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Elastic Analyses for a Circular Pressured Tunnel with Liner based on a New Model

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Abstract: The support of underground structures must be appropriately designed. New analytical solutions for a deep tunnel with liner in isotropic geomaterial have been obtained for a hydrostatic-pressure condition by using the complex potential theory proposed by Muskhelishvili. The construction sequence of the tunnel is properly modeled in the analytical solution. The sensitive analyses indicate that the flexible support is favorable to decrease the support pressure and the installation of the liner must be well-timed towards the different situations. And that the liner rigidity is increased and the liner is installed as early as possible, which both are advantageous to reduce the surrounding geomaterial deformation. The analyses also show that the structure has a strong dependency on the liner thickness. The present solutions contain previously known results as the special cases.

Keywords: *Tunnel, Complex Potential Theory, Liner, Elasticity*

1. Introduction

Tunnel is a main underground structure and widely used for transportation transfer, water passage, and other purposes such as electricity or communication cable installation. With the development and upgrade of infrastructures, tunnel construction is increasing all over the world and tunnel engineer is more aware of the importance of the safety and economics of tunnel construction. The subject of geomaterial-liner interaction has been studied by numerous researchers [1-5]. A common feature to all the openings is that the release of pre-existing stress upon excavation of the opening will cause the soil or rock to deform elastically at the very least. An understanding of the manner in which the soil and rock around a tunnel deform elastically due to changes in stress is quite important for underground engineering problems. In fact, the accurate prediction of the in situ stress field and deformability moduli through back-analysis of tunnel convergence measurements and of the 'Ground Reaction Curve' is essential to the proper design of support elements for tunnels [6].

The availability of many accurate and easy to use finite element, finite difference, or boundary element computer codes makes easy the stress-deformation analysis of underground excavations. However, Carranza-Torres and Fairhurst [7] note explicitly in their paper: ''...Although the complex geometries of many geotechnical design problems dictate the use of numerical modeling to provide more realistic results than those from classical analytical solutions, the insight into the general nature of the solution (influence of the variables involved etc.) that can be gained from the classical solution is an important attribute that should not be overlooked. Some degree of simplification is always needed in formulating a design analysis and it is essential that the design engineer be able to assess the general correctness of a numerical analysis wherever possible. The closed-form results provide a valuable means of making this assessment…''.

As an important analytical method, the complex variable theory [8] has been widely used to analyze the underground problem. A closed-form plane strain solution for stresses and displacements around tunnels of semi-circular or ''D'' cross-section and a semi-analytical elastic stress-displacement solution for notched circular openings in rocks based on the complex potential functions and conformal mapping representation are presented [9].

The paper attempts to present a close-form elastic solution for a circular lined tunnel under the hydrostatic-pressure condition and uniformly internal pressure (Fig.1). The work concentrates on the analyses of the effects of the relative rigidity and thickness of the liner and the so-called elastic deformation rate of the surrounding geomaterial on the stress-displacement field for various combinations of the mechanical and geometric parameters. In all the analyses the following assumptions have been made: (1) the geomaterial and the liner remain elastic materials; (2) the cross-section of the tunnel is circular; (3) plane strain conditions apply at any cross-section of the tunnel; (4) deep tunnel.

2. Analytical Solution

The elastic problem to be considered is that of a semi-infinite geomaterial containing a deep circular liner tunnel subjected to uniformly internal pressure p and in situ stress σ_0, as shown in Fig.1. It is assumed that the geomaterial and the liner are perfect bonded. The

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origin of the x- and y-axes is taken at the center of the tunnel. The inner and outer radii of liner are denoted by a and t, respectively, and the regions occupied by the geomaterial and the liner by index 1 and 2. In addition, it is assumed that the radii of the opening before the tunnel is excavated, after the opening elastic deformation finishes and before the liner is installed, will be referred to t''', t'' and t', respectively. The construction sequence of the tunnel is modeled with regard to Fig.2.

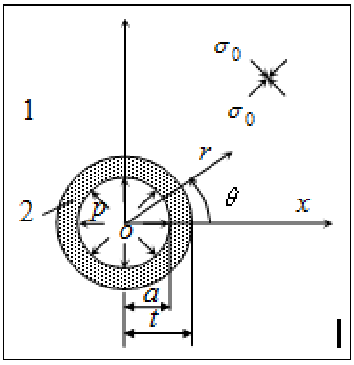

Figure1: *Circular lined tunnel under uniformly internal pressure and in situ stress.*

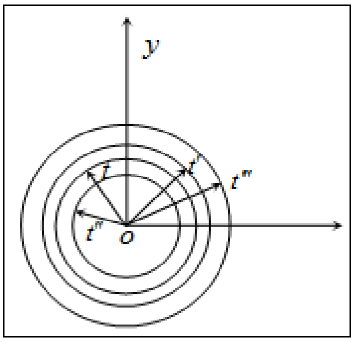

Figure 2: *Model for tunnel construction sequence.*

Then, we define

$$\varepsilon = (t' - t'')/t' \tag{1}$$

As the relative radius misfit between geomaterial and liner. This misfit is assumed of the order of admissible strains in linear elasticity.

Based on Kirsch's elasticity solution [10], t'' is expressed as follows

$$t'' = t'''[1-(1+\kappa_1)\sigma_0/4G_1] \tag{2}$$

The case of $\varepsilon = 0$ means that the liner is installed after the elastic deformation finishes, and $\varepsilon = \varepsilon_{\max}$ means that the region $r \le t'''$ in geomaterial is replaced by the liner. Thus, $\varepsilon_{\max}$ are written in the following form

$$\varepsilon_{\max} = \frac{(1+\kappa_1)\sigma_0}{4G_1} \tag{3}$$

Substituting (3) into (2) yields the following relation

$$t'' = (1-\varepsilon_{\max})t''' \tag{4}$$

Next, the elastic deformation rate of the surrounding geomaterial is defined by

$$\delta = \frac{t'''-t'}{t'''-t''} \times 100\% \tag{5}$$

Taking advantage of Eq.(4), δ can be rewritten

$$\delta = \frac{1}{\varepsilon_{\max}}(1-\frac{t'}{t'''}) \times 100\% \tag{6}$$

Inserting Eqs.(3),(4) and (6) into Eq.(1) gives

$$\varepsilon = \frac{(1-\delta)\varepsilon_{\max}}{1-\delta\varepsilon_{\max}} \tag{7}$$

We also define the dimensionless liner rigidity as follows

$$\Gamma = \frac{G_2}{G_1} \tag{8}$$

For the plane elastic problem shown in Fig.1, all the physical quantities are given in terms of two complex potentials $\varphi(z), \psi(z)$ and their derivatives.

Components of stresses and displacement in polar coordinate are written as follows

$$\sigma_\theta + \sigma_r = 2[\varphi'(z) + \overline{\varphi'(z)}] \tag{9}$$

$$\sigma_\theta - \sigma_r + 2\mathrm{i}\,\tau_{r\theta} = 2\mathrm{e}^{2\mathrm{i}\theta}[\bar{z}\varphi''(z) + \psi'(z)] \tag{10}$$

$$2G(u_r + \mathrm{i}u_\theta) = \mathrm{e}^{-\mathrm{i}\theta}[\kappa\varphi(z) - z\overline{\varphi'(z)} - \overline{\psi(z)}] + \text{(An arbitrary constant)} \tag{11}$$

Furthermore, components of resultant force is expressed as

$$F_x + \mathrm{i}F_y = -\mathrm{i}[\varphi(z) + z\overline{\varphi'(z)} + \overline{\psi(z)}] + \text{(An arbitrary constant)} \tag{12}$$

Wherein the overbar represents the complex conjugate, $z = x + \mathrm{i}\,y, \mathrm{i} = \sqrt{-1}$ is the imaginary unit. G is the shear modulus and κ is defined by Poisson's ratio γ as

$$\kappa = \begin{cases} \dfrac{3-\gamma}{1+\gamma} & \text{(plane stress)} \\ 3-4\gamma & \text{(plane strain)} \end{cases} \tag{13}$$

Constant terms in Eqs.(11) and (12) depend on the starting points from which F_x, F_y and u, v are measured, but they are nonessential in the analysis.

The stress complex potentials $\varphi_1(z), \psi_1(z)$ for domain 1 and $\varphi_2(z)$, $\psi_2(z)$ for domain 2 may be written as the following Laurent series.

$$\left.\begin{aligned}\varphi_1(z)&=\sum_{n=0}^{\infty}(K_{2n}z^{2n+1}+F_{2n}z^{-2n-1})\\ \psi_1(z)&=\sum_{n=0}^{\infty}(L_{2n}z^{2n+1}+H_{2n}z^{-2n-1})\end{aligned}\right\}\tag{14}$$

$$\left.\begin{aligned}\varphi_2(z)&=\sum_{n=0}^{\infty}(W_{2n}z^{2n+1}+V_{2n}z^{-2n-1})\\ \psi_2(z)&=\sum_{n=0}^{\infty}(Q_{2n}z^{2n+1}+R_{2n}z^{-2n-1})\end{aligned}\right\}\tag{15}$$

Note that symmetry dictates that all of the even power terms are missing and that the coefficients K_{2n}, F_{2n}, L_{2n}, H_{2n}, W_{2n}, V_{2n}, Q_{2n} and R_{2n} are real in the above expressions.

Considering that the stress field in liner is caused by the elastic misfit between the liner and the surrounding geomaterial as well as the internal pressure, moreover, the above expressions must satisfy the continuity conditions for the stress and displacement along the liner boundary, that is, along $z=t\,\mathrm{e}^{\mathrm{i}\theta}$ and for any θ

$$\left.\begin{aligned}F_{y1}+\mathrm{i}F_{x1}&=F_{y2}+\mathrm{i}F_{x2}\\ u_{r1}+\mathrm{i}v_{\theta1}&=u_{r2}+\mathrm{i}v_{\theta2}+t\varepsilon\end{aligned}\right\}\quad |z|=t\tag{16}$$

And along $z=a\,\mathrm{e}^{\mathrm{i}\theta}$

$$\sigma_{r2}-\mathrm{i}\tau_{r\theta2}=-p\quad |z|=a\tag{17}$$

Substituting Eqs.(14) and (15) into Eqs.(16) and (17), in conjunction with the stress state at infinity, the complex potentials in the liner may be expressed as

$$\varphi_2(z)=W_0z\ ;\ \psi_2(z)=R_0z^{-1}\tag{18}$$

Where

$$W_0=-\frac{p}{2}-\frac{(2\Gamma+\kappa_2-1)\,pt/2-2G_2t\varepsilon}{(1-\kappa_2-2\Gamma)a^{-2}t+2(\Gamma-1)t^{-1}}a^{-2}\tag{19}$$

$$R_0=\frac{(2\Gamma+\kappa_2-1)\,pt/2-2G_2t\varepsilon}{0.5(1-\kappa_2-2\Gamma)a^{-2}t+(\Gamma-1)t^{-1}}\tag{20}$$

Compared with the stress field in liner, the stress field in surrounding geomaterial is produced by three sources, i.e., the misfit between the liner and the surrounding geomaterial, the internal pressure and in situ stress. Thus, the complex potentials in the surrounding geomaterial may be derived as follows

$$\varphi_1(z)=K_0z\,;\psi_1(z)=H_0z^{-1}\tag{21}$$

Where

$$K_0=-\frac{\sigma_0}{2}\tag{22}$$

$$H_0=\frac{t^2}{2}[-\frac{p}{2}-\frac{(2\Gamma+\kappa_2-1)\,pt/2-2G_2t\varepsilon}{(1-\kappa_2-2\Gamma)a^{-2}t+2(\Gamma-1)t^{-1}}a^{-2}]+\frac{(2\Gamma+\kappa_2-1)\,pt/2-2G_2t\varepsilon}{0.5(1-\kappa_2-2\Gamma)a^{-2}t+(\Gamma-1)t^{-1}}+\sigma_0t^2\tag{23}$$

Substituting Eqs.(18) and (21) into Eqs.(9)-(11) yields the stress and displacement in domains 1 and 2, respectively.

$$\left.\begin{aligned}&\sigma_{r1}=2K_0+H_0r^{-2};\sigma_{\theta1}=2K_0-H_0r^{-2};\tau_{r\theta1}=0\\ &u_{r1}=[(\kappa_1-1)K_0r-H_0r^{-1}]/(2G_1)+f\,;u_{\theta1}=0\end{aligned}\right\}\tag{24}$$

$$\left.\begin{aligned}&\sigma_{r2}=2W_0+R_0r^{-2};\sigma_{\theta2}=2W_0-R_0r^{-2};\tau_{r\theta2}=0\\ &u_{r2}=[(\kappa_2-1)W_0r-R_0r^{-1}]/(2G_2);u_{\theta2}=0\end{aligned}\right\}\tag{25}$$

Where $f=f(r)$, which is a displacement function in geomaterial relative to the part deformation of the surrounding geomaterial before the liner installation. For $r=t'''$

$$f_0=f(r)=t'-t'''\qquad r=t'''\tag{26}$$

Consider the pre-existing displacement in geomaterial before excavation due to the in situ stress field

$$\left.\begin{aligned}u_{r1}^0&=\frac{(\kappa_1-1)K_0r}{2G_1}\\ u_{\theta1}^0&=0\end{aligned}\right\}\tag{27}$$

Referring to Eqs.(24) and (25), the displacement in liner and geomaterial due to exaction may be rewritten as

$$\left.\begin{aligned}\Delta u_{r1}&=u_{r1}-u_{r1}^0=\frac{-H_0r^{-1}}{2G_1}+f\\ \Delta u_{\theta1}&=0\end{aligned}\right\}\tag{28}$$

$$\left.\begin{aligned}\Delta u_{r2}&=u_{r2}+f_0=\frac{(\kappa_2-1)W_0r-R_0r^{-1}}{2G_2}+t'-t'''\\ \Delta u_{\theta2}&=0\end{aligned}\right\}\tag{29}$$

Let $r=t$ in Eqs.(24) and (28), the support pressure p_i and the surrounding geomaterial deformation u may be obtained as follows

$$p_i=2K_0+H_0t^{-2}\tag{30}$$

$$\Delta u_{r1}=\frac{-H_0t^{-1}}{2G_1}+f_0\tag{31}$$

wherein, f_0 may be yielded by measurement, thus, it is essential to observe the variation of u

$$u=\Delta u_{r1}-f_0=-H_0t^{-1}/(2G_1)\tag{32}$$

Taking $\varepsilon=\varepsilon_{max}$ in the above expressions, the solution in the paper is agreement with the result obtained by Ref.[11].

3. Sensitivity Analyses

The present problem may be treated as a plane strain problem. Here, we assume $\gamma_1=\gamma_2=0.3$, $p=0.1\sigma_0$ and $[(1+\kappa_1)\sigma_0]/(4G_1)=10^{-4}$.

3.1. Variations of support pressure with elastic deformation rate and dimensionless liner thickness

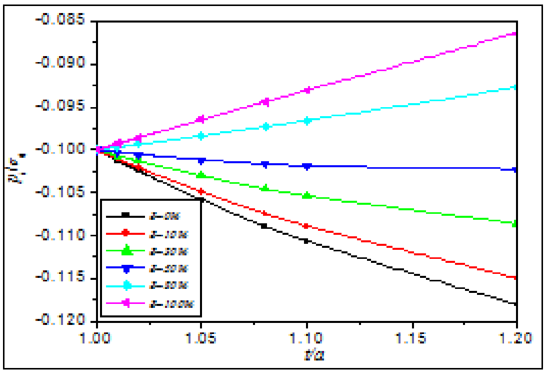

Figure 3: *Variations of support pressure with elastic deformation rate and dimensionless liner thickness for Γ=0.1.*

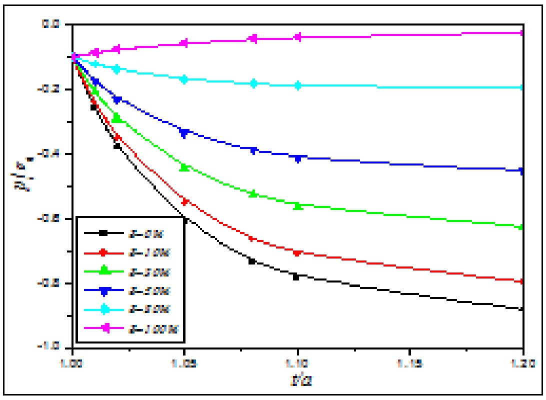

Figure 4: *Variations of support pressure with elastic deformation rate and dimensionless liner thickness for Γ=10.*

For soft liner, it is found from Fig.3 that the support pressure monotonically increases with increasing liner thickness when the elastic deformation rate δ is in the range of $\delta\le 50\%$. As for $\delta\ge 80\%$, a reverse trend is observed. For hard liner, the variation of p_i with the elastic deformation rate and the liner thickness is plotted in Fig.4. From this figure, we observe that the general trend is that the support pressure monotonically decreases with increasing liner thickness. Moreover, the trend becomes gentle with increasing elastic deformation rate. In this case, the increment of the liner thickness is unfavorable to decrease the support pressure. It is seen from Figs.3 and 4 that the liner is installed after the surrounding geomaterial partially deform, which may reduce the support pressure.

3.2. Variations of support pressure with elastic deformation rate and dimensionless liner rigidity

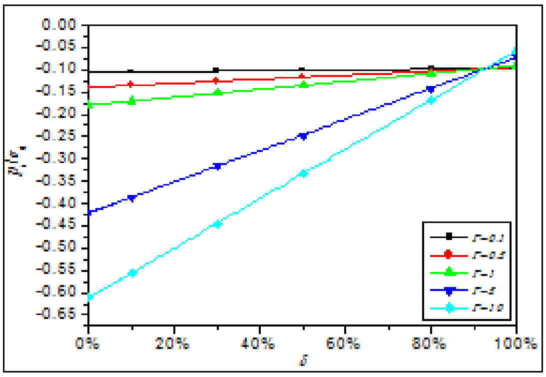

Figure 5: *Variations of support pressure with elastic deformation rate and dimensionless liner rigidity for t/a=1.05.*

Figure 5 shows the variation of p_i versus the elastic deformation rate and the dimensionless liner rigidity for a fixed-thickness liner. It is observed that the support pressure monotonically decreases with increasing elastic deformation rate. In addition, the increment of the liner rigidity may increase the support pressure in the rough range of $\delta\le 90\%$. As for $\delta\ge 90\%$, the reverse tendency takes place. Thus, the elastic deformation of the surrounding geomaterial partially happens before the liner is installed and the flexible support is adopted, which both are appropriate to decrease the support pressure.

3.3. Variations of support pressure with dimensionless liner thickness and rigidity

Figs.6 and 7 illustrates the variation of the support pressure with respect to dimensionless liner thickness and rigidity. For case of δ=0%, the support pressure increases with the increment of liner thickness and rigidity. When δ=100%, compared with δ=0%, the trend evolves to the contrary side.

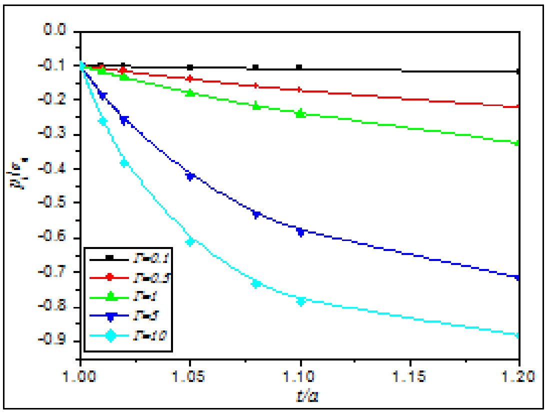

Figure 6: *Variations of support pressure with dimensionless liner thickness and rigidity for δ=0%.*

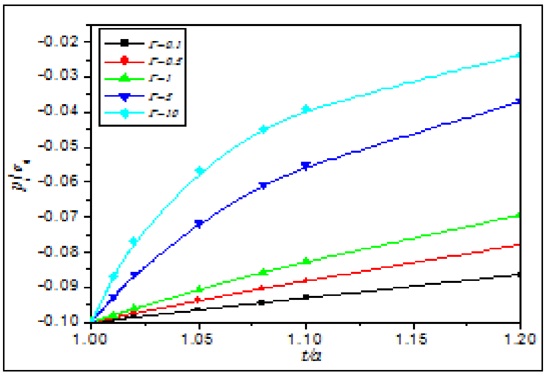

Figure 7: *Variations of support pressure with dimensionless liner thickness and rigidity for δ=100%.*

4. Conclusions

By using the complex potential theory proposed by Muskhelishvili, the construction sequence of the tunnel is appropriately modeled. The complex stress potentials are assumed to be in the form of Laurent series expansions, and the unknown coefficients are determined by the boundary conditions and the stress state at infinity. Finally, the close-form solutions for the stress and displacement field are explicitly derived. The sensitive analyses indicate that the flexible support is favorable to decrease the support pressure. In addition, the installation of the liner must be well-timed towards the different situations. The liner rigidity is increased and the liner is installed as early as possible, which both are advantageous to reduce the surrounding geomaterial deformation. The analyses also show that the structure has a strong dependency on the liner thickness. The present solutions contain previously known results as the special cases.

5. Acknowledgments

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