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Study on Downhole North-Seeking and Horizontal Hole Trajectory Measurement Technology Based on Inertial Navigation

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Abstract: In this paper, a system is designed to measure the angular velocity based on the triaxial fiber-optic gyroscope in order to solve the magnetic anomaly from secondary ores and metal interference in the underworking. In this system, with the magnetic factors excluded, the component of the earthly rotational angular velocity for the target point is obtained and then the corresponding north-seeking algorithm is derived. Meanwhile, with the limitations of longitude and latitude parameters removed, the geographical direction and angle to the North Pole for the target point are directly solved. In addition, the strapdown inertial navigation system consists of the triaxial fiber-optic gyroscope and accelerometer and it is used to collect the attitude data in the horizontal hole of the heading face. Then, the attitude integral calculation is deduced to solve the relative coordinates of the borehole trajectory. The data of the north-seeking system obtains the absolute spatial trend and coordinates of the downhole horizontal hole through correcting the coordinate system of the relative borehole trajectory. Moreover, this paper discusses the feasibility and limitations of the double-system integration and matching.

Keywords: *Downhole, Fiber-optic gyroscope, North-seeking, Borehole trajectory, Strapdown inertial navigation, Coordinates*

1. Introduction

Given the harsh conditions in mining the downhole resources, high requirements are set for the downhole construction technology. The advance borehole on the mine heading face involves the complicated construction, so it is demanding in the technology. The advance borehole effectively prevents the potential safety accidents such as the gas desorption, water emitting and collapse in the excavation and it plays an important role in evaluating the coal air content, hydrological environment and the roof pressure of coal petrography etc. [1]. The deflecting tool design (the screw motor) is used for drilling holes, so it is difficult to measure the long and narrow oylet and determine whether it has reached the target area. In addition, the loose and easily broken coal bed makes it rather difficult for the tool to do the thorough prospecting together with the complicated wall structure. These harsh conditions make it impossible for the traditional geophysical prospecting method to measure the hole plus the limitations of the working face machines including drilling rig, drill stem and anchor cable etc. Thus, a reliable method is urgently needed for the measurement of direction and trajectory. In recent years, with the development of the aviation attitude measurement technology, the inertial navigation system has developed rapidly to fight the magnetic-field interference, which provides new reference for the advance borehole measurement.

1.1. True-north measurement method of the downhole working conditions:

As the downhole environment is semi-closed with few reference objects, a few of methods can be used for finding the true-north direction in the narrow space. The traditional methods can be divided into two classes: geometric orientation and physical orientation.

In the geometric orientation, the spatial coordinate system of the ground is introduced into the downhole fixed point through the well-hole. Different navigation methods are used in different well-holes: foothills or inclined shafts, one vertical shaft and two vertical shafts. This kind of method is easy for measurers to grasp with reliable results achieved, but its accuracy needs improvement and the constant transmission of coordinates causes a bigger cumulative error. The physical orientation consists of magnetic instrument orientation, investment instrument orientation and gyroscopic theodolite orientation. The magnetic instrument first uses the electronic compass to detect the component of the geomagnetic field in the spatial coordinate system and then obtains the transformational relations of its own coordinate system and the geodetic coordinate system. Subsequently, the geographic North Pole is obtained after getting the direction of the geomagnetic North Pole and removing the magnetic declination. Easily influenced by the geomagnetic anomaly, this method cannot thoroughly solve the problem despite the magnetic compensation algorithm. The present advanced method is the gyroscopic theodolite measurement: the gyroscope rotates through the rotating gear and is relatively stable compared with space under the inertial effect. When the rotation axis revolves around the horizontal axis, the gyroscope moves on the horizontal plane under the vertical rotary force from the rotating earth and then the true north direction can be found through the set eyepieces. The accuracy of the equipment can be controlled within the range of minus or plus 20%, but it its operation is rather complicated for its big size.

1.2. True-north measurement method of the downhole working conditions:

The downhole borehole is mainly the MWD (measurement while drilling) for the limitations of space and measurement methods. Besides, some specific detectors are used for measuring the angle of the downhole borehole. China Coal Science and Industrial Corporation—Xi'an Research Institute, produces the YZG-series trajectory measuring meter for the mining drilling and it can be applied in the borehole trajectory MWD such as the measurement of horizontal holes, depression angle holes, gas drainage hole and exploration-and-discharge water hole etc. under the dusty underground coal mine filled with gas.

2. North-seeking system design based on the fiber-optic gyroscope:

2.1. Measurement method:

The fiber-optic gyroscope evolves into a new angular-rate sensor with the development of the fiber-optic sensor technology. It has the advantages of small size, high accuracy and easy maintenance. The fiber-optic gyroscope also has the measurement sensitive axis just like the traditional gyroscope and it can sense its relative angular velocity. Given that its reference coordinate system is not absolute, the fiber-optic gyroscope can sense its own relative angular change rate caused in the rotation of the earth in the earth coordinate system (*e* system), which is key to the north-seeking system[2-6].

In the geographic coordinate system, the signal sources of the fiber-optic sensor only produces small component when the earth rotates. In the case of the single-axis gyroscope north-seeking, when the gyroscope sensitive axis X is vertical to the carrier Z axis, the output of the gyroscope is:

$$\omega = \omega_e \cos\phi \cos L + \varepsilon(t) \tag{1}$$

In the equation, Φ is angle of equatorial plane and carrier Z axis, L is included angle of gyroscope sensitive axis and true-north direction, and ε is gyroscopic drift output [7-8]. The single-axis gyroscope north-seeking involves the local latitude, so the multi-axis gyroscope can effectively remove the parameters and measure the follow-up attitude of the equipment. When the sensitive axis Z of the triaxial gyroscope overlaps with the carrier Z axis, the output of the gyroscope sensitive axis X and axis Y is:

$$\omega_{x1} = \omega_e \cos\phi \cos L + \varepsilon(t_{x1})$$
$$\omega_{y1} = \omega_e \cos\phi \cos L + \varepsilon(t_{y1}) \tag{2}$$

After rotating the platform for 180 degrees, the following can be obtained:

$$L = a\tan\left(\frac{\omega_{x1} - \omega_{x2}}{\omega_{y1} - \omega_{y2}}\right) \tag{3}$$

2.2. Precise rotating platform design:

With the two-position north-seeking solution adopted in the design, the accurate displacement-limited fixation must be provided for the horizontal rotation of the rotary table in the geographic coordinate system because the positioning accuracy of the rotary table directly determines the north-seeking accuracy and the output may influence the properties of the gyroscope in the dynamic adjustment in particular. The vertical direction of the rotary table should overlap with the direction of the earth gravity and remove the absolute error of the orientation caused by the no coincidence in the coordinate system of the rotary table. In addition, given the biaxial sensitive rotation of the triaxial gyroscope, the coordinate systems of both the gyroscope and the rotary table also require catchment to ensure that the rotary axis of the rotary table overlaps with the sensitive axis of the gyroscope axis Z, thereby removing the relative error of the orientation caused from the no coincidence in the coordinate system of the gyroscope.

The bubble and gravity accelerometer are used in the dual correction for the horizontal direction alignment of the rotary table. In the case of centered bubbles, the output of the gravity accelerometer is slowly adjusted to the standard earth gravitational acceleration 1G. The rotary table should be locked and immovable after its direction is captured. The gyroscope is installed inside the measuring tube in the following position relationship: the axial direction of the tube overlaps with the gyroscope axis Z and the fixed plane of the tube is perpendicular to the gyroscope axis X. First, the tube is fixed with the chuck when the fixed plane overlaps with the plane of the rotary table. With the two-position (180 degrees) north-seeking solution adopted, the limitations of starting and terminal positions should be set up and then fixed with the locating pin in the rotation of the table. When the process control is not involved, the manual rotation is used in the turntable. In addition, the sensitivity of the sensitive axis in the direction Z of the gyroscope should be considered and controlled within the range of 8°/s while adjusting its rotating speed.

The precision bearing is used in the rotary table and the 180°positional accuracy between two positions is controlled within 0.5°. The tolerance of the turntable planeness is controlled within 0.5:200. The structural design is shown in Figure 1.

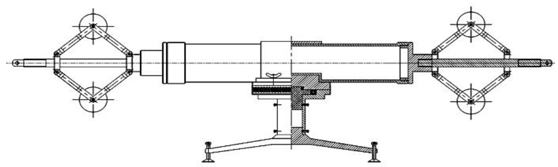


Figure 1. *Turntable structure diagram*

3. System design for trajectory measurement based on the strapdown inertial navigation:

3.1. Measurement methods:

The measurement devices consist of the triaxial fiber-optic gyroscope and the single-axis gravity accelerometer, which ensures that each coordinate axis has an angular speedometer in the relative coordinate system. According to the initial alignment relationship, the coordinate system for the spatial attitude measurement is established to obtain the dynamic data of the systematic attitude changes, namely, azimuth angle, pitch angle and face angle of the tube. Subsequently, the length is added in the attitude result, making it possible to obtain a series of the coordinate data in relation to the initial alignment. The layout of the gyroscope and accelerometer is shown in Figure 2.

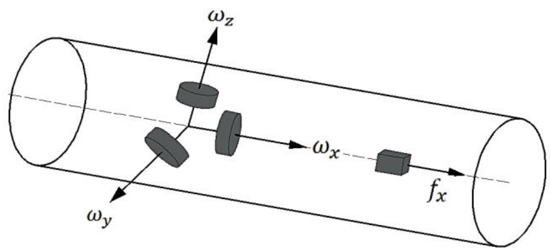


Figure 2. *Inertial navigation system consisting of gyroscope and accelerometer*

The sensitive axis of the single-axis gravity accelerometer overlaps with the tube axis to correct the pitch angle through the gyro calculation. The system for measuring the angular speed consists of triaxial fiber-optic gyros. This system is used for measuring the relative rotation of three coordinate axes and outputting the angular speed information.

3.2. Tube design for trajectory measurement:

The measurement tube relies on the inner space of the drill pipe to move forward in the horizontal hole. It is tubular with the support hands on both ends. And its diameter is smaller than the inner diameter of the drill pipe. This design ensures that the tube axis can be tangent to the drill pipe and that the real and entire movement trend is obtained. With springs, the support hands can approximate the tube wall and produce certain grabbing forces. The support hands are annular with wheels installed on the contact parts between them and the tube wall, ensuring the smooth passage through the drill pipe joint. Not influenced by the positions of the support hands, two separated odometer wheels are installed in the tail part to independently support two wheels attached to the tube walls. Moreover, rubber rings are installed on the support wheels to increase the friction and the bar is linked in the tail part to facilitate the flexible movement (moving forward or backward) inside the drill pipe. The overall structural is shown in Figure 3.

Figure 3. *Tube appearance*

With the magnetic steel installed on the odometer wheels, the odometer wheel can obtain the curve length St after the instrument enters the porthole through sensing the number of rotating circles.

3.3. Quaternion attitude calculation:

Different collocations correspond to different algorithms, and the simplest and most reliable algorithm is selected based on the present successful experience of the aviation system. In terms of this equipment, two algorithms provide reference: direction cosine algorithm and quaternion algorithm. In the direction cosine algorithm, the input of parameters is angle and distance. In this way, the gyro angular accelerometer is integrated and the angular data of the accelerometer is directly input. In the quaternion algorithm, the input data are the angular accelerometer and the gyro output signals are directly used in the operation. The quaternion algorithm is less time-consuming and produces less error than THE direction cosine algorithm, so the former, as the research subject of the instrument all the time, can achieve better accuracy. To transform the accelerated speed measured in the instrument coordinate system into the inertial coordinate system, the quaternion attitude algorithm requires the correction in every step. Thus, the calculation should be made through the quaternion transmission differential equation. As the gyro data output is quicker than the attitude algorithm in speed, the pre-compensation algorithm is used. Another advantage of the pre-compensation is to evaluate and calculate the error of the quaternion algorithm through using the actual calculation value and the predicted value.

As a reliable algorithm, the quaternion algorithm is validated in accuracy and calculation speed. The quaternion algorithm describes the rotation of a coordinate system in relation to the specific coordinate system by taking the gyro output data as the input. The algorithm is divided into two parts: scalar and vector. The scalar reflects half the cosine value of the intersection angle, while the vector reflects the rotational direction and the direction cosine in the reference coordinate system, which enables the quaternion algorithm to obtain two elements: direction and magnitude [9].

One real part λ and three imaginary part units p_1, p_2, and p_3 are introduced to establish the equation $q=\lambda+p_1i+p_2j+p_3k$. In the equation, are imaginary units regarded as the unit vectors of three coordinate axes.

When the initial attitude angle is substituted into the equation, the initial quaternion algorithm formula can be derived:

$$\begin{bmatrix}\lambda(0)\\p_1(0)\\p_2(0)\\p_3(0)\end{bmatrix}=\begin{bmatrix}\cos\frac{\psi_0}{2}\cos\frac{\theta_0}{2}\cos\frac{\gamma_0}{2}+\sin\frac{\psi_0}{2}\sin\frac{\theta_0}{2}\sin\frac{\gamma_0}{2}\\\cos\frac{\psi_0}{2}\cos\frac{\theta_0}{2}\sin\frac{\gamma_0}{2}+\sin\frac{\psi_0}{2}\sin\frac{\theta_0}{2}\cos\frac{\gamma_0}{2}\\\cos\frac{\psi_0}{2}\sin\frac{\theta_0}{2}\cos\frac{\gamma_0}{2}+\sin\frac{\psi_0}{2}\cos\frac{\theta_0}{2}\sin\frac{\gamma_0}{2}\\\sin\frac{\psi_0}{2}\cos\frac{\theta_0}{2}\cos\frac{\gamma_0}{2}+\cos\frac{\psi_0}{2}\sin\frac{\theta_0}{2}\sin\frac{\gamma_0}{2}\end{bmatrix} \quad (4)$$

The real-time matrix of the attitude is obtained by using the second-order or fourth-order Runge-Kutta Method. The attitude angle equation is derived:

$$\begin{cases}\theta_1=-\arcsin(2p_1p_3-2\lambda p_2)\\\psi_1=\arctan\left(\dfrac{2(p_1p_2+\lambda p_3)}{\lambda^2+p_1^{\ 2}-p_2^{\ 2}-p_3^{\ 2}}\right)\\\gamma_1=\arctan\left(\dfrac{2(p_3p_2+\lambda p_1)}{\lambda^2+p_3^{\ 2}-p_1^{\ 2}-p_2^{\ 2}}\right)\end{cases} \quad (5)$$

The attitude angles of all sampling points are obtained through the constant multiplied calculation [10-12].

3.4. Coordinate generation of the direction cosine algorithm:

By using the direction cosine, the quaternion algorithm is substituted to calculate azimuth angle ψ and pitch angle θ. Then the coordinate data of each sampling point in relation to the initial coordinate system are obtained by using the integral method [13-15].The formula is as following:

$$\begin{cases}X_t=X_{t-1}+\cos\theta_{t-1}\cos\psi_{t-1}(S_t-S_{t-1})\\Y_t=Y_{t-1}+\cos\theta_{t-1}\sin\psi_{t-1}(S_t-S_{t-1})\\Z_t=Z_{t-1}+\sin\theta_{t-1}(S_t-S_{t-1})\end{cases} \quad (6)$$

4. Experiment and error analysis:

4.1. North-seeking experiment:

The zero-offset 0.05°/h closed-loop interferometric fiber-optic gyroscope is used in the test experiment. The specific parameters of the gyroscope are shown in Table 1 and the specific parameters of the gravity accelerometer are shown in Table 2. When the control turntable rotates at 5°/s, the thermal balance lasts for 30 minutes and the angle between the initial position and the north-seeking direction becomes 45° while testing the north-seeking system. With sampling frequency and north-seeking time set at respectively 200Hz and 2 min, the collected data are discretized and processed through the high frequency filtering. After those steps, the final north-seeking accuracy exceeds 0.8° and can meet the anticipated requirements.

Table 1. *Specific parameters and performance of the gyroscope*

Type	Content
Rate range	360deg/s
Bias stability	0.05°/h
Readiness	0.5s
SF variation	0.01%
Acceleration(operation)	5g
Frequency range	0…0.5kHz
Temperature operating	-22°F…158°F

Table 2. *Specific parameters and performance of the gravity accelerometer*

Type	Content
Rate range	±2.0g
Zero position calibration	<10mg
Zero position stability(48h)	<0.05mg typ.
Switch repeatability	<0.05mg max.
Scale factor	1000±8mV/g
Zero temperature coefficient	<±0.1mg/°F typ.

4.2. Experiment of measuring the three-dimensional trajectory:

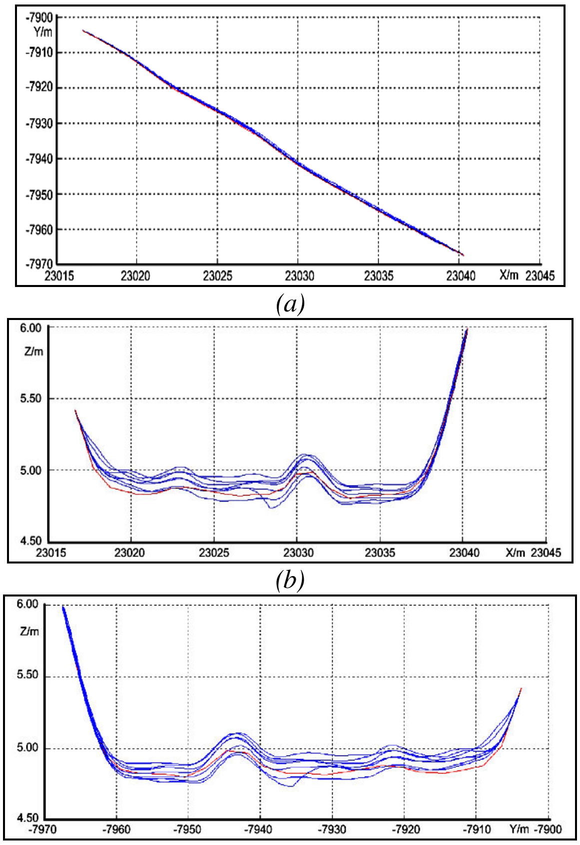
While testing the instrument in the downhole environment, high requirements are set for the anti-explosion performance of the instrument. Without benchmarking, the experiment is conducted on the spacious ground. In the experimental plan, the diameter of the MPP simulation pipeline is 150mm and its main mechanical property parameters are shown in Table 3. In the pipeline, steel frames are fixed on both sides and the central part bend on the ground in a compacted state.

Table 3. *Physical and mechanical properties of the pipeline*

Type	Content
Density	0.95g/cm3
Friction coefficient	<0.35
Roundness tolerance	3.6…5.0mm
Minimum radius of curvature	<75D m
Material	Modified polypropylene

NTS202 total station is used for dotting with the accuracy better than 0.1mm and the relative coordinates (23016.64, -7903.71, 5.421) are taken as the initial coordinates. When the prism is placed above the pipeline axis, the coordinate data with sampling intervals at 0.5m are obtained and the complete smooth curve is obtained through using Matlab fitting in the spacers. The stated instrument for measuring trajectory first stays at the pipe orifice for 10s and is then dragged to the other end at the uniform speed. After it is motionless for 10s, the instrument is pulled back in the opposite direction until it reaches the pipeline entrance. Subsequently, the instrument is kept motionless for 10s again, then the instrument is powered off and the data are uploaded. The sampling frequency of the instrument is still 200Hz and the AT45DB642 chip is read in the SPI mode to ensure the sampling rate. Then the data are uploaded to the computer through serial ports. The measurement data are first filtered through Kalman low pass filter, then the main program is compiled through Labview, and finally the algorithm and diagram are realized through Matlab.

The obtained trajectory coordinates are shown in Figure 4(a, b, c). Figure 4(a) is for X-Y plane, figure 4(b) is for Z-X plane, figure 4(c) is for Z-Y plane, the red curve is the coordinate of the total station, and the rest seven curves are data of repeated experiments. Through the data analysis, it is found: the repeatability rate is better than 25mm/100m, the benchmarking accuracy is better than 20mm/100m, and the overall accuracy is less than <0.20%.



(a)

(b)

(c)

Figure 4. *Comparison diagram of trajectory coordinates*

4.3. Error analysis:

Influenced by many downhole environment factors, large error fluctuations exist in the unpredicted range. The stability of the gyro data input directly determines the accuracy and repeatability of the systematic measurement. In terms of the hardware design and algorithm disposal, the following influence factors are summarized.

(1)The north-seeking accuracy and the attitude angle measurement accuracy are directly determined by non-linear errors such as the gyro zero-offset and scaling factors etc. The gyro resolution is the restrictive condition of the measurement accuracy and an insuperable hardware obstacle in the north-seeking. Meanwhile, the incurred cost in improving the accuracy limits the system optimization. As the scaling factors of the fiber-optic gyroscope are non-linear, the data such as the random noise require a lot of tests before the data scaling. Moreover, the scaled data in the experiment are still influenced by temperature, humidity, and vibration, so the repeatability needs to be improved.

(2)As the benchmark error of the mechanical positioning is introduced in the calculation error, the accuracy anomaly is caused. The gyroscope is used for north-seeking and attitude measurement, so the integral calculation is unavoidably involved. However, the mechanical error of any sampling period is enlarged when they are substituted into the integration formulas. The uncertain factors are random and arbitrary in nature, so it is not easy to detect and correct them. These errors can only be avoided by improving the accuracy of the mechanical design and the operation. Given the limitations of the volume in the turntable in the design, the multi-zone sensor correction is not set up. Moreover, the mechanical errors are not predictable and correctable, which limits the accuracy of the follow-up algorithms in terms of the operation.

(3)By using the dual-position north-seeking algorithm, the trajectory measurement of the quaternion algorithm and the direction cosine algorithm requires the constant measurement, which is very demanding for the system design. In addition, the high-precision sensors place high requirements on the environment, especially the collection circuit because the fluctuations of the power and AD collection are reflected in the data and cause the second collection error. The gyro wheels are used for length collection, but the frequent length loss directly causes the insufficient extension of the trajectory shape because of the downhole coal dust etc. Thus, the spatial position error is caused.

(4)As a measurement instrument and the sensor of earth gravity field, the accelerometer can detect various seismic signals and angular signals. According to specific needs, the suitable accelerated speed ranges are selected. The measurement interval of the single accelerometer is within ±90°and cannot realize the 360° measurement. Thus, several accelerometers must be used for continuous measurement to obtain the accurate data of the instrument at any angle. Meanwhile, given the data output interval of the accelerated speed, the suitable value intervals play great influences in the accuracy. The lab test results of the accelerometer are shown in Figure 5:

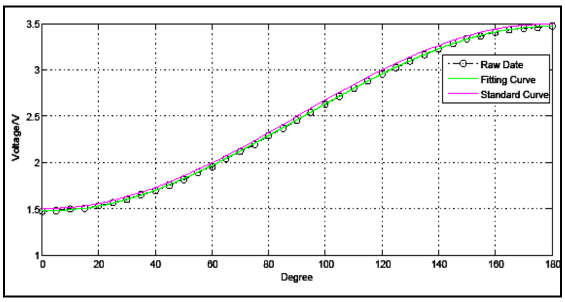
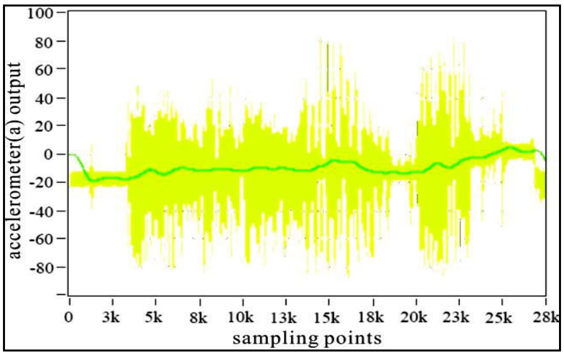


Figure 5. *Test results of accelerometer*

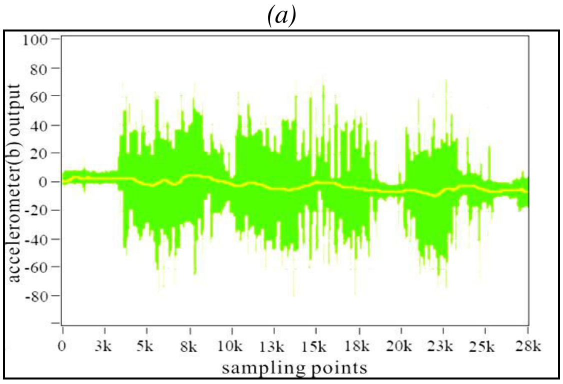
It can be seen from the measurement results that the output and measured curve chart are good in stability after the fitting. The area with a strong linear correlation is about ±40°, while the angular change rate is relatively small in the rest part, making the data collection difficult and having high requirements for the circuit sensitivity. In the instrument design, the effective values of this interval should be uttermost collected and the dip angle data collection is provided.

4.4. Filtering method:

While the instrument is running, large fluctuations or vibrations occur especially in the interfaces. Thus, the corresponding data require filtering and even removal. The integral method is used in the instrument calculation, so most of the data cannot be removed, otherwise the angular integration is not enough and the coordinate error will become more after enlargement. According to the construction requirements, the output interval of the sensor is derived reversely by using the maximum curvature radius and the kinematic speed of the instrument. In this way, the effective values of AD chip can be used to the utmost. Upon initially drilling the ground, the penetration angle and the unearthed Angle generally vary from 6° to 20°; in the pit, the penetration angle and the unearthed Angle are generally 0°. In the first stage and the final stage, the hole is drilled in skew lines, namely without vertical curvature and horizontal curvature. The drilling length in skew lines should not be less than 10m because it can ensure that the filtering bandwidth is 3~10 kHz. The low pass filter and filtering effects are shown in Figure 6.



(a)



(b)

Figure 6. *Comparison diagram before and after the filtering*

5. Conclusion

The fiber-optic gyroscope is used for the dual system measurement integrating the north-seeking and the trajectory, which realizes the downhole multi-factor measurement simultaneously and integrates the orientation and positioning. This method is workable. In the trajectory measurement of the fiber-optic gyroscope, high requirements are placed on the measurement accuracy of the downhole horizontal hole. This gyro method overcomes the barrier of poor electro-magnetic signals in the downhole environment and has a relatively independent measurement system. In addition, it has few interference factors and is convenient for measurement. Thus, the gyro method can replace the traditional electro-compass measurement method and be widely applied in the downhole measurement.

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