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Reliability Analysis of Slope Considering Earthquake Action based on Stochastic Finite Element Method

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Abstract: Based on the stochastic finite element method (FEM), slope reliability analysis is performed in this paper considering the inherent variability in parameters of rock and soil materials and the randomness of seismic effects, which is incorporated as the horizontal acceleration by pseudo-static method. Safety factor of the slope, which represents its reliability, is obtained by the shear strength reduction finite element method. Efficient Monte Carlo reliability analysis is accomplished using response surface method between the safety factor, the parameters of rock and soil mass and the horizontal acceleration of earthquake. This paper gives full play to the accurate simulation of FEM and the rationality of reliability analysis, and provides an effective method for slope safety assessment. A high slope example in rock engineering is presented to verify the feasibility and correctness of this method

Keywords: *slope, reliability, safety factor, horizontal acceleration, shear strength reduction*

1. Introduction

In recent years, the thought of uncertainty analysis has been accepted in practical projects gradually. For instance, the Chinese "Design code for engineered slopes in water resources and hydropower projects" (SL 386—2007) specifically states that a reliability analysis should be conducted when evaluating the stability of the Class I type slope, which can provide a rational basis of stability assessment of the slope. There are some common methods for the stability analysis of slope, for instance, First Order Reliability Method (FORM) [1], Response Surface Method (RSM) [2], Monte Carlo Simulations (MCS) [3], Adaptive Important Sampling Method [4] and Simple Analysis Method by J. M. Duncan [5]. In order to perform reliability analysis based on FEM calculation, Dianqing Li, Shuihua Jiang [6,7] proposed a method for the reliability analysis of slope stability using non-intrusive stochastic finite element method, which has been successfully applied to practical projects, combining the finite element analysis and the reliability evaluation.

Those investigations all focus on how to simulate random parameters discretely and optimize analysis method, only considering the influence of uncertainty of parameters of rock and soil materials on slope stability. However, in some actual projects, external loads, such as seismic action and seepage action, are main factors influencing the slope instability, which are also uncertain. Therefore, the effects of the uncertainty of external loads should not be ignored. Yingxiang Wu [8] takes seismic factors into account in the reliability analysis of slope stability. However, he just transforms the certain seismic wave into dynamic loads, and doesn't consider the uncertainty of seismic load in essence. Based on the stochastic finite element method, this paper proposed an approach for the reliability analysis of slope stability which introduces the inherent variability of parameters of rock and soil materials and the randomness of earthquake action. Utilizing the Pseudo-static method, earthquake action is considered as the horizontal acceleration in the simulating. Safety factor of slope is calculated by the shear strength reduction finite element method. Efficient Monte Carlo reliability analysis is then accomplished by applying the response surface method between the safety factor of slope, material parameters and the horizontal acceleration of earthquake. A high slope in rock engineering is taken as the numerical example to verify the feasibility and correctness of the proposed method.

2. Calculation of slope stability using FEM

2.1 Shear Strength reduction method

Shear strength reduction finite element method (SSRFEM)[9-11] cannot only get rid of the defect of the traditional limit equilibrium method which assumes that the rock and soil slices are rigid, but also consider the nonlinear constitutive relation of rock and soil materials and model complex loads and boundary conditions. By the means of reducing the shear strength parameters of rock and soil materials until the slope failure occurs, the strength reduction coefficient is regarded as the global safety factor of slope in this work, which is written as formula 1:

$$F_s = \frac{c}{c_i} = \frac{\tan\varphi}{\tan\varphi_i} = \frac{\int_A (c + \sigma\tan\varphi)dA}{\int_A \tau dA} \quad (1)$$

This safety factor has the identical physical significance with that calculated by the conventional limit equilibrium method. What's more, the shear strength reduction finite element method can take the influence of elastic-plastic constitutive relation of rock and soil materials into account, and simulate the failure process of slope and the shape of slip surface without assuming the shape of slip surface and the interaction force between strips in advance.

The key of SSRFEM is how to judge the failure of slope. Three methods are usually adopted: (1) the displacement mutation of feature points in the slope; (2) the un-convergence of finite element method calculation; (3) the connection of potential plastic zone of slip surface. (1) and (2) are easy to be implemented in finite element calculation, but the results are affected by the numerical algorithm, mesh quality and parameter input, which may have some artificial factors. In addition, these two methods have no clear physical significance, because local displacement mutation doesn't definitely mean the overall instability in practical engineering. (3) has the most specific significance, and with the development of visualization technique, program identification and automatically searching of the connected 3-D yield zone can be realized. In summary, a connected 3-D yield zone is regarded as the sign of slope failure in this paper, using WHUSSRFEM which is compiled by FORTRAN to implement the continuous computing of shear strength reduction finite element and the automatic recognition of connected 3-D yield zone. If the yield elements turn into a connected and closed 3-D yield zone inside the slope (which means that its horizontal projection is a closed zone) and the yield zone extends to the slope surface (which means that the projection of the zone on slope surface turns into a fill area), the program defines the yield zone as a 3-D region by searching the connection of yield elements and corresponding surface yield elements.

2.2 Pseudo-static method for earthquake load

Pseudo-static method applies the earthquake loads to the barycenter of potential unstable slide mass, which is equivalent to horizontal and vertical constant acceleration, with making the acceleration direction as the same direction of slope failure. Former researches show that the horizontal acceleration which points towards outside of the slope is the main cause of slope failure when subjected to earthquake action. Therefore, the horizontal earthquake acceleration is employed in this work to reflect the earthquake action. In the FEM calculation, the horizontal earthquake acceleration is applied to the barycenter of elements, making it equivalent to nodal forces. Parameter A is defined as horizontal earthquake acceleration 'a' to gravitational acceleration 'g' ratio, which is written as formula 2:

$$A = \frac{a}{g} \quad (2)$$

In this paper, A is treated as a random variable due to the randomness of seismic effects, as well as the inherent variability in parameters of rock and soil material. Many scholars have conducted studies on the value and distribution of A. This paper adopts the result of literature [17], with A's PDF and CDF subject to the following distribution. B and b are the undetermined coefficients, which can only be determined based on the site earthquake statistical data.

$$\text{PDF:} \quad f(A) = Bbe^{-bA} \quad (3)$$

$$\text{CDF:} \quad f(A) = 1.0 - Be^{-bA} \quad (4)$$

3. Slope reliability analysis

3.1 Response surface method

The performance function is always in high order, nonlinear and implicit when it comes to the reliability analysis of complex structures. Response surface method (SRM) uses explicit response surface to fit the real complex performance function, in order to transform complicated implicit performance function into simple explicit performance function in reliability analysis. Traditional response surface function is denoted by a quadratic polynomial without cross term:

$$f(\boldsymbol{x}) = a + \sum_{i=1}^{n} b_i x_i + \sum_{i=1}^{n} c_i x_i^2 \quad (5)$$

Where x= $[x_1, x_2, \ldots, x_n]^T$ is the random sample of random variable; and K= $[a, b_1, b_2, \ldots, b_n, c_1, c_2, \ldots, c_n]^T$ is the undermined coefficient vector with 2n+1 items.

As pointed out by Gomes [18], using the response surface method mentioned above to fit the real performance function requires less adjustment of the response surface and less invocation of performance function, and as a result, it can obtain better convergence and calculation accuracy. However, Zhang [19] pointed out that the collocation of the traditional response method can be negative when the coefficient of variation of random variable is large or its probability distribution is strongly non-normal, which makes no sense in the reality, such as the seismic horizontal coefficient derived in this paper will certainly be a positive value.

In order to get rid of such defect, response surface in the corresponding standard Gaussian space is constructed in this paper based on the suggestion proposed by Zhang [22].

$$f'(\boldsymbol{u}) = a + \sum_{i=1}^{n} b_i u_i + \sum_{i=1}^{n} c_i u_i^2 \qquad (6)$$

Where $u_i = \Phi^{-1}\left(F_i\left(x_i\right)\right)$ denotes the Rosenblatt transformation of random variable x_i, namely the standard normal random variable.

Similarly, the response surface is a quadratic polynomial with 2n+1 undermined coefficient while without cross terms, and it can obtain good convergence and enough calculation accuracy. Vector projection is used in this paper and 2n+1 collocations are selected to compute these coefficients, $[\mu_1, \mu_2, \ldots \mu_n]^{\mathrm{T}}$, $[\mu_1 + m\sigma_1, \mu_2, \ldots \mu_n]^{\mathrm{T}}$, $[\mu_1 - m\sigma_1, \mu_2, \ldots \mu_n]^{\mathrm{T}}$, …, $[\mu_1, \mu_2, \ldots \mu_n + m\sigma_n]^{\mathrm{T}}$, $[\mu_1, \mu_2, \ldots \mu_n - m\sigma_n]^{\mathrm{T}}$. Based on experience, a better accuracy can be obtained when m=2. Since the response surface is constructed in the standard Gaussian space (μ=0, σ=1), those 2n+1 collocations are $[0, 0, \ldots, 0]^{\mathrm{T}}$, $[2, 0, \ldots 0]^{\mathrm{T}}$, $[-2, 0, \ldots 0]^{\mathrm{T}}$,…, respectively. The collocations are transformed to the original space using the space mapping, and then substituted into real performance function to calculate the safety factor, and finally calculated through solving equations.

It's easy to analyze reliability using first-order reliability analysis or the Monte Carlo simulation, which requires negligible amount of calculation compared to the implicit performance function, after obtaining the explicit expression of response surface.

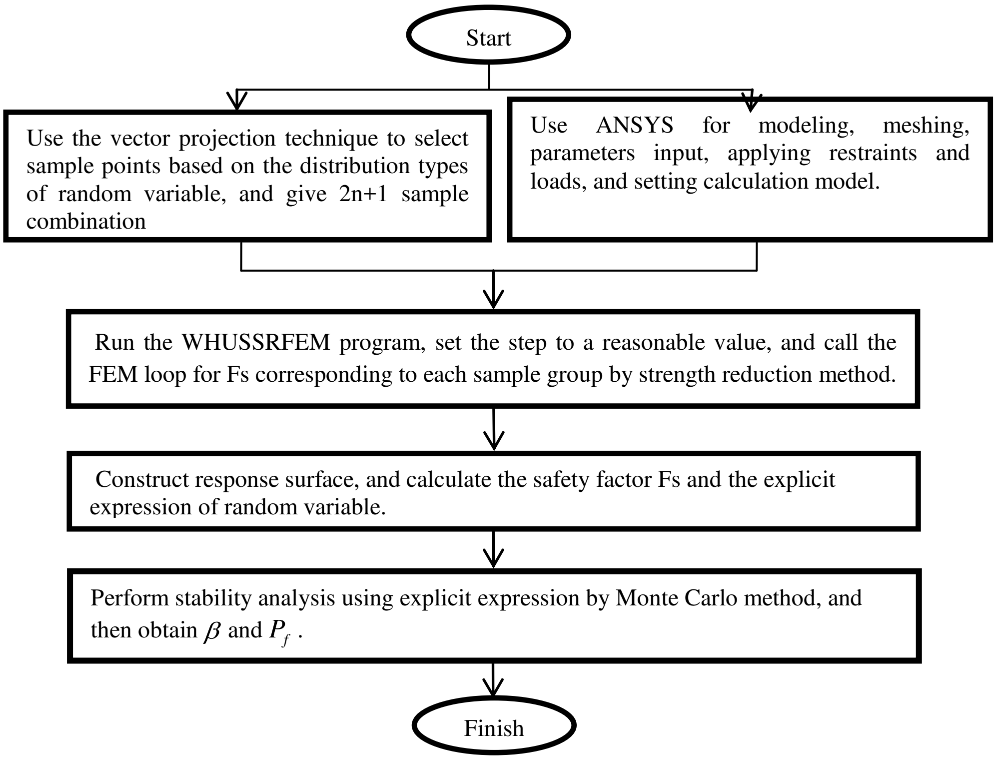

***Fig.1:** Flow chart of calculation*

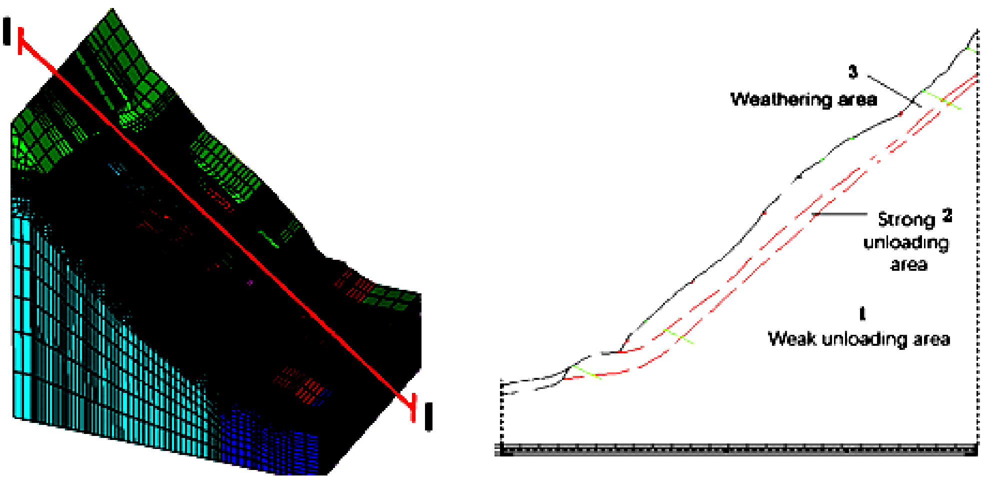

***Fig. 2:** Slope calculation model and condition*

Table 1: *Material Parameters*

Number	Unit weight /kN·m^{-3}	Elasticity modulus/GPa	Possion's ratio	*C*/MPa	φ
1	27	5.5	0.27	0.85	37
2	26	3.0	0.30	0.43	34
3	25	1.5	0.33	Random parameter	31

3.2 Calculation process

The aforementioned approach has been implemented in the strength reduction finite method program WHUSSRFEM [12], which can recognize and search yield zones in the slope. Preprocessing tasks for FEM which include modeling, meshing and parameter input are conducted based on ANSYS, while the analysis is performed by the elastic-plastic finite element method and a FEM program by Professor Chen [20], which can not only take complex geological conditions such as rock mass joint, fault and seepage into account, but also is capable of simulating some special conditions such as slope excavation and anchorage. Detailed flow chart of the calculation is shown in ***Fig. 1***.

4. Engineering instance

4.1 Calculation condition

A high rock slope at reservoir banks of a large hydropower station from literature [21] is employed to demonstrate the correctness and rationality of the method proposed in this paper. The relatesd slope located in the upstream about 780m from the dam axis in the Pubugou hydropower project,whose stability matters the running and the safety of the whole project.The slope height is 452 m,with 180210 elements and 174346 nodes in FEM model, calculation model and material parameters are shown in ***Fig. 2*** and ***Table 1***. In order to provide a better comparison with [21], other calculation and working conditions are kept the same with that in [21] except that seismic horizontal acceleration coefficient with random values is used in this work.

As illustrated above, B and b are undetermined coefficients that should be obtained according to the onsite seismic monitoring data. Based on some available statistical data, it is determined that B=0.05, b=20, μ_c=170, σ_c=20. And the cohesive force C obeys the law of normal stand distribution.

Table 2: *Safety factor computed under different sample combination*

Sample combination	Cohesive force *C* /MPa	horizontal earthquake acceleration *A*	Safety factor *Fs*
1	0.17	0.0347	1.16
2	0.13	0.0347	1.05
3	0.21	0.0347	1.23
4	0.17	0.0012	1.19
5	0.17	0.1892	1.08

4.2 Result of finite element calculation

Using vector projection technique to select sample combination, the safety factor is calculated and shown in ***Table 2***. Finite Element calculating process and the profiles of the connected yield zone are shown in ***Fig.3~7***.

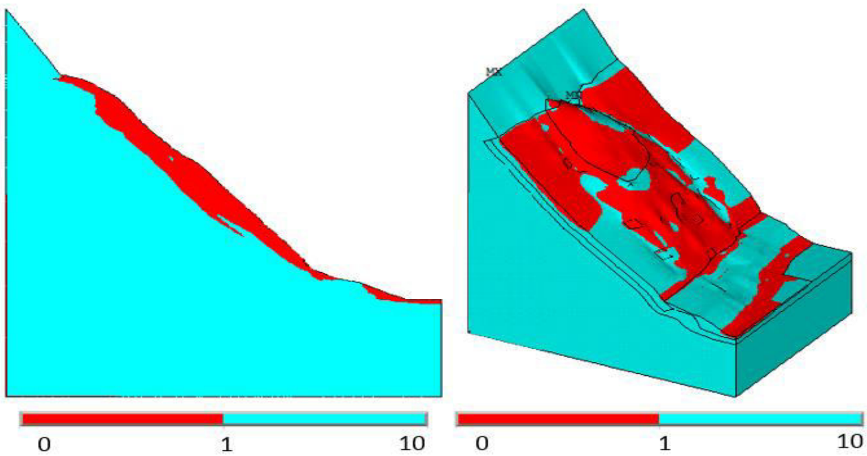

Fig. 3: *2D, 3D connection of yield zone for sample combination 1, Fs=1.16*

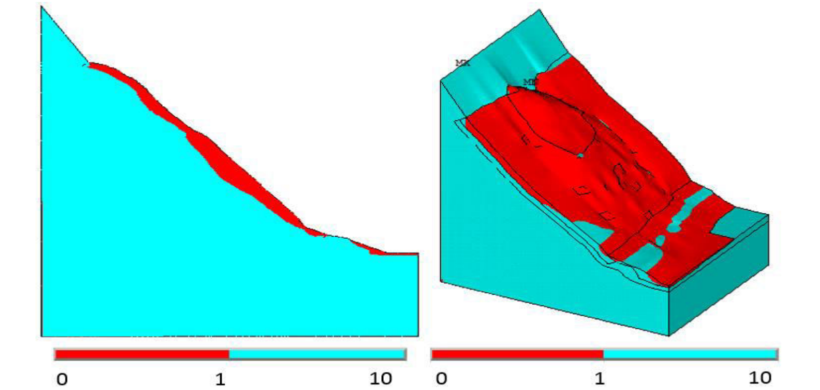

Fig. 4: *2D, 3D connection of yield zone for sample combination 2, Fs=1.05*

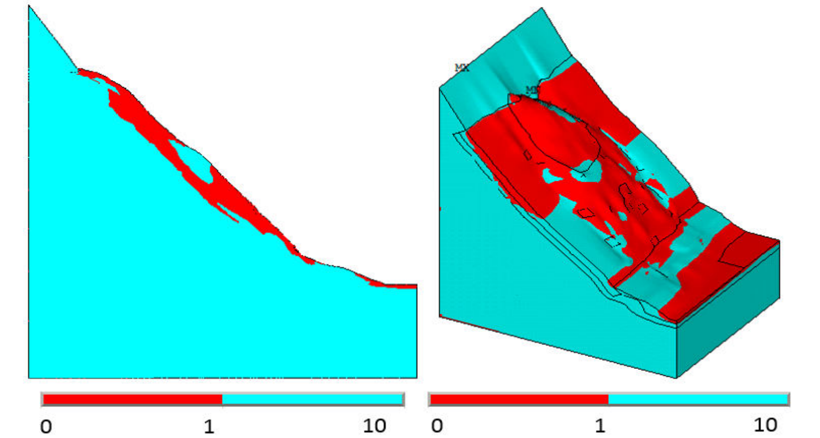

Fig.5: *2D, 3D connection of yield zone for sample combination 3, Fs=1.23*

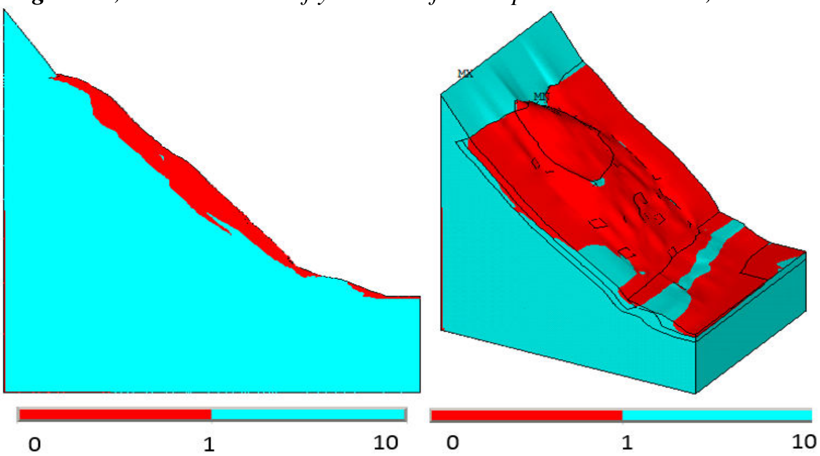

Fig. 6: *2D, 3D connection of yield zone for sample combination 4, Fs=1.19*

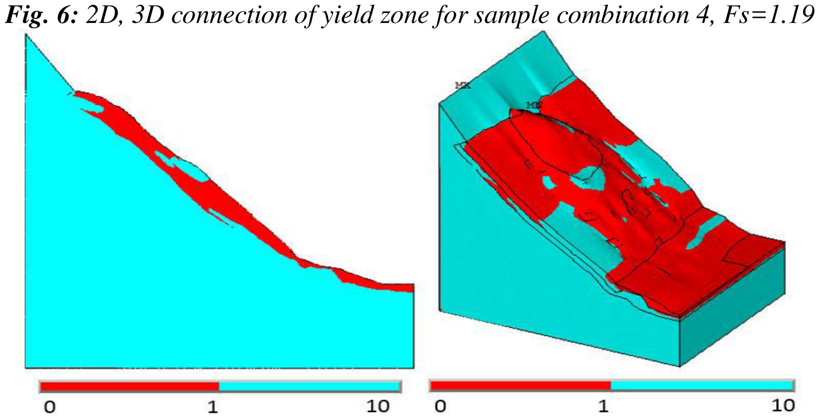

Fig. 7: *2D, 3D connection of yield zone for sample combination 5, Fs=1.08*

***Table3:** comparison of calculation results*

Condition	Method	Times of deterministic parsing	β	P_f
Paper [21] (with C)	Morgenstern-Price+MCM	200000	3.33	0.04%
This paper with A and C	SSRFEM+SRM+MCM	5	2.34	0.96%
This paper with C	SSRFEM+SRM+MCM	3	3.13	0.09%
This paper with A	SSRFEM+SRM+MCM	3	3.31	0.05%

4.3 Stability analysis

The response surface of safety factor and random variable can be constructed by solving the equations based on table 1. Its explicit expression is written as:

$$Fs = 1.16 + 0.045u_1 - 0.0275u_2 - 0.005u_1^2 - 0.0062u_2^2 \quad (7)$$

Or written as an expression about Fs, A and C:

$$FS = 1.16 + 0.045\Phi^{-1}\left(F_c(c)\right) - 0.0275\Phi^{-1}\left(F_A(A)\right) - 0.005\left[\Phi^{-1}\left(F_c(c)\right)\right]^2 - 0.0062\left[\Phi^{-1}\left(F_A(A)\right)\right]^2 \quad (8)$$

20000 times reliability analysis of C have been done using Monte Carlo Sampling Method and Rigid Equilibrium Limit Method in the paper [21]. However, the vector projection method is used to select 5 sample groups of A and C (select 3 groups when single factor) in this paper, and then the corresponding safety factors are chosen to construct response surface to arrive at the explicit expression of the safety factor and the random variable, and finally the expression is adopted to perform Monte Carlo reliability analysis. The comparison between the results of the two methods is shown in ***Table 3***.

The results indicate that the reliability analysis results of this work and literature [21] are close when neglecting seismic effects, but the calculation efficiency of the presented method is much higher. Much more, it is no need to assume the sliding surface in advance and the automatic recognition after the connection of yield zone.

By comparing the results of the three conditions, it indicates: the safety factor of slope decreases as strength parameters of rock mass decrease under the action of some earthquake horizontal acceleration, which is shown in ***Fig. 8***; the safety factor of slope decreases as the horizontal acceleration increases under the action of some strength parameters, which is shown in ***Fig. 9***; the transformation law of Fs is shown in ***Fig. 10*** with the consideration of A and C. And this paper takes 20 groups combination of A and C to verify the effectiveness of the response surface. The X-axis stands the Fs determined from FEM, while the Y-axis stands the Fs determined from RSM, which is shown in Fig. 11.And we can obtain the coefficient of correlation is 0.99,which verify the effectiveness of the response surface.

It's obvious that both the inherent variability of parameters of rock and soil mass and the randomness of earthquake actions can influence the failure probability of slope. Moreover, the failure probability increases one order of magnitude when both of them are considered comparing with only one of them considered respectively. Thus, it's necessary to take the randomness of external loads and the inherent variability of parameters of rock mass into account in the reliability analysis of slope stability.

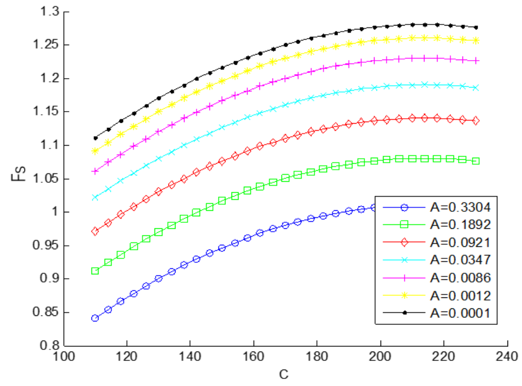

***Fig. 8:** The relationship between F_S and C*

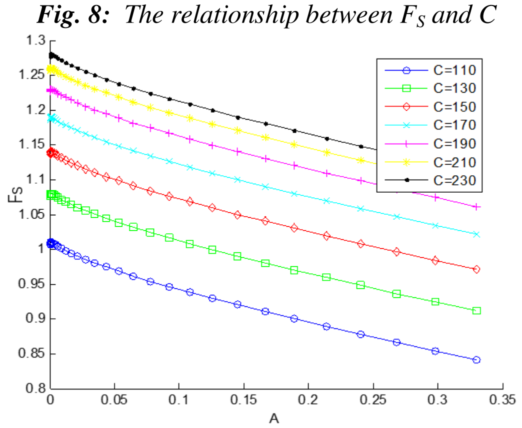

***Fig. 9:** The relationship between F_s and A*

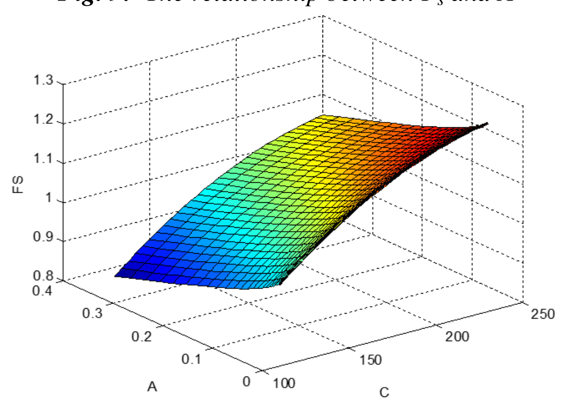

***Fig. 10:** The relationship between Fs and C、A*

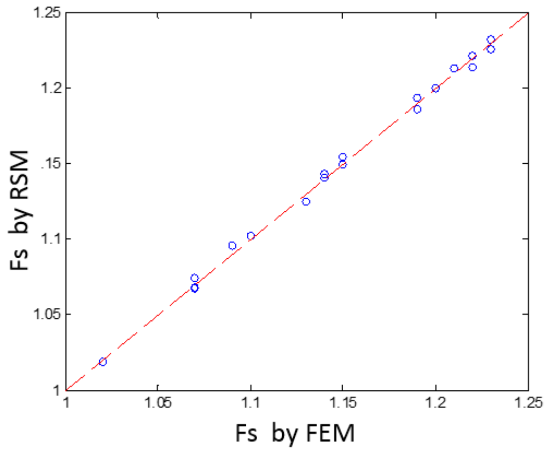

Fig. 11: *The correlation between FEM and RSM*

According to the above results, in this example, β =2.34, P_f =0.96%, which does not meet the reliability standard of slope compared with 3.7 degree (Class I structure, class one destruction) regulated by "Design code for engineered slopes in water resources and hydropower projects"(SL 386—2007). It means that the slope needs further reinforcement and risk treatment, which is consistent with the engineering treatment of actual project. Thus, the correctness of the proposed method is verified.

5. Conclusion

(1) The method presented in this paper adopts the safety factor obtained by strength reduction finite element method to represent stability and constructs response surface to achieve high-efficiency reliability analysis. It not only takes the uncertainty of the extrinsic factors besides the inherent variability of the material parameters of the slope into account, but also combines the advantage of the accuracy of FEM and the high-efficiency of the response surface. This method provides an effective approach to solve the stability problem of rock slope under complicated random working conditions.
(2) This paper studies the stability problem of slope under random stochastic seismic action and the result indicates that the inherent variability of parameters of rock mass and the randomness of earthquake action have considerable influence on the failure probability of slope. What's more, the failure probability increases one order of magnitude when both of them are considered comparing with only one of them considered respectively. Therefore it's necessary to take the uncertainty of main factors that affect safety of slope into account in reliability analysis of slope.
(3) This paper considers seismic action in the form of applying seismic acceleration based on the theory of pseudo-static method, which has simple principle and method with determination of the distribution of horizontal seismic acceleration by empirical formula and without consideration of dynamic and long-term effect of earthquake. However, mass seismic statistical data is the guarantee to obtain accurate results, which should be considered in future studies.

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