

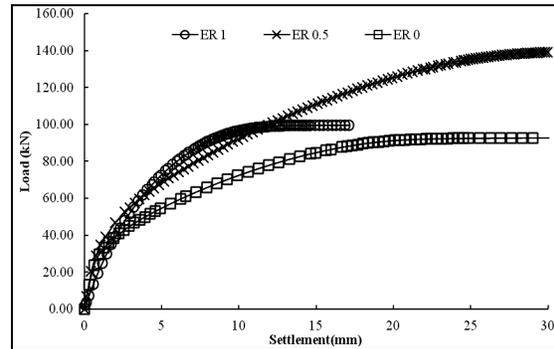


Figure 9. Effect of varying Embedment Ratio on Load-Settlement of Edge Beam Type-1

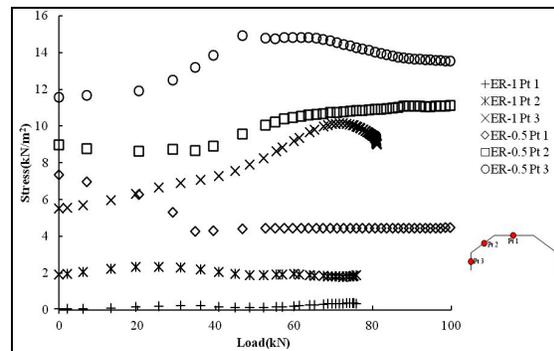


Figure 10. Stress levels at soil-shell interphase for varying embedment ratios

The Stress-Strain graph (Figure 11) for various embedment ratios of Edge Beam Type-1 shows that full embedment ratio gives maximum load bearing capacity to the footing. Although partial embedment is found to induce lower stresses in the shell-soil interphase for equivalent loading, whereas zero embedment has shown to induce great stresses in the soil region directly below the load. Thus, for soils with lower bearing capacity it may be beneficial to increase the embedment ratio to maximum.

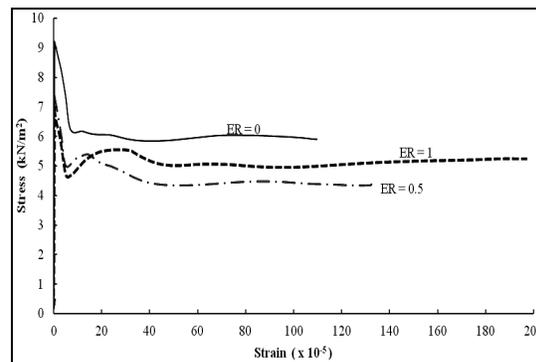


Figure 11. Stress-Strain Graph for Point-1 at various Embedment Ratios of Edge Beam Type-1

5. Conclusions

The 2-D Finite Element Analysis of HYPAR Shell Footings using PLAXIS was carried to study its load-settlement behaviour. It is observed that the effect of increasing the edge beam width as opposed to depth, greatly improves the load bearing and settlement characteristics.

The effect of soil plug width on the settlement properties has strengthened the importance of the properties of the cavity fill soil. Thus, demanding proper compaction of cavity soil for better load capacity in addition to possible increase in the width of the soil plug while designing HYPAR Shell Footings.

Stresses induced at the soil-shell interphase indicate that greater embedment produces lower stresses in the zone. While greater embedment contributes to greater load bearing capacity of the footing and decreased soil zone stresses the load-settlement characteristics were marginally better in footings with partial embedment.

The maximum load bearing characteristics were observed in shells with full embedment, with reduction in load bearing ability with a decreasing embedment ratio. The above observations help to determine an optimum cross-section of HYPAR Shell Footing for the purpose of least settlement and maximum load bearing requirement.

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