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New Algorithm for Tool Face Angle in Measurement While Drilling System

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Abstract: Coal-bed methane (CBM) Wells can be divided into vertical and horizontal well. The directional well is composed of vertical section and horizontal section. In the process of vertical section drilling, the drilling tool face angle is one of the most important parameters for the correction of borehole trajectory, however, the existing calculation method for tool face angle is too complicated and it is not the best suitable method for Measurement While Drilling (MWD) system which uses the Microprogrammed Control Unit (MCU) as the core. Based on this, it is deduced from this paper that two brand-new calculating formulas for tool face angle. According to the theoretical research and experiments, a kind of optimal combined algorithm for tool face angle is achieved. This optimal algorithm is suitable for MWD system with MCU as the core and can provide more accurate and efficient data support for real-time borehole trajectory correction.

Keywords: *MWD, tool face angle, Taylor expansion, triaxial accelerometer, optimal algorithm*

1. Introduction

The directional drilling technology is widely used in CBM development, mineral exploration, mining, oil recovery, trenchless pipe laying and other fields (Treadway Carl, 1996; Vanni D. et al., 2012; Yan Xiang-zhen et al., 2008; McKinnon D, 2003). In the process of directional drilling, the natural bending rules and artificial deflecting tools make the borehole drilled to the intended target, which is popular in recent years. For example CBM exploitation, as the gas quantitative efficiency is low, production cost is high and land area is large (Wang Fang-tian et al., 2011; Barraclough Scott, 1992), the cost of traditional mining method is greatly increased. Horizontal directional drilling can make the borehole trajectory coinciding with the shape of the coal seam. Compared with the traditional vertical Wells, horizontal directional drilling can increase the surface area of the gas recovery, thus improve the efficiency and quantity of the gas extraction and prolong the service life of gas well (Chen Jie et al., 2012). So the research on horizontal directional drilling technology and MWD system is much more important.

The horizontal directional well consists of vertical section and horizontal section. In the drilling process of vertical section, the drilling azimuth, apex angle and space coordinate are the core parameters for the control of borehole trajectory. To control these parameters accurately, the tool face angle in real time should be controlled, by adjusting the angle value of the drilling tool face angle to control the borehole trajectory (Guojun Wen et al., 2011). But most of the previous measuring methods need to put down the measurement instrument into the hole at a predetermined depth to measure the position of borehole, then pull up the instrument out of the hole and the measured data are stored for further processing by computer. The tool face angle cannot be measured in real time and the accuracy of the borehole trajectory is related to the data points measured. So this method is time-consuming and inconvenient.

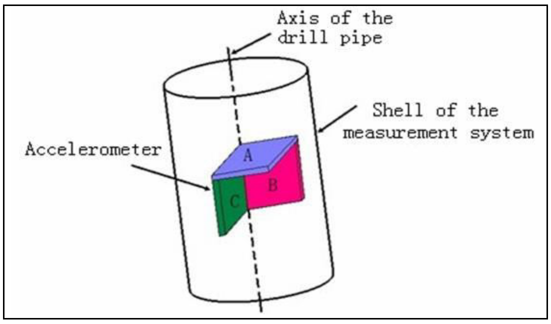

Figure 1: *The installation schematic diagram of accelerometers*

In the MWD system, it is necessary to measure the tool face angle in real-time and rectify the deviation according to the result of tool face angle measurement. Although traditional strapdown inertial navigation system contains formula to calculate tool face angle, the calculation formulas are very complex and not suitable for MWD system with the Microprogrammed Control Unit (MCU) as the core. Based on this, here the researchers use three mutually perpendicular uniaxial accelerometers (or a three-axis accelerometer) to form a three-axis accelerometer measurement system to measure and calculate the tool face angle in the process of drilling time, and then deduce a tool face angle calculation formula that is suitable for practical application with high calculation efficiency and precision. The three-axis accelerometer measurement system is shown in Figure 1. The three single axis accelerometers are labeled as A, B, C and both of them are perpendicular to each other. And accelerometer A is perpendicular to the pipe axis and accelerometer B and C are parallel with the pipe axis. As known, the tool face angle can be divided into magnetic north tool face angle and high edge tool face angle, and both of them can control borehole trajectory. The tool face angle discussed in this article is high edge tool face angle.

2. Formulas for Tool Face Angle

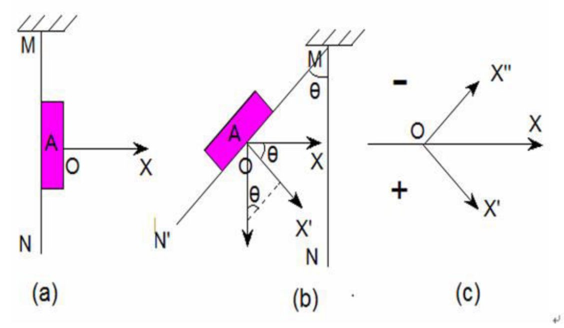

Figure 2: *The working principle of the single axis accelerometer*

Because the flexible single axis quartz accelerometer is used to measure and calculate the tool face angle, it is necessary to study the performance of the single axis accelerometer firstly. As shown in Figure 2 (a), accelerometers A is installed in the vertical plane. The direction of its sensitive axis (OX) points to the horizontal plane. As shown in Figure 2 (b), when the accelerometer A rotates a θ degree clockwise around MN, the sensitive axis will rotate a θ degree too. After the rotation, the direction of the sensitive axis is changed as OX'. As a rule, if the sensitive axis of accelerometer A is vertical downward, the output value by the sensor is the local acceleration of gravity. Assumed that the output value of the accelerometer is gx when it rotates in the direction shown in Figure 2 (b). Depending on the trigonometric function relation, it can be achieved that sinθ=gx/g. Through the solution of the trigonometric function equation the value of θ can be obtained and θ is called the pitch angle (or roll angle) of the accelerometer. As shown in Figure 2 (c), if OX rotates a θ degree clockwise (θ is the pitching angle or roll angle and 0<θ<90), shown as OX', then this direction is defined as the positive direction of the sensitive axis and the value of θ is positive. If OX rotates a θ degree anticlockwise (θ is the pitching angle or roll angle and 0<θ<90), shown as OX'', then this direction is defined as the negative direction of the sensitive axis and the value of θ is negative.

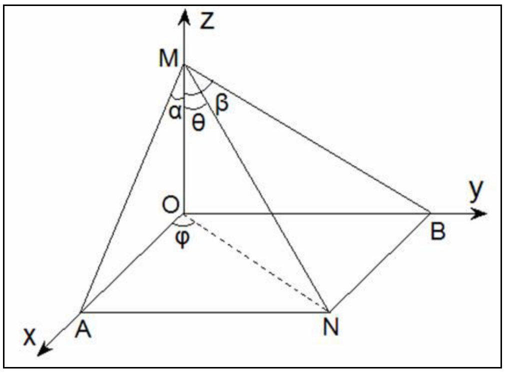

Figure 3: *Instrument coordinate system*

The instrument coordinate system established is shown in Figure 3 under the installation mode in Figure 1 (Lu Gui-ying et al., 2010). The x, y, z axis are three axes of this space coordinate system. Take the line MN as the high side and the projections of MNO in yOz and xOz plane are BOM and AOM respectively.

In Figure 3 α and β are the pitch angle and roll angle respectively and deduced by the measurements of accelerometer B and C, θ respects the vertex angle while φ respects the tool face angle. Then the vertex angle θ, pitching angle α and roll angle β are used to deduce tool face angle φ. When this measurement system is used to calculate the tool face angle, the vertex angle θ is needed. Because the measurement method for vertex angle is logical and it is a must to be measured in the MWD system, in this thesis the author only discussed the calculation accuracy and efficiency of the tool face angle and the vertex angle will not be discussed.

2.1 The first kind of formula for tool face angle

This formula is generally useful in the strap down inertial navigation system. As shown in Figure 3,

$$\tan\varphi = \frac{AN}{OA} = \frac{OB}{OA} = \frac{\tan\beta \times OM}{\tan\alpha \times OM} = \frac{\tan\beta}{\tan\alpha} \quad (1)$$

$$\varphi = \arctan\frac{\tan\beta}{\tan\alpha} + 180^{\circ}\cdot n$$

(n is an integer)

In the process of calculating the tool face angle by formula (1), it can be assumed that the result of the tanφ is positive. So, there are two results within the range of [0~2π], that is to say, the value of φ may be in the range of [0~π/2] or in the range of [π-3π/2]. So, the precise tool face angle cannot be obtained from formula (1).

According to Figure 3, the relationship between the quadrant which the tool face angle in and the sensitive direction of the accelerometer B and the accelerometer C can be obtained (as shown in Figure 4).

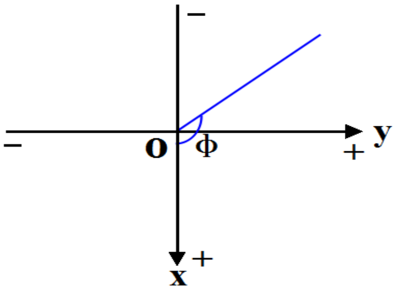

***Figure 4**: the relationship between the quadrant and the sensitive direction*

In Figure 4, the x axis stands for the sensitive axis of the accelerometer B and the y axis stands for the sensitive axis of the accelerometer C, the ON is the high side. So, we can get the rules as follow.

If the x axis is positive and the y axis is positive, the value of the tool face angle will be within the range of [0~π/2].

If the x axis is negative and the y axis is positive, the value of the tool face angle will be within the range of [π/2~π].

If the x axis is negative and the y axis is negative, the value of the tool face angle will be within the range of [π~3π/2].

If the x axis is positive and the y axis is negative, the value of the tool face angle will be within the range of [3π/2~2π].

So, if φ is calculated using formula (1), the precise tool face angle will be figured out under the help of the relation between the tool face angle values and sensitive direction of the accelerometers(are shown in Table 1). In this table, the x axis stands for the sensitive axis of the accelerometer B and the y axis stands for the sensitive axis of the accelerometer C, the '+' means that the sensitive direction is positive and the '-' means that the sensitive direction is negative.

***Table 1:**The relation between the sensitive direction and the value scope of the tool face angle*

x	+	-	-	+
y	+	+	-	-
The tool face angle	$\arctan\frac{\tan\beta}{\tan\alpha}$	$180+\arctan\frac{\tan\beta}{\tan\alpha}$	$180+\arctan\frac{\tan\beta}{\tan\alpha}$	$360+\arctan\frac{\tan\beta}{\tan\alpha}$

If the outputαby the accelerometer B and the outputβby the accelerometer C are both positive, the tool face angle will be arctan tanβ/tanα. If the outputαby the accelerometer B is negative and the outputβby the accelerometer C is positive, the tool face angle will be180°+ arctan tanβ/tanα.

If the outputαby the accelerometer B and the outputβby the accelerometer C are both negative, the tool face angle will be 180°+ arctan tanβ/tanα. If the outputαby the accelerometer B is positive and the outputβby the accelerometer C is negative, the tool face angle will be 360°+ arctan tanβ/tanα.

2.2 The second kind of formula for tool face angle

This formula is deduced by the authors. As shown in Figure 3,

$$\cos\varphi=\frac{OA}{ON}=\frac{OM\times\tan\alpha}{OM\times\tan\theta}=\frac{\tan\alpha}{\tan\theta}\quad(2)$$

$$\varphi=\arccos\frac{\tan\alpha}{\tan\theta}+180^{\circ}\cdot n$$ (n is an integer)

In formula (2), if obtained result cosφ is positive, there would be two different values of the tool face angle in the range of [0~360°] shown in Figure 5 The same conclusion can be obtained if cosφ is negative. So the tool face angle cannot be deduced by using formula (2) directly and uniquely as the same as using formula (1).

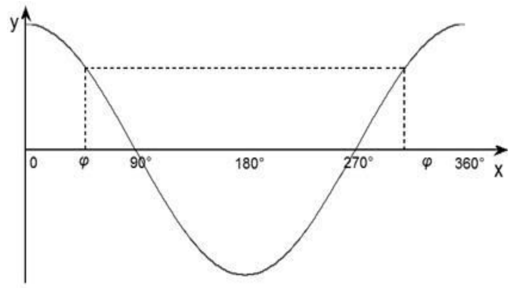

***Figure 5**: The calculation result by formula (2)*

If φ is calculated using formula (2), the precise tool face angle will be figured out under the help of the relation between the tool face angle values and sensitive direction of the accelerometers(are shown in Table 2). In this table, the x axis shands for the sensitive axis of the accelerometer B and the y axis stands for the sensitive axis of the accelerometer C, the '+' means that the sensitive direction is positive and the '-' means that the sensitive direction is negative.

***Table 2**: the relation between sensitive direction and the tool face angle values in formula (2)*

x	+	-	-	+
y	+	+	-	-
The tool face angle	$\arccos\frac{\tan\alpha}{\tan\theta}$	$\arccos\frac{\tan\alpha}{\tan\theta}$	$360-\arccos\frac{\tan\alpha}{\tan\theta}$	$360-\arccos\frac{\tan\alpha}{\tan\theta}$

If the outputαby the accelerometer B and the outputβby the accelerometer C are both positive, the tool face angle will be arccos tanα/tanθ. If the outputαby the accelerometer B is negative and the outputβby the accelerometer C is positive, the tool face angle will be arccos tanα/tanθ.

If the outputαby the accelerometer B and the outputβby the accelerometer C are both negative, the tool face angle will be 360°-arccos tanα/tanθ.
If the outputαby the accelerometer B is positive and the outputβby the accelerometer C is negative, the tool face angle will be 360°-arccos tanα/tanθ.

2.3 The third kind of formula for the tool face angle

This formula is also deduced by the authors (As shown in Figure 3).

$$\sin\varphi = \frac{\tan\beta}{\tan\theta} \quad (3)$$

$$\varphi = \arcsin\frac{\tan\beta}{\tan\theta} + 180^\circ \cdot n \quad \text{(n is an integer)}$$

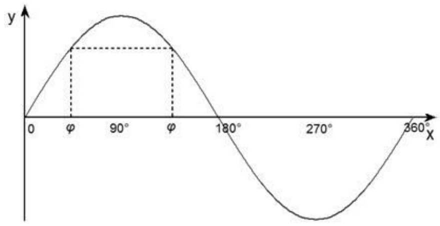

***Figure 6**: The calculation result by formula (3)*

In formula (3), if obtained result sinφ is positive, there would be two different values of the tool face angle in the range of [0~360°] shown in Figure 6. The same conclusion can be obtained if sinφ is negative. So the tool face angle cannot be deduced by using formula (3) directly and uniquely as the same as using formula (1) and formula (2).

If φ is calculated by formula (3), the precise tool face angle will be figured out under the help of the relationship between the tool face angle values and sensitive direction of the accelerometers in table 3. In this table, the x axis stands for the sensitive axis of the accelerometer B and the y axis stands for the sensitive axis of the accelerometer C, the '+' means that the sensitive direction is positive and the '-' means that the sensitive direction is negative.

***Table 3:** the relation between sensitive direction and the tool face angle values in formula (3)*

x	+	-	-	+
y	+	+	-	-
The tool face angle	$\arcsin\frac{\tan\beta}{\tan\theta}$	$180-\arcsin\frac{\tan\beta}{\tan\theta}$	$180-\arcsin\frac{\tan\beta}{\tan\theta}$	$360+\arcsin\frac{\tan\beta}{\tan\theta}$

If the outputαby the accelerometer B and the outputβby the accelerometer C are both positive, the tool face angle will be arcsin tanβ/tanθ.
If the outputαby the accelerometer B is negative and the outputβby the accelerometer C is positive, the tool face angle will be 180°-arcsin tanβ/tanθ.
If the outputαby the accelerometer B and the outputβby the accelerometer C are both negative, the tool face angle will be 180°-arcsin tanβ/tanθ.
If the outputαby the accelerometer B is positive and the outputβby the accelerometer C is negative, the tool face angle will be 360°+arcsin tanβ/tanθ.
Among the above mentioned three calculation formulas for tool face angle, formula (1) is the most commonly used one, formula (2) and formula (3) are deduced by the author. If φ is calculated using one of the above mentioned three formulas, the quadrant in which the tool face angle value is, can be ascertained according to the sensitive direction of the accelerometers and then the tool face angle will be figured out. However, as the core processor of the MWD system is the MCU (Microprogrammed Control Unit), whose operation speed is low. And a single chip can handle basic operations such as addition, subtraction, multiplication, division, but is not able to handle complex calculation such as differential, integral, inverse trigonometric function, etc. So these three formulas cannot be directly used in MWD system.

3. Research on the Accuracy and the Efficiency of the Three Formulas

To use the three formulas in the MWD system, the trigonometric functions and the formulas all are needed to be converted to basic operations through Taylor expansion formula, so that they can be processed by the MCU. In the MWD system, it is necessary to ensure that the selected calculation formula is short in operation time in the MCU and is high in calculating precision. It is critical to discuss the precision and efficiency of the above three calculation formulas for the tool face angle.

3.1 The analysis of the accuracy and efficiency of calculation for the three formulas

Due to each of the formulas contains a division of a tangent functions by another one (tanA/tanB), the accuracy and efficiency of calculation are the same for the three formulas. As a result, it is not necessary to analyze the accuracy and efficiency of calculation of Taylor expansion of this part in the formulas. More attention will be paid to the calculation accuracy and efficiency of the arctangent, arccosine and arcsine for the three formulas. Shown as follows:
Taylor expansion of formula (1):

$$\arctan\frac{\tan\beta}{\tan\alpha}=\frac{\tan\beta}{\tan\alpha}-\frac{\tan^3\beta}{\tan^3\alpha}/3$$
$$+\frac{\tan^5\beta}{\tan^5\alpha}/5-...$$

The domain of formula (1) is a set of real numbers.

Taylor expansion of formula (2):

$$\arccos\frac{\tan\alpha}{\tan\theta}=\pi/2-\frac{\tan\alpha}{\tan\theta}-\frac{\tan^3\alpha}{\tan^3\theta}/(2\times3)$$
$$-1\times3\times\frac{\tan^5\alpha}{\tan^5\theta}/(2\times4\times5)-...$$

The domain of formula (2) is -1≤tanα/tanθ≤1.

Taylor expansion of formula (3):

$$\arcsin\frac{\tan\beta}{\tan\theta}=\frac{\tan\beta}{\tan\theta}+\frac{\tan^3\beta}{\tan^3\theta}/(2\times3)+$$
$$1\times3\frac{\tan^5\beta}{\tan^5\theta}/(2\times4\times5)+$$
$$1\times3\times5\frac{\tan^7\beta}{\tan^7\theta}/(2\times4\times6\times7)...$$

The domain of formula (3) is-1≤tanβ/tanθ≤1.

As the number of terms of Taylor expansion is infinite, several front items are usually taken as the approximate value instead of real value in practice. The more items are used, the higher approximation precision is. Also, the more times the basic computing will be, hence resulting in the slower computing speed. In order to improve the computing speed of the single chip microcomputer in the MWD, less number of items should be used on the premise of ensuring the required calculation precision.

First of all, let's analyze the relationship between the calculation accuracy and item numbers of Taylor expansion for the three formulas. It is important to notice that the unit of the calculate value by using Taylor expansion is radian; it should convert to degree when needed.

(1) The three curves shown in Figure 7 are respectively the relative error curves between the approximate value and the real value when the front 3, 4 or 10 items of the Taylor expansion of formula (1) are used. In this Figure, the x-coordinate respects the value of tanβ/tanα, while the y-coordinate respects the value of relative errors between the approximate value and the real value.

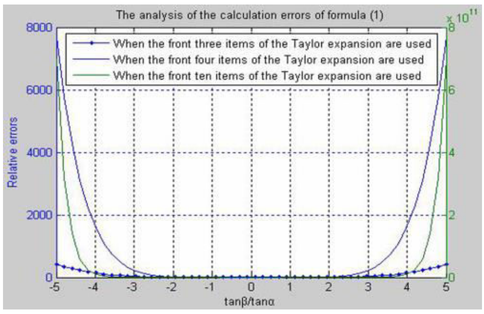

***Figure 7:** The analysis of the accuracy of Formula (1)*

Figure 7 illustrates that when ｜tanβ/tanα｜ is small, the accuracy of all the three curves is high. However, the error is bigger and bigger along with the increment of ｜tanβ/tanα｜.

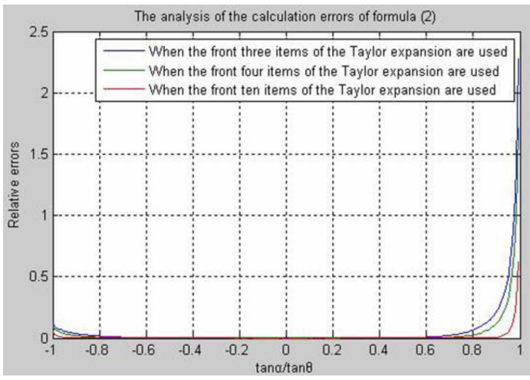

***Figure 8:**The analysis of the accuracy of Formula (2)*

(2) The three curves shown in Figure 8 are respectively the relative error curves between the approximate value and the real value when the front 3, 4 or 10 items of the Taylor expansion of formula (2) are used. In this Figure, the x-coordinate respects the value of tanα/tanθ, while the y-coordinate respects the value of relative errors between the approximate value and the real value. When 0≤│tanα/tanθ│≤0.7 (the tool face angle is about 45° to 135° or 225°to 315°), all the three approximation errors are small; when 0.7≤│tanα/tanθ│≤1 (The tool face angle is about 0°to 45°, 135° to 225°or 315°to 360°), accuracy is gradually improved along with the increase of the number of items used, but there is still a larger error existed.

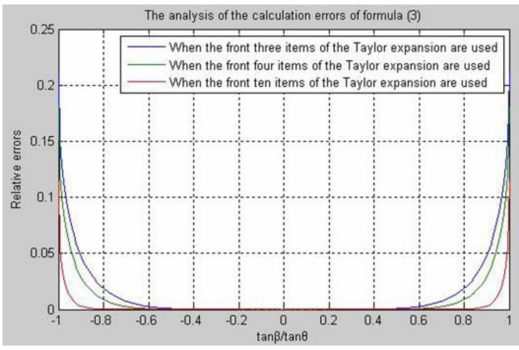

***Figure 9：** The analysis of the accuracy of Formula (3)*

(3) The three curves shown in Figure 9 are respectively the relative error curves between the approximate value and the real value when the front 3, 4 or 10 items of the Taylor expansion of formula (3) are used. In this Figure, the x-coordinate respects the value of tanβ/tanθ, while the y-coordinate respects the value of relative errors between the approximate value and the real value. When 0≤│tanβ/tanθ│＜0.7 (the tool face angle is about 0° to 45°, 135° to 225° or 315°o 360°), all the three approximation errors are small; when 0.7≤│tanβ/tanα│≤1 (the tool face angle

is about 45°to 135° or 225°to 315°), accuracy is gradually improved along with the increase of the number of items used, but there is still a larger error existed.

According to the above analysis of the calculation accuracy of the three formulas, the precision of the three formulas are high respectively in a certain scope. Also the calculation accuracy is lower beyond this range.

Although the accuracy is improved with the increase of the numbers of items of the Taylor expansions, the results also have large error, and the more the items are used, the longer the calculation time is. Therefore, considering the similar operation time in application, different formula should be chosen according to different interval of the formula to ensure the computation time is the least as well as the accuracy is the highest at the same time.

3.2 Comparison of the calculation accuracy of the three formulas

It is necessary to compare the calculation accuracy of the three formulas in different intervals of the calculation results under similar MCU operation time. (In case of 0°to 90°)

As it is known, addition and subtraction operation need 1 machines cycle, while multiplication and division operation need 4 machines cycles in MCU. If the front 4 items of formula (1) and formula (2) or the front 3 items of formula (3) are used, the operations of addition, subtraction, multiplication, and division in MCU are similar. Thus the operation time is similar too.

3.2.1 If the value of φ is between 0~45°

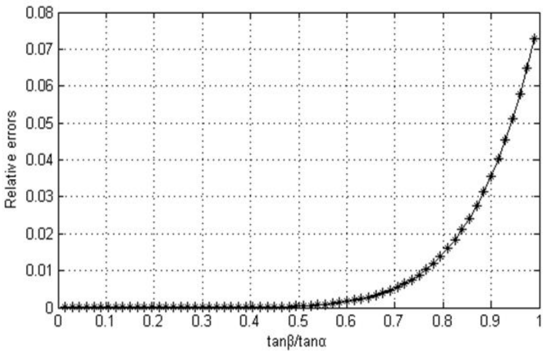

***Figure 10**：The relative error of the calculation result using the approximate value of formula (1) if φ is between 0~45°*

To formula (1)

$$\varphi = \arctan\frac{\tan\beta}{\tan\alpha}$$

$$= \frac{\tan\beta}{\tan\alpha} - \frac{\tan^3\beta}{\tan^3\alpha}/3 + \frac{\tan^5\beta}{\tan^5\alpha}/5 - \ldots$$

If 0＜tanβ/tanα＜1 and the value of φ is between 0~45°, the front 4 items of the Taylor expansion of formula (1) are as shown in Figure 10. In this Figure, the x-coordinate respects the value of tanβ/tanα between (0, 1), while the y-coordinate respects the value of relative errors between the approximate value and the real value. The maximum relative error between calculated value and true value is about 8%, especially when tanβ/tanα＞0.84, the relative error is more than 2% and calculation accuracy is low.

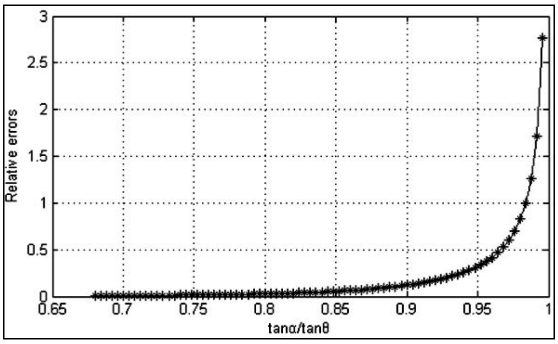

***Figure 11**：The relative error of the calculation result using the approximate value of formula (2) if φ is between 0~45°*

To formula (2)

$$\varphi = \arccos\frac{\tan\alpha}{\tan\theta} = \pi/2 - \frac{\tan\alpha}{\tan\theta} -$$

$$\frac{\tan^3\alpha}{\tan^3\theta}/(2\times3) - 1\times3\times\frac{\tan^5\alpha}{\tan^5\theta}/(2\times4\times5) - \ldots$$

If $\tan\alpha/\tan\theta > \sqrt{2}/2$ and the value of φ is between 0~45°, the front 4 items of the Taylor expansion of formula (2) are as shown in Figure 11. In this Figure, the x-coordinate respects the value of tanα/tanθ between $(\sqrt{2}/2, 1)$, while the y-coordinate respects the value of relative errors between the approximate value and the real value. The maximum relative error between calculated value and true value is about 30%, especially when tanα/tanθ＞0.98, the relative error is more than 10% and calculation accuracy is low.

To formula (3)

$$\varphi = \arcsin\frac{\tan\beta}{\tan\theta}$$

$$= \frac{\tan\beta}{\tan\theta} + \frac{\tan^3\beta}{\tan^3\theta}/(2\times3) + 1\times3\frac{\tan^5\beta}{\tan^5\theta}/(2\times4\times5) \quad \mathrm{I}$$

$$+1\times3\times5\frac{\tan^7\beta}{\tan^7\theta}/(2\times4\times6\times7)\ldots$$

f $0 < \tan\beta/\tan\theta < \sqrt{2}/2$ and the value of φ is between 0~45°, the front 3 items of the Taylor expansion of formula are as shown in Figure 12. In this Figure, the x-coordinate respects the value of tanβ/tanθ between $(0, \sqrt{2}/2)$, while the y-coordinate respects the value of relative errors between the approximate value and the real value. The relative error between calculated value and true value is within 0.7% and the calculation accuracy is high.

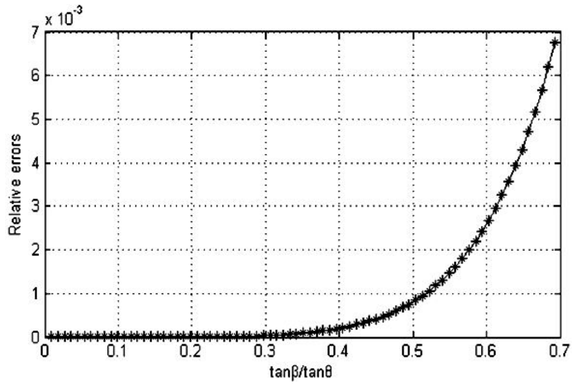

Figure 12 : *The relative error of the calculation result using the approximate value of formula (3) if φ is between 0~45°*

If the value of φ is between 0~45°and the front 4 items of formula (1) and formula (2) or the front 3 items of formula (3) are used, the formula (3) has the highest calculation accuracy.

3.2.2 If the value of φ is between 45°~90°

To formula (1)

$$\varphi = \arctan\frac{\tan\beta}{\tan\alpha}$$

$$= \frac{\tan\beta}{\tan\alpha} - \frac{\tan^3\beta}{\tan^3\alpha}/3 + \frac{\tan^5\beta}{\tan^5\alpha}/5 - ...$$

If tanβ/tanα > 1 and the value of φ is between 45°~90°, formula (1) is divergent. The approximate value can't be expressed by Taylor expansion.

To formula (2)

$$\varphi = \arccos\frac{\tan\alpha}{\tan\theta} = \pi/2 - \frac{\tan\alpha}{\tan\theta}$$

$$-\frac{\tan^3\alpha}{\tan^3\theta}/(2\times3) - 1\times3\times\frac{\tan^5\alpha}{\tan^5\theta}/(2\times4\times5) - ...$$

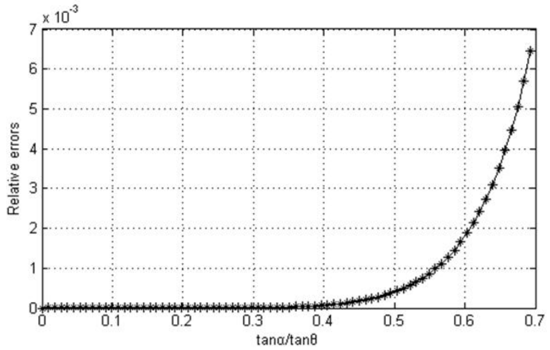

Figure 13: *The relative error of the calculation result using the approximate value of formula (2) if φ is between 45°~90°*

If $0 < \tan\alpha/\tan\theta < \sqrt{2}/2$ and the value of φ is between45°~90°, the front 4 items of the Taylor expansion of formula (2) are as shown in Figure 13. In this Figure, the x-coordinate respects the value of tanα/tanθ between $(0, \sqrt{2}/2)$, while the y-coordinate respects the value of relative errors between the approximate value and the real value. The relative error between calculated value and true value is within 0.7% and the calculation accuracy is high.

To formula (3)

$$\varphi = \arcsin\frac{\tan\beta}{\tan\theta} = \frac{\tan\beta}{\tan\theta} + \frac{\tan^3\beta}{\tan^3\theta}/(2\times3) +$$

$$1\times3\frac{\tan^5\beta}{\tan^5\theta}/(2\times4\times5) + 1\times3\times5\frac{\tan^7\beta}{\tan^7\theta}/(2\times4\times6\times7)...$$

If $\tan\beta/\tan\theta > \sqrt{2}/2$ and the value of φ is between 45°~90°, the front 3 items of the Taylor expansion of formula (3) are as shown in Figure 14. In this Figure, the x-coordinate respects the value of tanβ/tanθ between $(\sqrt{2}/2, 1)$, while the y-coordinate respects the value of relative errors between the approximate value and the real value. The maximum relative error between calculated value and true value is about 20%, especially when tanβ/tanθ > 0.9, the relative error is more than 10% and calculation accuracy is low.

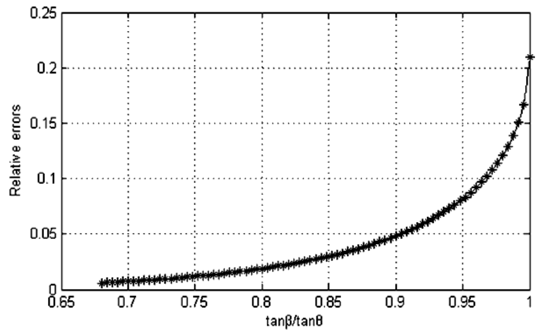

Figure 14 : *The relative error of the calculation result using the approximate value of formula (3) if φ is between 45°~90°*

Through the analysis above, if the value of φ is between 45°~90° and the calculating speed of the three formulas are similar in the MCU, the precision of formula (2) is highest.

The analysis above takes the value of φ between 0° to 90° as an example, if φ is not in this intervals, the same results will be achieved due to the cycle properties of trigonometric function. Then according to the different measurement results, different formulas should be chosen in calculation to ensure the accuracy of computation and also can reduce the operation time. Therefore, in MWD, the system is able to handle the more data points per unit time, and the accuracy of MWD is also greatly improved.

According to the theoretical analysis above, the optimal rules as follows should be employed to program for MCU.

If $|\tan\alpha/\tan\theta| > \sqrt{2}/2$ or

$0<|\tan\beta/\tan\theta|<\sqrt{2}/2$, the front 3 items of formula (3) is used as the approximate value in calculation and the calculation precision is within 7%;

$$\varphi=180^{\circ}\cdot n+\arcsin\frac{\tan\beta}{\tan\theta}$$

$$\arcsin\frac{\tan\beta}{\tan\theta}\approx\frac{\tan\beta}{\tan\theta}+\frac{\tan^3\beta}{\tan^3\theta}/(2\times3)$$

$$+1\times3\frac{\tan^5\beta}{\tan^5\theta}/(2\times4\times5)$$

(n is an Integers)

If $0<|\tan\alpha/\tan\theta|<\sqrt{2}/2$ 0r $|\tan\beta/\tan\theta|>\sqrt{2}/2$, the front 3 items of formula (2) is used as the approximate value in calculation and the calculation precision is also within 7%.

$$\varphi=180^{\circ}\cdot n+\arccos\frac{\tan\alpha}{\tan\theta}$$

$$\arccos\frac{\tan\alpha}{\tan\theta}\approx\pi/2-\frac{\tan\alpha}{\tan\theta}-\frac{\tan^3\alpha}{\tan^3\theta}/(2\times3)$$

$$-1\times3\times\frac{\tan^5\alpha}{\tan^5\theta}/(2\times4\times5)$$

(n is an Integers)

Then the quadrant and the value of the tool face angle will be figured out through the output value of accelerometer B and C.

If the output of the accelerometer B and C are both positive, the tool face angle is in the first quadrant, the value scope is between 0°~90°.

If the output of the accelerometer B is negative and accelerometer C is positive, the tool face angle is in the second quadrant, the value scope is between 90°~180°.

If the outputs of the accelerometer B and C are both negative, the tool face angle is in the third quadrant, the value scope is between 180°~270°.

If the output of the accelerometer B is positive and accelerometer C is negative, the tool face angle is in the fourth quadrant, the value scope is between 270°~360°.

3.3 Experimental verification

For verifying the optimal rules abovementioned, many rigorous tests were performed with the experimental circuit and instrument platform which is shown in Figure 15. To ensure the accuracy of measurements of flexible quartz accelerometer, the method that de-noises the output signal based on multiwavelet analysis is used *(Zhao Chi-hang et al., 2013).*

In this experiment, firstly the researchers take the front four items of the Taylor expansion of formula (1) and formula (2) and the front three items of the Taylor expansion of formula (3) respectively to calculate the approximate value of the tool face angle. The relative errors between the calculated results and the real value are recorded as E1, E2 and E3 respectively. Then researchers use the similar method to calculate the tool face angle by the optimal combined algorithm above. The relative error between the calculated results and the real value is recorded as E4.

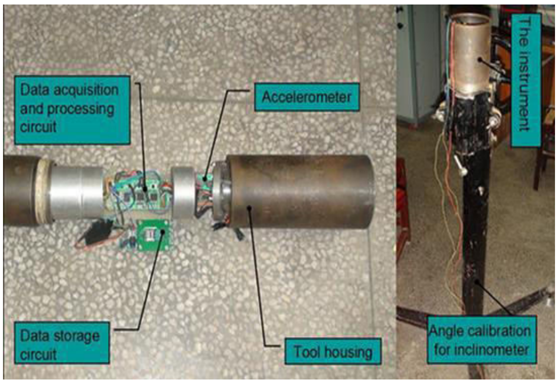

Figure 15: *The indoor experimental platform*

Table 4: *Experimental results*

Real value of tool face angle/°	Real Value of vertex angle θ /°	Measured Value of pitching angle α/°	Measured Value of roll angle β/°	Relative error of E1（%）	Relative error E2/（%）	Relative error E3/（%）	Calculated value by formulas (4) and (5) /°	Relative error E4/（%）
7	1	0.99	0.12	1.28	184.65	1.55	6.91	1.55
13	2	1.94	0.45	0.42	69.38	0.02	13.00	0.02
28	3	2.64	1.42	0.87	10.08	0.76	28.21	0.76
39	4	3.09	2.51	1.44	3.07	0.81	38.69	0.81
46	1	0.70	0.70	—	0.22	4.13	45.90	0.22
60	2	1.01	1.75	—	0.49	2.12	59.70	0.49
75	3	0.8	2.92	—	0.60	8.90	74.55	0.60
80	4	0.68	3.92	—	0.29	13.73	80.23	0.29
88	1	0.05	1.1	—	0.33	6.07	87.71	0.33
100	2	-0.36	2	—	0.37	8.86	100.37	0.37
128	3	-1.84	2.34	—	0.24	1.22	127.69	0.24
134	3	-2.09	2.19	—	0.13	0.30	133.83	0.13
157	4	-3.7	1.59	0.15	2.64	0.24	156.77	0.24
173	1	-1.01	0.12	0.13	6.23	0.06	173.11	0.06
181	2	-1.89	-0.05	0.28	13.02	0.24	181.43	0.24
206	3	-2.58	-1.26	0.01	3.23	0.58	204.81	0.58

231	4	-2.5	-3.09	—	0.21	0.52	231.49	0.21
243	1	-0.44	-1	—	0.37	3.35	243.91	0.37
278	2	0.3	-2.07	—	0.22	2.52	278.62	0.22
312	3	1.95	-2.12	—	0.53	1.09	310.34	0.53
337	4	3.5	-1.54	0.22	2.46	0.12	337.39	0.12
349	4	3.89	-0.74	0.07	3.09	0.10	349.36	0.10

In Table 4, E1 is the relative errors between the calculated value and the real value when the front 4 items of Taylor expansion of formula (1) are used. And Formula (1) is commonly used algorithm in tool face angle calculation, but this formula is divergent and the value of φ cannot be calculated by its Taylor expansion if $\tan\beta/\tan\alpha > 1$. Therefore, "—" appears in Table 4, namely the Taylor expansion is divergent and this formula cannot be used to calculate the errors. So if formula (1) is used, other more complex algorithms have to be used in MWD system. As a result, the formula (1) is not the best for the MWD system with slow MCU running speed. For formulas (2) and (3), the Taylor expansion can be employed to calculate the approximate value of φ, however, in this experiment when the item number used is less, the calculation error is bigger and the maximum errors are up to 184.65% and 13.73% respectively.

If the combination method of formula (2) and formula (3) is used, the maximum relative error of E4 is only 1.55%. As the value of the tool face angle is more accurate and the operation time is shorter, the drilling trajectory of CBM well will be adjusted more in the real time through the value of tool face angle in the MWD.

Compared with the theoretical analysis of the errors, the experiment errors are bigger. It should be concluded from two aspects. On the one hand, the output value by accelerometers existing a certain measurement error; on the other hand, there is a rounding error in the measuring system. And the smaller the tool face angle value is, the larger the deviation caused by the system error. In this experiment the maximum absolute error is 0.45°, which can meet the accuracy requirements of the engineering application.

4. Conclusions

Through the analysis and experiments in this paper, it can be known that the commonly used method for calculating the tool face angle is relatively complex, thus it is not the best choice for MWD system with the single chip processor as the core. Through using of the combined algorithm derived in this paper, the computational efficiency and calculation accuracy of tool face angle will be the best running result, which can provide real-time borehole trajectory correction with more accurate and efficient data support.

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