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Study on a Corrected Method to Calculate Rigidity-Gravity Ratio for Super High-Rise Building

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Abstract: With the huge depth-width ratio and uneven distribution of lateral stiffness and mass, the super high-rise building could not meet the premise of rigidity-gravity ratio formula provided by Chinese code "Technical specification for concrete structures of tall building (JGJ3-2010)", so it's overall stability has been concerned widely. According to the principle of overall stability which is evaluated by rigidity-gravity ratio, the corrected method to rigidity-gravity ratio was deduced in this paper. The corrected method was used for an engineering example, its result was compared with P-delta effect calculation and buckling analysis, the consistent conclusions were obtained. The researches show that the calculation of rigidity-gravity ratio should consider the influence of actual gravity load and horizontal load distribution forms, the overall stability of super high-rise building should be comprehensively evaluated through the corrected rigidity-gravity ratio checking, P-delta effect calculation and Integral buckling analysis.

Keywords: *super high-rise building, overall stability, rigidity-gravity ratio, P-delta effect, buckling analysis*

1. Introduction

With the increasing of height and depth-width ratio of the high-rise building, the gravity load has become larger and larger, and the lateral stiffness has become smaller and smaller. Under the wind load and horizontal seismic action, the P-delta effect caused by gravity of structure increase significantly, which may lead to instability and collapse of structure [1]. Therefore, the most important point for stability design of the high-rise building is the control of the P-delta effect level. Many researchers had done a lot of studies on overall stability of high-rise building [2-4]. However, understanding the overall stability of a structure is not an easy task. Presently, Chinese code "Technical specification for concrete structures of tall building JGJ3-2010" [5] (hereinafter referred to as "Tall building specification") mainly adopt rigidity-gravity ratio (the ratio of the lateral stiffness and gravity load, $EJ_d / (H^2\sum_{i=1}^{n} G_i)$) to measure the stability and control P-delta effect of structure. Past engineering experience has shown that the P-delta effect could be controlled within a certain range and the overall stability of structure could be with appropriate safety reserves for average high-rise buildings whose rigidity-gravity ratio can meet limit value provided in "Tall building specification". However, for the super high-rise buildings with high depth-width ratio, and irregularity of stiffness and mass distribution, the calculation results of rigidity-gravity ratio according to the method in Chinese code are often very near to or even lower than the lowest limit value [6-7]. If only rely on the rigidity-gravity ratio checking method as Chinese code to measure the overall stability of super high-rise buildings could not acquire accurate and reasonable results.

In this paper, the correction calculation method of rigidity-gravity ratio for super high-rise buildings was deduced according to the relationship between rigidity-gravity ratio and P-delta effect. Then, the method has been applied to a practical engineering, and the calculation results are consistent with the P-delta effect computation and buckling analysis, relevant methods can provide reference for overall stability evaluation of super high-rise building.

2. The correction calculation method of rigidity-gravity ratio

2.1 Relationship between rigidity-gravity ratio and P-delta effect

High-rise building structure can be considered as a cantilever bar model with the medium slenderness ratio and divided into shear, bending and bending-shear type according to its overall instability form[6]. For the super high-rise building, the common instability form is bending-shear type. According to the provisions of "Tall building specification", for bending-shear type structure, when the rigidity-gravity ratio meets formula (1), its P-delta effect could be ignored in elastic computation; when meets formula (2), the additional internal force and displacement caused by P-delta effect should be concerned.

$$EJ_d \geq 2.7H^2\sum_{i=1}^{n} G_i \qquad (1)$$

$$EJ_d \geq 1.4H^2\sum_{i=1}^{n}G_i \quad (2)$$

$$EJ_d = \frac{11qH^4}{120u} \quad (3)$$

Where, EJ_d is the equivalent lateral stiffness of structure, it can be calculated by formula (3) based on the assumption that a structure can be regarded as a cantilever bending member under inverted triangular load action; H is the total height of structure; G_i is the design value of floor gravity load; q is The maximum level of inverted triangular load; u is the top horizontal displacement of structure.

The above formulas (1) and (2) are based on a curved cantilever bar model under a vertical concentrated load at the top. According to Euler formula, the equivalent concentrated buckling load P_{cr} could be derived from formula (4). In the practical structural computing, the P_{cr} should be replaced by the sum of floor critical gravity load $\left(\sum_{i=1}^{n}G_i\right)_{cr}$ as formula (5). On the basis of formula (4) and (5), we can deduce the approximate calculation formula of critical gravity load, as shown in formula (6).

$$P_{cr} = \frac{\pi^2 EJ_d}{4H^2} \quad (4)$$

$$P_{cr} = \frac{1}{3}\lambda_{cr}\left(\sum_{i=1}^{n}G_i\right) = \frac{1}{3}\left(\sum_{i=1}^{n}G_i\right)_{cr} \quad (5)$$

$$\left(\sum_{i=1}^{n}G_i\right)_{cr} = \frac{3\pi^2 EJ_d}{4H^2} \quad (6)$$

According to the approximate lateral displacement calculation formula of bending-shear type structure, the relationship between rigidity-gravity ratio and P-delta effect could be set up as formula (7), we can find that P-delta effect has increased gradually with the decrease of $EJ_d/(H^2\sum_{i=1}^{n}G_i)$ characterized by hyperbolic curve[8]. When structure satisfies the requirement of formula (1) or formula (2) , the P-delta effect at the elastic stage could be controlled within 5% or 10% respectively ($\Delta^*/\Delta \leq 1.05$ or $\Delta^*/\Delta \leq 1.1$), even at the elastic-plastic stage, considering the structural stiffness reduction of 50%, P-delta effect also can be controlled within 10% or 20% [5].

$$\Delta^* = \frac{1}{1-\sum_{i=1}^{n}G_i \Big/ \left(\sum_{i=1}^{n}G_i\right)_{cr}}\Delta = \frac{1}{1-\frac{0.135}{EJ_d/(H^2\sum_{i=1}^{n}G_i)}}\Delta \quad (7)$$

In the above formula, λ_{cr} is the critical load parameter, Δ^* is the lateral displacement considering P-delta effect, Δ is the lateral displacement with no P-delta effect.

From the above analysis, the checking formulas derivation process of rigidity-gravity ratio in "Tall building specification" is based on the following two assumptions: (1) the gravity loads of structure are evenly distributed along the height; (2) the vertical arrangement of structure remains the uniform distribution [9]. But for high-rise building, with the increasing of floor, the plane size of structure and section size of vertical members are often decreasing gradually, and the form of anti-lateral force members are transited from steel-concrete composite members to reinforced concrete members. As the gravity load and vertical arrangement of structure are not evenly distributed, the assumptions of structure simplified to a uniform cantilever bar model with a vertical concentrated load on the top will cause a certain error. On the other hand, the actual wind load and horizontal seismic action forms are different from the inverted triangular distribution, the computing formula of $EJ_d = \frac{11qH^4}{120u}$ may cause a certain error as well. In conclusion, as for the rigidity-gravity ratio checking of super high-rise building, traditional cantilever bar model should to be corrected, and the EJ_d should be computed with the actual horizontal load distribution form.

2.2 Correction of calculation model and equivalent critical gravity load

Because the structural arrangement and horizontal load distribution of super high-rise building have significant differences to simplified model provided in "Tall building specification", the calculation model should be revised to a correction cantilever bar mode under "n" vertical concentrated load which is closer to the actual situation, as shown in figure 1. The critical load parameter can be calculated by formula (8) provided in reference [10].

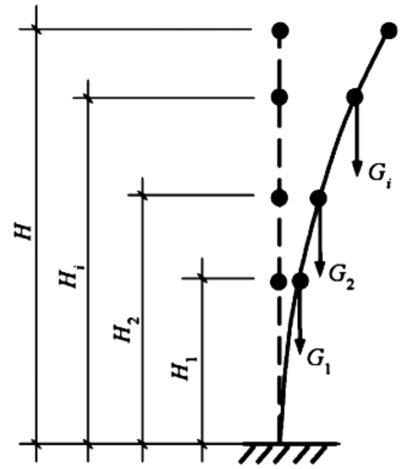


***Figure1.** Corrected cantilever bar model*

$$\lambda_{cr} = \frac{EJ_d\pi^2}{4H^2\sum_{i=1}^{n}G_i\left(\frac{H_i}{H}\right)^2} \quad (8)$$

Where, H_i is the vertical distance from mass G_i to the ground. Put formula (4) into formula (8), the correction formula of the equivalent concentrated buckling load P_{cr} can be deduced as follow formula.

$$P_{cr} = \lambda_{cr} \sum_{i=1}^{n} G_i \left(\frac{H_i}{H} \right)^2 \tag{9}$$

Assign $\alpha = \sum_{i=1}^{n} G_i \left(\frac{H_i}{H} \right)^2 \Big/ \sum_{i=1}^{n} G_i$ and plug it into formula (9), the following formula could be acquired:

$$P_{cr} = \lambda_{cr} \sum_{i=1}^{n} G_i \left(\frac{H_i}{H} \right)^2 = \alpha \lambda_{cr} \sum_{i=1}^{n} G_i = \alpha \left(\sum_{i=1}^{n} G_i \right)_{cr} \tag{10}$$

Plug formula (10) into formula (4), the expression of equivalent critical gravity load of super high-rise buildings is obtained, as shown in formula (11).

$$\left(\sum_{i=1}^{n} G_i \right)_{cr} = \frac{\pi^2 EJ_d}{4 \alpha H^2} \tag{11}$$

Compare formula (11) with formula (4), we can draw the conclusions that the critical gravity load of super high-rise buildings is $1/3\alpha$ times as magnitude as regular structure. On the same time, when meet the same control level of P-delta effect, the value of rigidity-gravity ratio calculated by the methods of "Tall building specification" should be multiplied by $1/3\alpha$.

2.3 Correction of the equivalent lateral stiffness

As the previous analysis, the actual wind load and horizontal seismic action forms of super high-rise buildings are not inverted triangular distribution, the computing formula of the equivalent lateral stiffness EJ_d provided in "Tall building specification" should be corrected as following formula[6], adopt the actual horizontal load distribution forms.

$$EJ_d = \sum_{i=1}^{n} \frac{P_i h_i^2 H}{6} (3 - \beta_i) / u \tag{12}$$

In the above formula, P_i is the horizontal concentrated load, h_i is the height of floor, $\beta_i = H_i / H$, u is the top horizontal displacement of structure under inverted triangular load action.

3. Verification of correction calculation method of rigidity-gravity ratio

3.1Calculation of P-delta effect

In order to prove the accuracy of the above correction method, the P-delta effect of structure could be calculated under the action of wind load and frequent earthquake. The method is: check whether the increments of the lateral displacements and overturning moment are consistent with the values provided in "Tall building specification" regardless of the stiffness reduction.

3.2 Integral buckling analysis

To further verify the stability of the super high-rise building, the integral linear buckling analysis and nonlinear buckling analysis should be carried out, the controlling index are [8]: (1) The linear buckling critical load coefficient of the whole structure should be greater than 10. (2) The nonlinear buckling critical load coefficient of the whole structure should be greater than 5 on the premise that initial imperfection and geometric nonlinearity have been taken into account. (3) The buckling of main anti-literal force members should lag behind the overall buckling of structure.

4. Engineering example analysis

In order to validate the correction method of rigidity-gravity ratio, the correction rigidity-gravity ratio, P-delta effect, buckling critical load coefficient of a super high-rise building engineering example have been computed by three software of SAP2000, SATWE and ETABS.

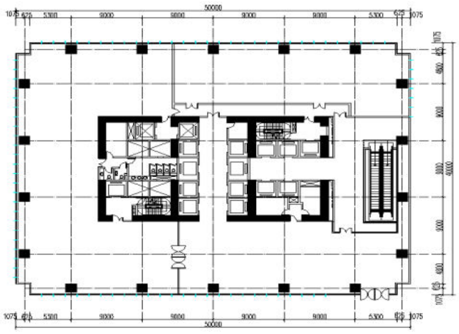
4.1 General situation of engineering

The super high-rise building located in Chongqing is composed of super-tall tower, podium and basement, 63 floors above the ground, 4 floors underground, tower height of 287.9m, skirt building height of 33.3m, depth-width ratio of 6.4, plane size of standard floor of 50m×40m, layer height of standard floor of 4.5 m, plan and section drawings are shown in figure 2. The frame-core tube system is applied in the tower, and reinforced concrete frame structure system is applied in skirt building. To improve the lateral resistance, 3 strengthening layers are arranged in 20th, 33th and 48th floor. With the increase of floor number, the section size of frame columns has gradually reduced from 1400mm×1400mm to 1000mm×1000mm, the strength grade of concrete has gradually reduced from C60 to C40. The strength grade of steel is Q345B.

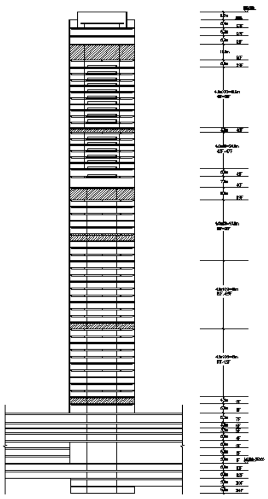
The seismic fortification intensity of Chongqing is 6 degree (0.05g), seismic fortification category is key fortification (b-class). The site classification is I_1 class. The basic wind pressure is $0.4 kN/m^2$. In order to guarantee the reliability of the computing results, three above software has carried out the modal analysis respectively. The first three order periods are shown in table 1. From the calculation results of table 1, the periods are very close shows that the relevant results of above software could be used for analysis and comparison.

Table 1: *Comparison of periods*

software	T_1(s)	T_2	T_3
SATWE	6.677	5.896	3.885
ETABS	6.313	5.354	3.097
SAP2000	6.807	6.125	3.938



(a) Standard floor plan of tower



(b) Section of building

***Figure2**. Plan and section drawings of structure*

4.2 Calculation of correction rigidity-gravity ratio

The distribution curve of gravity load is presented in Figure 3, the rule of curve shows that the values decrease gradually along the height of building and have a certain mutation at electromechanical layers. The distribution curves of frequent earthquake action and actual wind load are given in Figure 4 and Figure 5, their distribution characteristics are different from inverted triangle, and mainly have a certain mutation at the strengthening layers. According to the above description, we can find that this project could not satisfy the prerequisites of "uniform distribution of gravity load and inverted triangular distribution of horizontal load along the height" provided in "Tall building specification". Therefore, the rigidity-gravity ratio and equivalent lateral stiffness EJ_d should be corrected in accordance with the above-mentioned methods. Because the project is in low earthquake intensity area, wind is the control load and the structure has a weaker stiffness in the Y direction, this paper only give the calculation results of the structure of Y direction under wind load as shown in table 2. The results indicate that the rigidity-gravity ratio could not meet the limit of 1.4 calculated by method of "Tall building specification". In terms of the correction method provided in this paper, the calculated value is greater than 1.4, the stability of structure meet the requirements, but the adverse impact of the P-delta effect should be considered.

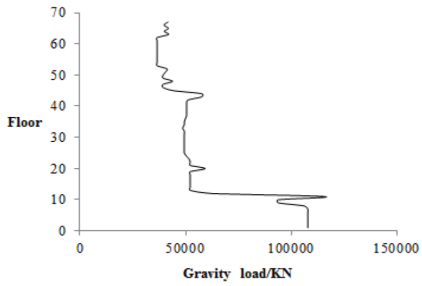


***Figure3**. Floor gravity load*

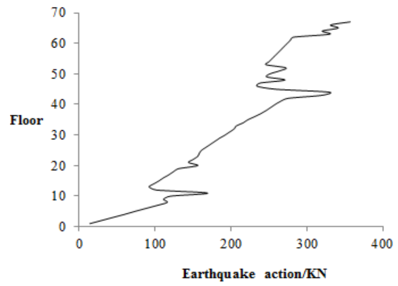


***Figure4**. Earthquake action*

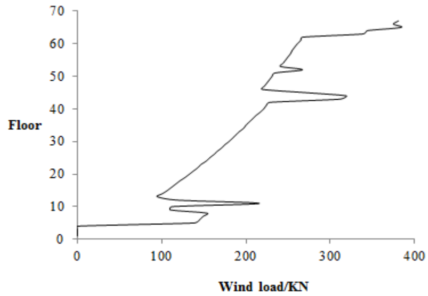


***Figure5**. Wind load*

***Table 2:** Calculation of rigidity-gravity ratio*

Software	$\sum_{i=1}^{n} G_i$ (KN)	$EJ_d=\sum_{i=1}^{n}\frac{P_i h_i^2 H}{6}(3-\beta_i)/u$ $(KN\cdot m^2)$	$EJ_d/(H^2\sum_{i=1}^{n} G_i)$	$\alpha=\sum_{i=1}^{n} G_i\left(\frac{H_i}{H}\right)^2 \Big/ \sum_{i=1}^{n} G_i$	Correction rigidity-gravity ratio
SATWE	3767644	4.22×10^{10}	1.35	0.28	1.61
ETABS	3785621	4.30×10^{10}	1.37	0.27	1.69

4.2 Calculation of P-delta effect

To verify the correction method's reliability of rigidity-gravity ratio, the increments of the top lateral displacements and overturning moment regardless of the stiffness reduction have been calculated by ETABS program software. The increments are between 5%-10% indicates that the corresponding values of rigidity-gravity ratio should be between 1.4-

2.7. The results have verified the correctness of the correction method, as shown in table 3.

***Table 3:** P-delta effect of structure*

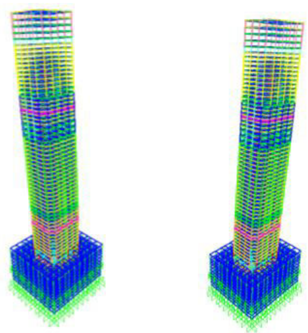
Load type	Top lateral displacement (mm)		Percentage of displacements increments	overturning moment (KN.m)		Percentage of overturning moment increments
	No P-delta effect	Considering P-delta effect		No P-delta effect	Considering P-delta effect	
wind	117	125	6.8%	1.92×10^6	2.04×10^6	6.3%
Frequent earthquake	83	85	2.4%	1.34×10^6	1.38×10^6	3.0%

4.3 Integral buckling analysis

The linear and geometric nonlinear buckling analyses of whole structure have been executed by SAP2000 program software. In the process of calculation, shell elements are used to simulate the shear walls and floor slabs; link elements are used to simulate the beams and columns; to accelerate the computation speed, the secondary members such as floor beams have been deleted.

4.3.1 Linear buckling analysis

In the process of linear buckling analysis, the load type of "1.0 dead load +1.0 live load" has been adopted. According to the computing results, the buckling of main anti-literal force members lag behind the overall buckling of structure. The first two order buckling modes are given in figure 6, respectively along the Y and X direction, the critical load coefficients are 15.11 and 18.61(greater than 10). The results demonstrate the overall buckling will not occur at elastic stage.



(a) The first mode *(b) The bulking mode*

***Figure 6**. Buckling modes of structure*

4.3.2 Nonlinear buckling analysis

In the process of nonlinear buckling analysis, the initial imperfection has been defined as the lateral displacements curve of the first bulking mode and it's top horizontal displacement is H/500(H is the height of the building). The calculation results show that the nonlinear buckling critical load coefficient of this project is 6.7. Because the coefficient is greater than 5, the overall buckling of structure will not occur at elastic-plastic stage as well.

4.4 Summaries

From the above analyses, we can find that the correction rigidity-gravity ratio of this project is between 1.4 and 2.7, the corresponding increments of the top lateral displacements and overturning moment are between 5% and 10%, and the critical load coefficients of linear buckling and nonlinear buckling are greater than 10 and 5 respectively. The results of several above methods are consistent and indicate that overall stability of the structure meets the safety requirements, but needs to consider the impact of P-delta effect.

5. Conclusions

(1) The rigidity-gravity ratio of super high-rise building should adopt correction method which has considered the influence of actual gravity load and horizontal load distribution form. The values of rigidity-gravity ratio by the method of "Tall building specification" should be multiplied by $1/3\alpha$ ($\alpha = \sum_{i=1}^{n} G_i \left(\frac{H_i}{H}\right)^2 \Big/ \sum_{i=1}^{n} G_i$) and the equivalent lateral stiffness should be calculated by formula as follows:

$$EJ_d = \sum_{i=1}^{n} \frac{P_i h_i^2 H}{6}(3-\beta_i)/u$$

(2) The overall stability of super high-rise building could be comprehensively evaluated through the correction rigidity-gravity ratio checking, P-delta effect calculation and integral buckling analysis as provided in the paper.

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