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Analysis of Deformation of Tunnel Surrounding Rock based on Three Dimensional Finite Element and Improved Grey Theory

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Abstract: Elastoplastic numerical calculation model for three dimensional finite element for tunnel is set up based on PLAXIS 3D and displacement prediction model for tunnel surrounding rock is established based on improved grey prediction theory; with stress distribution, strain pattern and displacement variation pattern in excavation process of tunnel analyzed, relation between parameter and deformation studied, calculation procedure and method for improved grey prediction theory presented so as to obtain settlement and displacement of arch crown in front of tunnel face within certain extent in tunnel excavation process by prediction. Suitong Tunnel is taken as test subject to carry out contrastive analysis relation among three displacement-time curves namely value of simulating calculation, predicted value and measured value and perform cross validation of the above mentioned three values. The validation result shows that the model is able to properly embody deformation pattern of surrounding rock, with prediction precision of displacement value satisfying requirement of specification and having larger scope of application in prediction of surrounding rock stability than that of other methods. The abstract section is mandatory, with a word limit of 200 words. The scope, aims, results and conclusions may be summarized here. Avoid inserting any reference in this section.

Keywords: *Numerical calculation, Grey prediction theory, Displacement prediction, Tunnel face*

1. Introduction

Disturbance of natural state of equilibrium of original rock by tunnel excavation leads to redistribution and deformation of stress on surrounding rock, not only adversely affecting stability of surrounding rock of tunnel body but also being closed associated with rationality and economic efficiency of supporting structure and parameter for surrounding rock in construction, therefore, it is very important to make a study of displacement variation and deformation monitoring and deformation pattern of tunnel surrounding rock in construction process so as to carry out scientific analysis and prediction thereof, which is not only the key to integration of safety with economic efficiency in tunnel works to the full extent but also the foundation for numerical calculation and back analysis of stability of surrounding rock.

Although simulation analysis of construction method and supporting parameter for tunnel construction may be carried out by two dimensional numerical simulation, it is unable to entirely present “time-space effect” of construction process, “space effect” due to forwarding tunneling face and “time effect” as a result of rheological property of rock body co-determine the deformation development of surrounding rock in excavation process [1-4]. Therefore, three dimensional finite element analysis of surrounding rock deformation of tunnel is able to remedy the deficiency of two dimensional numerical simulation and field monitoring in an effective way, the result of which shall be of practical significance to improvement in design and guidance of construction to a certain extent.

In addition, as a consequence of rheological property of surrounding rock, its load and displacement are changed over time, scholars desire to find out deformation pattern from measured data and predict development trend in future and deformation in surrounding rock in various time-spaces in tunnel. At present, there are many methods for analysis and prediction for surrounding rock deformation both here and abroad, such as grey prediction, BP neural network, regression analysis, time series analysis, functional approaching etc. grey prediction has been widely applied to prediction study of deformation on surrounding rock in tunnel due to its unique advantages including less data to be measured, short period of time, slight fluctuation and without special requirement for data to be measured[5-9].

With regard to above mentioned problems, the paper takes Suitong Tunnel in Wujing Expressway as test subject and utilizes PLAXIS 3D to set up three dimensional finite element numerical calculation model for tunnel and obtain value of simulating calculation, predicted value and measured value (Measurement method of Reference Tunnel relevant regulatory requirements) of deformation of surrounding rock in tunnel based on grey prediction theory method and monitored data in field and then carry out cross validation and contrastive analysis of the above mentioned three values, the results of which show that finite element model and grey prediction model are effective and rational.

2. Three Dimensional Finite Elements Numerical Calculation

2.1 Establishment of model

According to data in engineering geology and design for the tunneling, the calculation model takes certain section as study subject. In order to ensure accuracy of calculation, taking into account the boundary conditions for the tunnel model (sphere of influence), modeling is in strict accordance with requirement of construction, the model dimension: (direction X) 100m (more than 5 times tunnel diameter) in horizontal direction and 120 m (direction Y) in vertical direction and longitudinal direction, 40m (3 times tunnel direction) in longitudinal direction (direction Z). In coordinate system, axis Z is for tunneling direction, XY plane is normal to tunnel excavation direction, the boundary around model is subject to normal restriction, with bottom restricted along three directions and surface freed. The tunnel is flat horseshoe in shape, with building line being 10.5m in net width and 5.0m in net height. Finite element model mesh is dissected into 9320 units and 25513 nodes as shown in figure 1. The model adopts Mohr-Coulomb strength failure criterion[10-12], the expression for Mohr-Coulomb principal stress is as follows:

$$f\left(\sigma_1,\sigma_2,\sigma_3\right)=\frac{1}{2}\left(\sigma_1-\sigma_3\right)-\frac{1}{2}\left(\sigma_1+\sigma_3\right)\sin\varphi-c\cos\varphi=0 \quad (1)$$

Where: $\sigma_1,\sigma_2,\sigma_3$ are principle stress; c is cohesion; φ is internal friction angle.

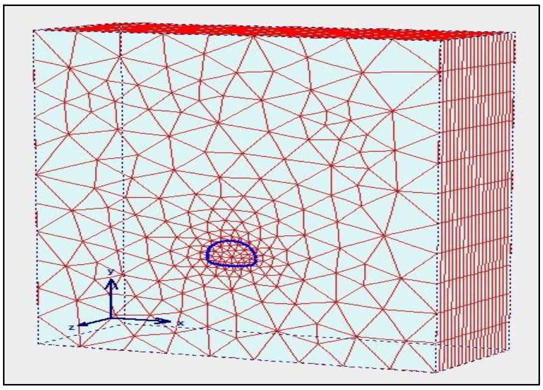


Figure 1 *Three-dimensional finite element calculation model*

2.2. Model Parameter

The physical and mechanical parameters of surrounding rock adopted in the paper area as shown in table 1.

Table 1 *Physico-mechanical parameters of rock mass*

Material	Density/g·cm^{-3}	Elastic modulus/GPa	Poisson's ratio	Cohesive force / MPa	Internal friction angle /°	The axial compressive strength /MPa	The axis tensile strength /MPa
Rocks of Grade Ⅴ	2.10	1.20	0.30	0.30	30	/	/
Anchor(Circumfer	/	45	/	/	/	/	/
Primary support	2.30	28	0.20	0	40	10.0	10.0
Secondary lining	2.50	29.5	0.20	/	/	12.5	12.5

2.3 Simulation of Construction Process

The paper carries out modeling calculation based on excavation sequence adopted for construction field of Suitong Tunnel face moves forward by step length of 2m along direction of axis Z per day, excavation process is as shown in figure 2. Adopting PLAXIS 3D to perform simulating calculation of 24 dynamic excavation steps in construction to properly reflect mechanical behavior of surrounding rock in dynamic construction process in tunnel.

The concrete excavation steps: In-situ stress field→Step 1-step 5 for excavation: Excavation of upper bench and completion of primary support, release of 30% load→Step 6-7: Excavation of upper and lower benches and completion of primary support at the same time, release of 70% load→Step 8-11: Excavation of inverted arch, release of 30% load→Step 12: Primary support for upper semi-section, construction of inverted arch, release of 70% load→Repeat of above mentioned 12 steps, the entire simulation process contains 24 excavation steps in construction.

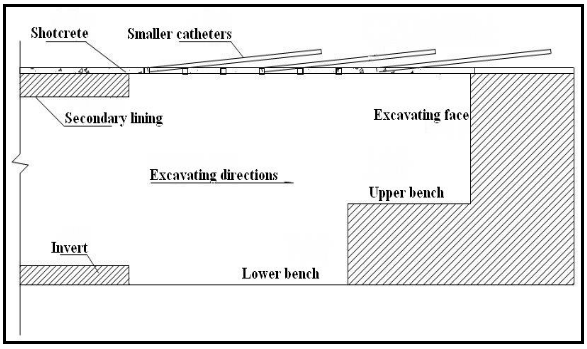
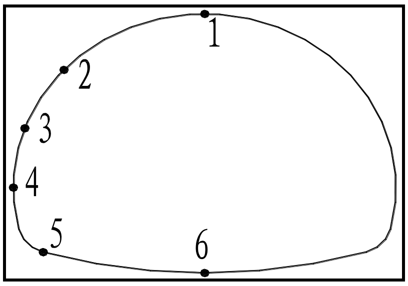


Figure 2 *Tunnel excavation construction*

2.4. Calculation Analysis

The paper selects several key points on certain typical section in the section to make a study of their displacement variation (the paper makes a study of deformation of No.1, 3 and 4 points), these key points are as shown in figure 3.



According to measured data, the convergence values of horizontal convergence of haunch of upper bench haunch and horizontal convergence of side wall of lower bench and crown settlement in the typical section are 15.930mm,25.940mm,16.110mm respectively; Convergence values of simulating calculation thereof are 16.110mm,26.782mm,16.690mm respectively. It is observed from comparison between table 2 and table 3 that calculated value of deformation is slightly higher than measured value of deformation(Main reason is the lag measured by impact); taking influence of space effect and disturbance effect of later deformation and other factors on surrounding rock into consideration, calculated value and measured value show similar effect.

Table 2.*Measured deformation values of a section*

Convergence of upper bench		Convergence of lower bench		Crown settlement	
Time /d	Displacement/mm	Time /d	Displacement/mm	Time /d	Displacement/mm
1	1.700	1	0.240	1	6.980
2	3.540	2	4.620	2	7.990
3	4.620	3	9.120	3	9.300
4	6.350	4	10.490	4	11.990
5	8.040	5	12.560	5	12.900
6	9.960	6	12.960	6	13.990
7	10.240	7	15.620	7	14.060
8	11.990	8	18.420	8	14.090
9	13.240	9	18.210	9	14.160
10	13.280	10	20.260	10	14.210
11	13.590	11	20.660	11	14.300
12	14.210	12	21.240	12	14.860
13	14.320	13	21.420	13	14.860
14	14.390	14	21.990	14	15.090
15	14.420	15	22.350	15	15.160
16	14.640	16	22.660	16	15.290
17	14.940	17	23.040	17	15.360
18	15.110	18	23.450	18	15.450
19	15.320	19	23.650	19	15.760
20	15.540	20	24.010	20	15.800
21	15.710	21	24.260	21	15.910
22	15.840	22	24.850	22	15.990
23	15.930	23	25.940	23	16.110

Table 3.*Calculated deformation values of a section*

Convergence of upper bench		Convergence of lower bench		Crown settlement	
Time /d	Displacement/mm	Time /d	Displacement/mm	Time /d	Displacement/mm
1	1.718	1	0.053	1	7.446
2	4.773	2	3.970	2	8.950
3	8.425	3	10.413	3	10.634
4	10.925	4	15.442	4	12.100
5	12.164	5	17.949	5	13.278
6	14.247	6	22.378	6	14.343
7	15.822	7	25.429	7	14.910
8	15.823	8	25.432	8	14.911
9	15.864	9	25.559	9	14.902
10	15.913	10	25.882	10	14.893
11	15.923	11	26.018	11	14.895
12	16.124	12	26.724	12	15.295
13	16.221	13	26.927	13	15.302
14	16.193	14	26.893	14	15.625
15	16.193	15	26.896	15	15.628
16	16.160	16	26.855	16	15.907
17	16.153	17	26.838	17	15.915
18	16.131	18	26.811	18	16.192
19	16.149	19	26.832	19	16.188
20	16.127	20	26.806	20	16.433
21	16.137	21	26.816	21	16.430
22	16.115	22	26.790	22	16.689
23	16.110	23	26.782	23	16.690

3. 3. Grey theory prediction

In grey prediction, GM (1, 1) model is fundamental model for grey theory and most widely used grey theory model. Since GM (1, 1) model is small in fluctuation and modeling sequence shall satisfy requirement of equal time interval.

In monitoring process of tunneling, field monitoring data is based on non-equal time interval in most cases, data is in fluctuation to a great extent, which leads to greater error. Considering the aforementioned factors, the paper set up improved DGM (1,1) model based on reality and GM (1,1), the model has overcome problems in conventional GM (1,1) model, such as low prediction precision and poor prediction stability, having very important theoretical value and practical significance to improve long term prediction precision of displacement of surrounding rock.

3.1. Improved DGM (1, 1) model

Fundamental form of DGM (1,1) model is as follows:

$$x^{(1)}(k+1)=\beta_1 x^{(1)}(k)+\beta_2 \quad (2)$$

Suppose non-negative sequence:

$$x^{(0)}=\left\{x^{(0)}(1),x^{(0)}(2),\cdots,x^{(0)}(n)\right\} \quad (3)$$

Sequence formed by its primary accumulation:

$$x^{(1)} = \left\{x^{(1)}(1), x^{(1)}(2), \cdots, x^{(1)}(n)\right\} \quad (4)$$

Where:

$$x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i), k = 1, 2, \cdots, n; \quad (5)$$

If $\hat{\beta} = (\beta_1, \beta_2)^T$ is parameter sequence, and

$$Y = \begin{bmatrix} x^{(1)}(2) \\ x^{(1)}(3) \\ \vdots \\ x^{(1)}(n) \end{bmatrix}, B = \begin{bmatrix} x^{(1)}(1) & 1 \\ x^{(1)}(2) & 1 \\ \vdots & \vdots \\ x^{(1)}(n-1) & 1 \end{bmatrix} \quad (6)$$

Then, parameter sequence by least square estimation of grey differential equation $x^{(1)}(k+1) = \beta_1 x^{(1)}(k) + \beta_2$ satisfies: $\hat{\beta} = (B^T B)^{-1} B^T Y$

Suppose $\hat{\beta} = [\beta_1 \beta_2](B^T B)^{-1} B^T Y$, then take $x^{(1)}(1) = x^{(0)}(1)$, its prediction model is:

$$\hat{x}^{(1)}(k+1) = \beta_1^k x^{(0)}(1) + \frac{1-\beta_1^k}{1-\beta_1}\beta_2 (k = 1, 2, \cdots n-1) \quad (7)$$

Prediction function is obtained by reduction and generation algorithm:

$$\hat{x}^{(0)}(k+1) = \alpha^{(1)} \hat{x}^{(1)}(k+1)$$
$$= \hat{x}^{(1)}(k+1) - \hat{x}^{(1)}(k); (k = 1, 2, \cdots n-1) \quad (8)$$

3.2. Model Precision Verification

1) Residual test

Suppose original sequence $x^{(0)}$ and residual sequence

$$e^{(0)} = \left\{ e^{(0)}(t) = x^{(0)}(t) - \hat{x}^{(0)}(t) \middle| t = 1, 2, \cdots n \right\} \quad (9)$$

Relative error of prediction:

$$q = \left| \frac{\varepsilon^{(0)}(k)}{X^{(0)}(k)} \right| \times 100\% \quad (10)$$

2) Posterior-variance-test

Posterior error ratio:

$$c = S_2 / S_1 \quad (11)$$

Where:

$$S_1^2 = \frac{1}{n}\sum_{k=1}^{n}\left[x^{(0)}(k) - \bar{x}\right]^2, \bar{x} = \frac{1}{n}\sum_{k=1}^{n} x^{(0)}(k)$$
$$S_2^2 = \frac{1}{n}\sum_{k=1}^{n}\left[\varepsilon^{(0)}(k) - \bar{\varepsilon}\right]^2, \bar{\varepsilon} = \frac{1}{n}\sum_{k=1}^{n} \varepsilon^{(0)}(k) \quad (12)$$

Small error probability:

$$p = p\left\{\left|\varepsilon(k) - \bar{\varepsilon}\right| < 0.6745 S_1\right\} \quad (13)$$

3.3. Prediction and analysis of data

The prediction adopts horizontal convergence value of haunch of upper bench and convergence value of side wall of lower bench and crown settlement value on certain section as basic test data to set up grey prediction model based on aforementioned formulas Eq(2)~(8) to obtain corresponding time response function by calculation as follows:

1) Horizontal convergence time response function for upper bench:

$$\hat{x}^{(1)}(k+1) = 105.5431e^{-0.1448k} + 16.9783(k+1) - 120.8214 \quad (14)$$

2) Horizontal convergence time response function for lower bench:

$$\hat{x}^{(1)}(k+1) = 148.8518e^{-0.1726k} + 25.9383(k+1) - 174.5501 \quad (15)$$

3) Crown settlement time response function:

$$\hat{x}^{(1)}(k+1) = 46.7546e^{-0.1992k} + 16.2937(k+1) - 56.0683 \quad (16)$$

Deformation value by prediction is obtained by calculation according to formulas Eq(14)~(16) as shown in the following table 4.

***Table 4.**Predictive deformation values of a section*

Convergence of upper bench		Convergence of lower bench		Crown settlement	
Time /d	**Displacement/mm**	**Time /d**	**Displacement/mm**	**Time /d**	**Displacement/mm**
1	1.700	1	0.240	1	6.980
2	2.754	2	2.336	2	7.849
3	4.671	3	6.078	3	9.374
4	6.330	4	9.227	4	10.624
5	7.765	5	11.877	5	11.648
6	9.007	6	14.107	6	12.487
7	10.081	7	15.983	7	13.175
8	11.010	8	17.561	8	13.738
9	11.815	9	18.890	9	14.200
10	12.511	10	20.007	10	14.578
11	13.113	11	20.948	11	14.888
12	13.634	12	21.739	12	15.142
13	14.084	13	22.405	13	15.350
14	14.474	14	22.965	14	15.520
15	14.812	15	23.437	15	15.660
16	15.104	16	23.833	16	15.774

17	15.356	17	24.167	17	15.868
18	15.575	18	24.448	18	15.945
19	15.764	19	24.684	19	16.008
20	15.928	20	24.883	20	16.060
21	16.069	21	25.050	21	16.102
22	16.192	22	25.191	22	16.137
23	16.298	23	25.310	23	16.165

4. Calculation Analysis

4.1. Results Chart

Calculated results and result chart are shown in Figure 4, Figure 5 and Figure 6:

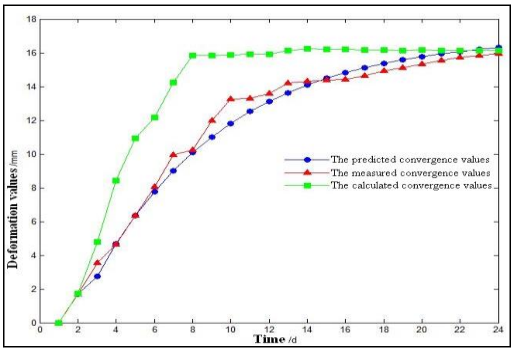


Figure 4 *Convergence curves comparison of the predicted values, calculated values and measured values on upper bench*

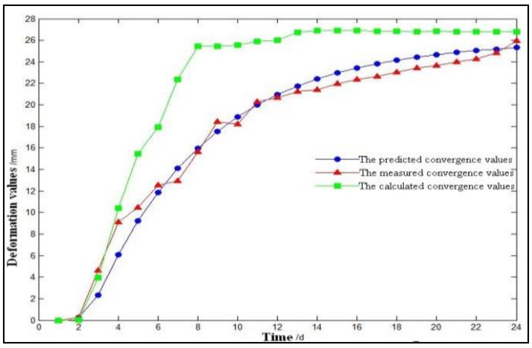


Figure 5 *Convergence curves comparison of the predicted values, calculated values and measured values on lower bench*

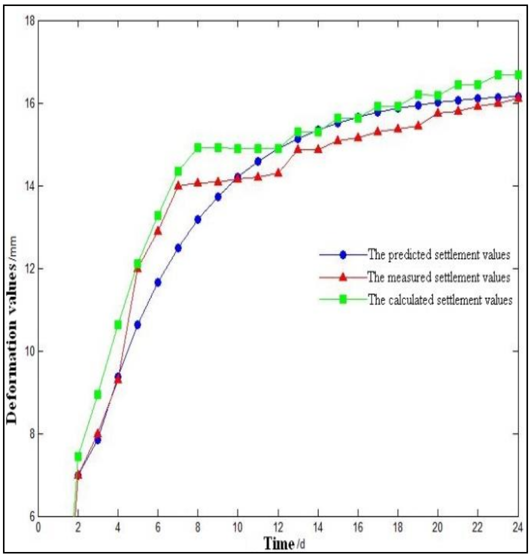


Figure 6 *Crown settlement curves comparison of the predicted values, calculated values and measured values*

4.2. Data Analysis and Validation

It is observed from table 2, table 3 and table 4 that the measured convergence values for horizontal convergence of upper bench, horizontal convergence of lower bench and crown settlement on the section are 15.930mm, 25.940mm and 16.110mm respectively and that the convergence values by numerical calculation are 16.110mm, 26.782mm and 16.690mm respectively and that the convergence values by prediction are 16.298mm, 25.310mm and 16.165mm respectively, which are similar. Prior to excavation to the monitored section, the surrounding rock in the section has already generated horizontal convergence displacement due to excavation of surrounding rock in front part, however, monitoring and measuring are unable to measure displacement generated previous to excavation, therefore, calculated value is larger than measured value (maximum difference is 30%).

Table 5 *Accuracy grade of Grey Prediction Model*

Accuracy grade	Difference ratio of posterior/C	Small error probability/P
Grade Ⅰ	C≤0.35	P≥0.95
Grade Ⅱ	0.35＜C≤0.5	0.8≤P＜0.95
Grade Ⅲ	0.5＜C≤0.65	0.7≤P＜0.8
Grade Ⅳ	C＞0.65	P＜0.7

Table 6.*Precision evaluation index*

Measuring line position	Project	Difference ratio of posterior /C	Small error probability/P	The average relative error/RMRE/%	Correlation coefficient/R^2
Convergence of upper bench	Predicted values	0.131	1.000	4.202	0.986
	Calculated values	0.368	0.957	22.588	0.865
Convergence of	Predicted values	0.166	1.000	7.310	0.986

lower bench	Calculated values	0.390	0.870	28.436	0.884
Crown settlement	Predicted values	0.250	1.000	3.343	0.953
	Calculated values	0.097	1.000	4.715	0.991

It is observed from formulas Eq(9)~(13) and above mentioned model precision inspection that considering hysteresis of monitored value and in addition to greater error in calculated values for horizontal convergence on upper and lower benches, the other predicated values and calculated values on the section satisfy the requirement of posterior error. Their average relative error is close to "0", which may be considered as no deviation and indicate that calculated value and predicted value are able to properly reflect practical situation, verifying that the prediction model is higher in prediction precision as well as practical and effective in actual engineering.

5. Conclusions

(1) Three dimensional finite element numerical calculation model is set up based on PLAXIS 3D to obtain value by simulating calculation, value by prediction and value of measurement according to grey prediction DGM (1,1) model and monitored data on site and then carry out cross validation and contrastive analysis of the above mentioned three values, the result shows that finite element model and grey prediction model are effective and rational.

(2) It is observed from the case in engineering mentioned in the paper that the surrounding rock in the section has already generated horizontal convergence displacement due to excavation of surrounding rock in front part before excavation to the monitored section, monitoring and measuring are unable to measure displacement generated previous to excavation, therefore, calculated value is larger than measured value.

(3) When DGM (1,1) prediction model for deformation on surrounding rock in tunnel is set up based on grey theory, the original large database shall be subject to constant updating so as to improve and set up new prediction model by metabolism of database and enhance prediction precision and real-time of data prediction in a constant way.

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References

[1] Sun Jun and Zhu Hehua, "Mechanical simulation and analysis of behaviour of soft and weak rocks in the construction of a tunnel opening," *Rock and Soil Mechanics*, vol. 15, no. 4, pp. 20–32, Dec. 1994.

[2] Wang Mingnian, Li Zhiye and Guan Baoshu, "Research on controlling measures for ground surface settlement of three little distance parallel shallow embedded tunnels," *Rock and Soil Mechanics*, vol. 23, no. 6, pp. 821–824, Dec. 2002.

[3] Yang Longcai, Zhou Shunhua and Yao Yanming, "Ground surface settlement during excavation of variety span tunnel," *Journal of Tongji University*, vol. 31, no. 4, pp. 408-412, Apr. 2003.

[4] She Jian and He Chuan, "3D Elastoplastic numerical simulation of surrounding rock displacement in soft surrounding rock section during construction process," *Chinese Journal of Rock Mechanics and Engineering*, vol. 25, no. 3, pp. 623-629, Mar. 2006.

[5] Tao Zhiping, "Study on tunnel deformation mechanism at landslide site and disaster predicting and controlling," Ph.D. dissertation, Dept. Civil. Eng., Southwest Jiaotong University, Chendu, China, 2003.

[6] Wu Yi-ping and Li Yawei. "Application of Grey-ENN model to prediction of wall-rock deformation in deep buried tunnel," *Rock and Soil Mechanics*, vol. 29, no. S1, pp. 263-266, Nov. 2009.

[7] Bi Weiguo and Tian Gang, et al. "On Grey Theory to Predict Tunnel Deformation," *Moden Tunnelling Technology*, vol. 48, no. 6, pp. 53-57, Dec. 2011.

[8] Xu Jiancong, "Research on Grey -Cusp - Catastrophic Destabilization Prediction Model of Tunnel Surrounding Rock and Primary Support System," *Chinese Journal of Rock Mechanics and Engineering*, vol. 27, no. 6, pp. 1181-1187, Jun. 2008.

[9] Ding Wantao, Li Shucai and Wang Shugang, "Prediction of Grey System Model on Convergence of Surrounding Rock Deformation of Longtan Tunnel," *Rock and Soil Mechanics*, vol. 27, no. S1, pp. 118-121, Oct. 2006.

[10] Pan Changshi, *Numerical Methods of Tunnel Mechanics*, Beijing, China: China Railway Publishing House, 1995.

[11] Zhou Shunhua, *Excavation Theory*, Beijing, China: China Railway Publishing House, 1997, pp. 90–98.

[12] Shang Yuequan and Huang Runqiu, *Numerical Methods in Engineering Geology*, Chengdu, China: Chengdu University of Science and Technology Press, 1991.