www.cafetinnova.org

Indexed in
Scopus Compendex and Geobase Elsevier,
Geo-Ref Information Services-USA, List B of Scientific Journals, Poland, Directory of Research Journals

ISSN 0974-5904, Volume 09, No. 02

International Journal of Earth Sciences and Engineering

April 2016, P.P.512-520

Fitting Statistical Distributions for Speeds and Headways for Peak and Non-Peak Mixed Traffic Flows in Mangalore City, Karnataka, India

VARGHESE GEORGE, PURUMALA SNEHITH, ISHAN S TAMHANE, SREEJA REDDY, RANJUSHA P G, MARSH M BANDI AND AKASH ANAND
Department of Civil Engineering, National Institute of Technology Karnataka, Mangalore-575025, India
Email: *varghese@nitk.ac.in, purumalasnehith@gmail.com*

Abstract: Statistical methods can be employed effectively in analyzing and interpreting observed phenomenon such as traffic arrivals, headways, and vehicular speeds. This paper describes the application of statistical distributions in fitting observed data on speed and headways for specific midblock sections in Mangalore city. As part of this exercise, the relevant data were collected, and attempts were made to fit the normal, log-normal, Erlang, negative exponential and the shifted exponential distributions to the observed traffic data. The best fitting statistical distributions were identified using the Chi-square tests and the Kolmonogrov-Smirnov (K-S) tests for goodness of fit. The findings of this study will further assist researchers in adopting the best approach in modeling urban road traffic especially in mixed traffic conditions comprising various combinations of fast and slow moving vehicles.

Keywords: *statistical distributions, speed, headway, Mangalore*

1. Introduction

Statistical distributions are useful in describing a wide variety of phenomena where there is high element of randomness. Discrete and continuous distributions are of immense use in explaining traffic behavior.

The primary scope of this study is focused on performing a study on the speed and headway characteristics for mid-block sections on the road sections identified above. As part of this exercise, the relevant data were collected, and attempts were made to fit the normal, log-normal, Erlang, negative exponential and the shifted exponential distributions to the observed traffic data. The best fitting statistical distributions were identified using the Chi-square tests and the Kolmonogrov-Smirnov (K-S) tests for goodness of fit. Selected road sections in the City of Mangalore were considered for this study.

Mangalore city, the gateway to the State of Karnataka, is located on 12.87° N latitude and 74.88° E longitude. The city spans over an area of 132.45 km^2 (51.14 sq mi), with the major activity centers including the New Mangalore Port, the central business district, the educational centers, and pilgrimage-centers located in various parts of the city. The District of Mangalore is provided with a road network of 1133km with four national highways passing through the sub-urban areas. National Highway NH-66, that connects Panvel (in the State of Maharashtra) to Edapally Junction (near Cochin in the State of Kerala), passes through Mangalore in a north–south direction, while NH-48 runs eastward connecting Mangalore to Bangalore. NH-169 runs in the north-east direction connecting Mangalore to Solapur via Shimoga, and NH-234, a 715-km long National Highway connects Mangalore to Villupuram, in the State of Tamilnadu.

The important road junctions in Mangalore city include Hampankatta, Jyothi, Navabharath, PVS, Bunts Hostel, Kankanady, Nanthoor and Bendoorwell. Among the roads in the city, the stretches connecting Bunts Hostel Circle to PVS Circle and Hampankatta to Navabharath Circle (KS Rao road) were considered in the present study.

This study on identifying the best fitting statistical distributions for speeds and headways on urban road sections was performed in order to simulate the flow of traffic at a later stage. The findings of this study will further assist researchers in adopting the best approach in modeling urban road traffic especially in mixed traffic conditions comprising various combinations of fast and slow moving vehicles.

2. A Brief Review of Literature

Kinzer [1] discussed the application of the Poisson distribution in the field of road traffic in 1933, Adam [2] illustrated the use of the above distribution with numerical examples. The Poisson distribution was later widely adopted in the study of traffic at road junctions by Greenshield et al. [3]. The expression for the Poisson distribution is given as follows:

$$P(x) = e^{-\lambda}\ \lambda^{x}/x! \qquad \text{(Eq. 1)}$$

Where, λ = average occurrence of an event; x = 0, 1, 2, n represents the number of successes or occurrence in 'n' trials; e = base of natural logarithm = 2.71828.

Gerlough and Barnes [4], and Gerlough [5] observe that the Binomial distribution could be used to describe the distribution of vehicle arrivals in congested traffic where the mean/variance was found to be greater than 1. This is due to the reason that the variance is lesser in congested flow condition. The expression for Binomial distribution is as follows:

$$P(x) = {}^{n}C_{x}\, p^{x}\, q^{n-x} \quad \text{(Eq. 2)}$$

Where, x = 0, 1, 2, n represents the number of successes; n = total number of trials; p = average probability of occurrence of an event; q = average probability of non-occurrence of an event.

In road section with traffic flows at junction controlled by signals, it is often found that the flow at mid-sections is affected by cyclic variations. In such cases, the negative binomial distribution is found to provide a better fit. In such conditions, the variance tends to be higher and hence the mean/variance ratio will be lesser than 1, according to Gerlough and Huber [6]. The expression for negative binomial distribution is given as:

$$P(x) = {}^{x+k-1}C_{k-1}\, p^{k}\, q^{x} \quad \text{(Eq. 3)}$$

Where, 0, 1, 2, n represents the number of successes; $p = m/s^2$; $k = m^2/(s^2-m)$; $q = (1-p)$; m = mean of observed data; s^2 = variance of the observed data.

3. Theoretical Foundations

The negative exponential, shifted negative exponential, Erlang, normal and log-normal distributions can be used to describe continuous variables such as headways. The expression for the negative exponential can be derived from the Poisson as:

$$P(h \geq t) = e^{-t/T} \quad \text{(Eq. 4)}$$

Where, t = interval of observation; T = mean of the interval probability distribution.

However in many cases of random flow conditions, the negative exponential fails to describe the observed data satisfactorily. This is due to the reason that vehicles have a definite physical size that limits their movement at higher traffic densities. Moreover drivers tend to maintain a safe distance in front of their vehicle. In such situations, the shifted exponential distribution is found to provide a better fit to the observed frequencies.

According to the above explanation, we need to shift the starting point of the curve towards the right, such that there is a minimum gap of approximately τ seconds. The shifted negative exponential distribution is generally expressed as:

$$P(h \geq t) = e^{-(t-\tau)/(T-\tau)} \quad \text{(Eq. 5)}$$

Where, τ = minimum allowable headway.

In the case of non-random flow conditions it is required to fit other distributions such as the Erlang distribution, normal distribution, log-normal distribution and other similar distribution functions. The expressions for the Erlang, normal, and log-normal distributions are given below.

Erlang Distribution:

$$P(h \geq t) = \sum_{i=0}^{k-1} (kt/T)^{i} \left(e^{-(kt/T)}/i!\right) \quad \text{(Eq. 6)}$$

Where, $k = T^2/s^2$; T = mean of the observed intervals; and s^2 is the variance of the observed intervals. When the Erlang distribution is used, the value of k is rounded off to the nearest integer.

Normal Distribution:

$$P(h \geq t) = 1/2\,(1 + \mathrm{erf}((x-\mu)/(\sigma\sqrt{2}))) \quad \text{(Eq. 7)}$$

Where, μ = mean of the distribution; σ = standard deviation of the distribution.

Log-Normal Distribution:

In the log-normal distribution, the logarithm of the stochastic variable, rather than the variable itself, is distributed according to the normal distribution. This distribution is used in headways, especially for traffic in platoons. One advantage of using this distribution is the ability to make a quick test of fitting by graphical means.

$$P(h \geq t) = 1/2 + (1/2 * \mathrm{erf}((\ln x-\mu)/(\sigma\sqrt{2}))) \quad \text{(Eq. 8)}$$

Where, μ = mean of the distribution; σ = standard deviation of the distribution.

The goodness of fit of the best fitting distribution for an observed phenomenon can be determined using the Chi-squared test and the Kolmogrov-Smirnov test. The values determined are compared to the standard values provided in statistical tables for the required level of significance.

The Chi-square value is computed based on the actual and simulated frequencies as given below:

$$\chi^2_{obs} = \sum (fi^2/Fi) - \sum fi \quad \text{(Eq. 9)}$$

Where, fi = observed frequency in each class interval; and Fi = theoretical expected frequency in each class interval.

The Kolmogorov-Smirnov test (K-S test) is a non-parametric test that can be used to test the goodness of fit. The test is performed using the values A and B, where A is the relative frequency given as, $A = (Fi/\sum fi)$, and B is the cumulative probability for intervals greater than t, where t = value of the lower class-limit for the continuous variable such as the headway. The sum of the absolute differences between A and B are then found for each class interval, and are compared to the tolerable values for *K-S* tests given in statistical tables for the given frequency of observation for the preferred level of significance.

4. Methodology

The methodology adopted in this study comprises the following steps:

- Performing a reconnaissance survey of the study area and identification of locations for traffic surveys.
- Obtaining road dimensions, lane-widths, and width of foot-paths.
- Conducting video-graphic studies for pre-peak and peak periods.
- Analysis of video-graphic data and tabulation of data for speed and headway distributions.
- Estimation of total flows for peak and non-peak hours based on conversion factors specified by IRC [7].
- Study of cumulative speed distribution curves and headways and fitting of most suitable statistical distributions including normal, Log-normal, exponential, shifted-exponential, and Erlang distributions.

4.1 Data Collection and Tabulation

The width of the carriageway, the lane widths, and the width of footpaths were observed, and video-graphic surveys were performed in the best possible locations identified in the reconnaissance survey.

The video graphic data was analyzed using electronic stop-watches, and the speeds and headways of vehicles were computed. The data extracted was tabulated adopting suitable interval ranges using the expression:

i = (range/ (1+3.222log$_{10}$ N)) (Eq. 10)

Where, range = difference between the highest and the lowest values observed; and N = number of observations.

The distance between relevant land marks were observed. The survey was performed for pre-peak and peak hours on 05-08-2015 (Wednesday) and 06-08-2015 (Thursday). The details of the observations made were tabulated. Table 1a, 1b, 1c, 1d provide details on speed distribution at the locations; Bunts to PVS Circle and KS Rao road. Table 2a, 2b, 2c, 2d provide details on headway distributions for the above locations.

Table 1a: *Speed distributions: Bunts to PVS Circle (Pre-peak period)*

Sl. No:	Speed Range (kmph)	Mid Value	Frequency
1	1 - 5.5	3.25	0
2	5.5 – 10	7.75	6
3	10 - 14.5	12.25	19
4	14.5 - 19	16.75	75
5	19 - 23.5	21.25	83
6	23.5 - 28	25.75	60
7	28 - 32.5	30.25	20
8	32.5 - 37	34.75	12
9	37 - 41.5	39.25	5
10	41.5 - 46	43.75	1
		Total	281

Table 1b: *Speed distributions: Bunts to PVS Circle (Peak period)*

Sl. No:	Speed Range (kmph)	Mid Value	Frequency
1	1-4	2.5	2
2	4-7	5.5	5
3	7-10	8.5	16
4	10-13	11.5	103
5	13-16	14.5	199
6	16-19	17.5	155
7	19-22	20.5	72
8	22-25	23.5	26
9	25-28	26.5	5
10	28-31	29.5	1
		Total	584

Table 1c: *Speed distributions: Hampankatta to Navabharath Circle or KS Rao Road (Pre-peak period)*

Sl. No:	Speed Range (kmph)	Mid Value	Frequency
1	5-9.5	7.25	26
2	9.5-14	11.75	61
3	14-18.5	16.25	79
4	18.5-23	20.75	69
5	23-27.5	25.25	24
6	27.5-32	29.75	10
7	32-36.5	34.25	4
8	36.5-41	38.75	2
9	41-45.5	43.25	1
		Total	276

Table 1d: *Speed distributions: Hampankatta to Navabharath Circle or KS Rao Road (Peak period)*

Sl. No:	Speed Range (kmph)	Mid Value	Frequency
1	4 - 7.5	5.75	3
2	7.5 - 11	9.25	18
3	11-14.5	12.75	60
4	14.5-18	16.25	87
5	18-21.5	19.75	55
6	21.5-25	23.25	25
7	25-28.5	26.75	4
8	28.5-32	30.25	1
9	32-35.5	33.75	1
10	35.5-39	37.25	1
		Total	255

Table 2a: *Headway distributions at Bunts to PVS Circle (Pre-peak period)*

Sl. No:	Headway (Sec)	Mid Value	Frequency
1	0-2	1	135
2	2-4	3	95
3	4-6	5	53
4	6-8	7	35
5	8-10	9	20
6	10-12	11	14
7	12-14	13	10
8	14-16	15	1
		Total	363

Table 2b: *Headway distributions at Bunts to PVS Circle (Peak period)*

Sl. No:	Headway (Sec)	Mid Value	Frequency
1	0-0.75	0.375	12
2	0.75-1.5	1.125	45
3	1.5-2.25	1.875	90
4	2.25-3	2.625	130
5	3-3.75	3.375	123
6	3.75-4.5	4.125	91
7	4.5-5.25	4.875	69
8	5.25-6	5.625	45
9	6-6.75	6.375	28
10	6.75-7.5	7.125	18
11	7.5-8.25	7.875	12
12	8.25-9	8.625	12
		Total	675

Table 2c: *Headway distributions at KS Rao road (Pre-peak period)*

Sl. No:	Headway (Sec)	Mid Value	Frequency
1	0-2	1	289
2	2-4	3	156
3	4-6	5	48
4	6-8	7	20
5	8-10	9	9
6	10-12	11	4
7	12-14	13	2
		Total	528

Table 2d*: Headway distributions at KS Rao road (Peak period)*

Sl. No:	Headway (Sec)	Mid Value	Frequency
1	0-2	1	81
2	2-4	3	186
3	4-6	5	111
4	6-8	7	57
5	8-10	9	34
6	10-12	11	8
7	12-14	13	3
		Total	480

4.2 Analysis of Data

The following sections provide details on the analysis of data for speeds and headways at pre-peak and peak periods for the road sections considered for the present study.

4.2.1 Bunts Hostel Circle to PVS Circle

A. Speed Distributions

Attempts were made to fit various types of continuous distributions such as the Erlang distribution, normal and log normal distributions.

It was found that for the pre-peak and peak periods, the Erlang distribution was found to fit the observed data. The fitting of the theoretical distribution to the observed data was verified and confirmed using a parameter-less approach such as Kolmogorov-Smirnov test (K-S test).

The m/s^2 value for the pre-peak period was found to be 0.54159 and the value for k was obtained as $k = T^2/s^2$ =12. Table A.1a provides details on the computations performed for comparing the Erlang distribution with respect to the observed data for the pre-peak period. Table A.1b provides similar details for the peak-period of flow where the values for m/s^2 and k were found out to be 1.1043 and 18 respectively.

In Table A.1a, for the pre-peak period, it was found that the m/s^2 value of 0.54159 was significantly lesser than 1.00, which indicated the presence of a cyclic variations in vehicular flow due to the intermittent flow of incoming vehicles from the previous junction namely, the Bunts Hostel Circle.

In Table A.1b, for peak flow conditions from Bunts Hostel Circle to PVS Circle, the m/s^2 value of 1.1043 indicates the beginning of congested flow of traffic. The k value was found to be 18, which is greater when compared to the k-value of 12 in the case of pre-peak flow conditions on the same road stretch.

B. Headway Distributions:

A number of attempts were made to fit various types of continuous distributions such as the Erlang distribution, negative exponential and the shifted-negative exponential distributions.

It was found that for the pre-peak period, the negative exponential distribution fitted the observed data. The fitting of the theoretical distribution to the observed data was verified and confirmed using the Chi-square test.

The m/s^2 value for the pre-peak period was found to be 0.3707. Table A.2a provides details on the computations performed for comparing the negative exponential distribution with respect to the observed data for the pre-peak period.

However, in the peak flow period, it was found that the Erlang distribution gave a better fit with m/s^2 and k values of 1.176 and 4 respectively. See Table A.2b. The goodness of fit was confirmed using a parameter-less approach such as the Kolmogorov-Smirnov test (K-S test).

4.2.2 Hampankatta Circle to Navabharath Circle

A. Speed Distributions

It was found that for the pre-peak and peak period, the Erlang distribution was found to fit the observed data in the case of the road section between Hampankatta Circle and Navabharath Circle.

The m/s^2 value for this approach was found to be 0.42459 and the k value was found to be 8. This was verified using a parameter-less approach such as the Kolmogorov-Smirnov test (K-S test). See Table A.3a.

For the peak flow conditions too, the Erlang distribution was found to give the best fit. The values for m/s^2 and k were found out to be 0.81 and 14 respectively. See Table A.3b.

In Table A.3a, for the pre-peak period, it was found that the m/s^2 value of 0.424 was significantly lesser than 1.00, which indicated the presence of cyclic variations in vehicular flow. This was due to the fact that traffic flow to this road link mainly comes from traffic flow arriving through a branch road, namely, the GHS Cross Road joining the main stream. The traffic on the main stream, without considering the inflow from the branched road was found to be approximately 50% of the total flow. The inflow on the main road and the contributions from the side road were found to be heavily influenced by the signals operated in the previous junctions.

During the pre-peak period, the incoming traffic from Hampankatta Junction enters the link only when a green signal is available at the junction. Due to this reason, during pre-peak period, the m/s^2 value is found to be 0.4 indicating high cyclicity.

Also, in the pre-peak period, the mean speed on this road stretch was found to be 17.36 km/hr and the total flow was found to be 419 PCU/hr.

In the case of analysis performed for the peak period for the above direction of flow, it was found that the values for m/s^2 and k were 0.8125, and 14 respectively, while the mean speed was found to be 16.62 km/hr for a total flow of 550 PCU/hr. See Table A.3b.

Comparing the pre-peak to peak flow conditions, it can be seen that, there was only a minor reduction in the mean speed which indicates that there is no significant variation between the peak and non-peak flows.

During the peak-period, the increase in m/s^2 value from 0.4 to 0.8 indicates that, cyclic variation due to the impact of signal installed in the previous junction has reduced considerably. The vehicles arriving from Hampankatta junction as well as from the branch road (GHS Road) have resulted in the increase in the m/s^2 value, resulting in an increase of about 10% of the traffic. The increase in the traffic flow has consequently reduced the variance of flow, resulting in a higher value of k. Also it is seen that the presence of a bus-bay and a traffic divider at Hampankatta Junction has compelled the drivers to take the side-road via GHS Road.

B. Headway Distribution:

It was found that for the pre-peak period, the negative exponential distribution was found to fit the observed data. The m/s^2 value for this approach was found to be 0.56. This was confirmed using chi-square test. See Table A.4a.

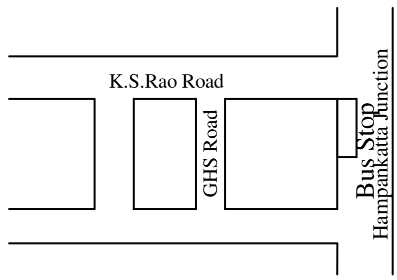

Figure 1: *Site Location at K S Rao Road*

In the case of the peak flow conditions, the Erlang distribution was found to give a better fit with m/s^2 and k values of 0.673 and 3 respectively. This was confirmed using a parameter-less approach such as Kolmogorov-Smirnov test (K-S test). See Table A4b. As discussed above, it is seen that the presence of a bus-bay and a traffic divider at Hampankatta Junction has compelled the drivers to take the side-road via GHS Road.

5. Conclusions

Comparisons based on speed distributions for the road section:

In the analysis for speed for the road link connecting Bunts Hostel Circle to PVS Circle in Mangalore, it was observed that the mean/variance value for the pre-peak period was 0.54, while for the peak period it was 1.10. This indicated the existence of cyclic variations in vehicular flow in the pre-peak period and congested flow in the peak period. On verification of the traffic flow on-site, it was observed that the cyclic variations were due to intermittent flow of incoming vehicles at the previous junction.

For the above road section, it was seen that the Erlang distribution could be used to provide a proper fit between the observed data and the theoretical data with K values of 12 for the pre-peak period and 18 for the peak period.

In the analysis for speed for the road link connecting Hampankatta Junction to Navabharath circle, the mean/variance value for the pre-peak period was 0.42, while for the peak period it was 0.81. In both the cases, the mean/variance value is significantly lesser than 1.0, indicating that cyclic variations were predominant for both the flow conditions. On verification of the traffic flow at the location, it was observed that the cyclic variations were due to the intermittent arrival of vehicles through a branch road namely the GHS cross road joining the main stream which contributed 50% of the total flow on the link. The arrival of vehicles on this branch road was influenced by the signals operated in the previous junctions. The remaining 50% of the flow was also influenced by the signal operated at Hampankatta Junction which resulted in a low mean/variance value. In this road section, the Erlang distribution was used to fit the pre-peak and peak period observations with

K values of 8 and 14 respectively. The road section was found to be congested due to difficulties faced by vehicles due to intending enter onto the road due to the presence of a bus lane very close to the junction. Thus it was found that the increase in flow during the peak period was marginal when compared to the pre-peak period.

Comparisons based on headway distributions for the road section:

In the study of headways for the road link between Bunts Hostel Circle and PVS Circle, the mean/variance value for the pre-peak period was 0.37, while for the peak period it was 1.17. In this case too, cyclic variations in vehicular flow in the pre-peak period and congested flow in the peak period were observed. The negative exponential was used to fit the pre-peak data while the Erlang distribution with a *K*-value of 4 was used to fit the peak flow conditions.
In the analysis of headways for the road link between Hampankatta Circle and Navabharath Circle, the mean/variance value for the pre-peak period was 0.56, while for the peak period it was 0.67 which indicates that cyclic variations were observed for both the flow conditions. Similar observations were made in the study of speed distributions described above. In this road section, the negative exponential was used to fit the pre-peak observations, while the Erlang distribution with a *K*-value of 3 was used to fit the peak flow conditions.

Overall observations:

In the study of speeds and headways, it can be seen that similar conclusions were arrived at for pre-peak and peak flows which confirmed the existence of cyclic variations in the two road sections. These observations necessitate a relook into the traffic management priorities for Hampankatta Junction which was earlier much more congested.

The above findings can be applied for the subsequent modeling using VISSIM. Also the results helps to generate the required number of vehicles for the simulation of observed flow in the city, thus, can give a better solution to the traffic flow problems.

References

[1] Kinzer, J. P., "Application of the Theory of Probability to Problems of Highway Traffic", B.C.E thesis, Polytechnic Institute of Brooklyn (June 1, 1933); also in Proc. Inst. Traffic Eng., 5:118-124 (1934).
[2] Adams, W. F., "Road traffic considered as a random series", J.Inst. Civil Eng., 4:121-130 (1936).
[3] Greenshields, B. D., Shapiro, D., and Ericksen, E. L., "Traffic Performance at Urban Street Intersections" Tech. Rep. No. 1, Yale Bureau of Highway Traffic, New Haven, Conn. (1947).
[4] Gerlough, D. L., and Barnes, F. C. The Poisson and other probability distributions in highway traffic. In "Poisson and Other Distributions in Traffic" Eno Foundation (1971).
[5] Gerlough, D. L., Use of Poisson distribution in highway traffic, pp. 1-58. In "Poisson and Traffic", Eno Foundation (1955).
[6] Gerlough, Daniel L., and Huber, Matthew J. (1975), "Traffic Flow Theory: A Special Report No.165", Transportation Research Board, National Research Council, Washington D.C.
[7] IRC, Guidelines for Capacity of Urban Roads in Plain Areas, IRC: 106-1990, Indian Roads Congress, New Delhi, 1990.

Table A.1a: *Fitting Erlang distribution for speeds: Bunts to PVS Circle (pre- peak)*

Speed Range (kmph)	Mid Value (xi)	Frequency (fi)	Observed cumulative frequency (fi)	Relative cumulative frequency (fi/∑fi) (a)	Erlang Probability (b)	K-S difference (a-b)
1 - 5.5	3.25	0	281	1	1	1.009E-12
5.5 - 10	7.75	6	281	1	0.99992	8.003E-05
10 - 14.5	12.25	19	275	0.9786477	0.9886915	0.0100438
14.5 - 19	16.75	75	256	0.911032	0.8875551	0.0234769
19 - 23.5	21.25	83	181	0.6441281	0.6396453	0.0044828
23.5 - 28	25.75	60	98	0.3487544	0.3554833	0.0067288
28 - 32.5	30.25	20	38	0.1352313	0.155975	0.0207437
32.5 - 37	34.75	12	18	0.0640569	0.0561959	0.0078611
37 - 41.5	39.25	5	6	0.0213523	0.0172368	0.0041155
41.5 - 46	43.75	1	1	0.0035587	0.0046343	0.0010756
Total		281				0.078608

Note:
Mean speed = 21.7304 kmph; m/s^2 = 0.54159; k = 12
K-S table value for 281 observations = 1.36 / (Number of observations)$^{0.5}$ = 0.0811
K-S observed = 0.0786
Since K-S observed is lesser than K-S table value, the null hypothesis that there is no significant difference between the observed data and the theoretical distribution was accepted for a significance level of 5%.

***Table A.1b:** Fitting Erlang distribution for speeds: Bunts to PVS Circle (peak)*

Speed Range (kmph)	Mid Value (xi)	Frequency (fi)	Observed cumulative frequency (fi)	Relative cumulative frequency (fi/∑fi) (a)	Erlang Probability (b)	K-S Difference (a-b)
1-4	2.5	2	584	1	1	4.44E-16
4-7	5.5	5	582	0.9965753	0.9999984	0.0034231
7-10	8.5	16	577	0.9880137	0.9984094	0.0103957
10-13	11.5	103	561	0.9606164	0.9564751	0.0041413
13-16	14.5	199	458	0.7842466	0.7611144	0.0231321
16-19	17.5	155	259	0.4434932	0.4425009	0.0009923
19-22	20.5	72	104	0.1780822	0.1845872	0.006505
22-25	23.5	26	32	0.0547945	0.0574204	0.0026259
25-28	26.5	5	6	0.010274	0.0139782	0.0037042
28-31	29.5	1	1	0.0017123	0.0027787	0.0010663
Total		584				0.055986

Note:
Mean speed = 15.7534 kmph; m/s^2 = 1.1043; k = 18
K-S table value for 584 observations = 1.36 / (Number of observations)$^{0.5}$ = 0.0563
K-S observed = 0.0560
Since K-S observed is lesser than K-S table value, the null hypothesis that there is no significant difference between the observed data and the theoretical distribution was accepted for a significance level of 5%.

***Table A.2a:** Fitting negative exponential for headways: Bunts to PVS Circle (pre- peak)*

Headway Range (sec)	Mid Value (xi)	Observed Frequency (fi)	Lower Class Limit (t_L)	Total Observed Frequency $\geq t_L$	Theoretical Exponential prob of $h \geq t$	Cumulative Theoretical frequency	Theoretical frequency in each class (Fi)	fi^2/Fi
0-2	1	135	0	363	1	363	146.18	124.67
2-4	3	95	2	228	0.5973	216.82	87.3	103.37
4-6	5	53	4	133	0.3568	129.52	52.16	53.853
6-8	7	35	6	80	0.2131	77.36	31.15	39.325
8-10	9	20	8	45	0.1273	46.21	18.59	21.516
10-12	11	14	10	25	0.0761	27.62	11.14	17.594
12-14	13	11	12	11	0.0454	16.48	9.62	12.57
14-16	15		14	1	0.0271	9.84		
Total		363				887		372.92

Note:
Mean time headway = 3.881543 s; m/s^2 = 0.37075.
Chi-square table value for 5 degrees of freedom = 11.1
Chi-square observed = 9.9277
Since the Chi-square value for observed data is lesser than the table value, the null hypothesis that there is no significant difference between the observed data and the theoretical distribution was accepted for a significance level of 5%.

***Table A.2b:** Fitting Erlang distribution for headways: Bunts to PVS Circle (peak period)*

Headway Range (sec)	Mid Value (xi)	Frequency (fi)	Observed cumulative frequency (fi)	Relative cumulative frequency (fi/∑fi) (a)	Erlang probability (b)	K-S difference (a-b)
0-0.75	0.375	12	675	1	1	0
0.75-1.5	1.125	45	663	0.9822	0.9898	0.0076
1.5-2.25	1.875	90	618	0.9155	0.9136	0.0018
2.25-3	2.625	130	528	0.7822	0.7619	0.0203
3-3.75	3.375	123	398	0.5896	0.5789	0.0106
3.75-4.5	4.125	91	275	0.4074	0.4079	0.0005
4.5-5.25	4.875	69	184	0.2725	0.2707	0.0018
5.25-6	5.625	45	115	0.1703	0.1713	0.0009

6-6.75	6.375	28	70	0.1037	0.1043	0.0006
6.75-7.5	7.125	18	42	0.0622	0.0615	0.0006
7.5-8.25	7.875	12	24	0.0355	0.0353	0.0001
8.25-9	8.625	12	12	0.0177	0.0198	0.0021
Total		675				0.0474

Note:

Mean time headway = 3.6294 s; m/s^2 = 1.1756; k = 4

K-S table value for 675 observations = 1.36 / (Number of observations)$^{0.5}$ = 0.0523

K-S observed = 0.0474

Since K-S observed is lesser than K-S table value, the null hypothesis that there is no significant difference between the observed data and the theoretical distribution was accepted for a significance level of 5%.

Table A.3a: *Fitting Erlang distribution for speeds: KS Rao Road (pre-peak)*

Speed Range (kmph)	Mid Value (xi)	Frequency (fi)	Observed cumulative frequency (fi)	Relative cumulative frequency (fi/∑fi) (a)	Erlang Probability (b)	K-S Difference (a-b)
5-9.5	7.25	26	276	1.0000	0.9974	0.0026
9.5-14	11.75	61	250	0.9058	0.9225	0.0167
14-18.5	16.25	79	189	0.6848	0.6780	0.0068
18.5-23	20.75	69	110	0.3986	0.3802	0.0184
23-27.5	25.25	24	41	0.1486	0.1695	0.0209
27.5-32	29.75	10	17	0.0616	0.0632	0.0016
32-36.5	34.25	4	7	0.0254	0.0205	0.0049
36.5-41	38.75	2	3	0.0109	0.0059	0.0049
41-45.5	43.25	1	1	0.0036	0.0016	0.0020
Total		276				0.0788

Note:

Mean speed = 17.3261 kmph; m/s^2 = 0.42459; k = 8

K-S table value for 276 observations = 1.36 / (Number of observations)$^{0.5}$ = 0.0818

K-S observed = 0.0788

Since K-S observed is lesser than K-S table value, the null hypothesis that there is no significant difference between the observed data and the theoretical distribution was accepted for a significance level of 5%.

Table A.3b: *Fitting Erlang distribution for speeds: KS Rao Road (peak)*

Speed Range (kmph)	Mid Value (xi)	Frequency (fi)	Observed cumulative frequency(fi)	Relative cumulative frequency (fi/∑fi) (a)	Erlang probability (b)	K-S difference (a-b)
4 - 7.5	5.75	3	255	1	0.9999	0.0001
7.5 - 11	9.25	18	252	0.9882	0.9943	0.0061
11 - 14.5	12.75	60	234	0.9176	0.9119	0.0056
14.5 - 18	16.25	87	174	0.6823	0.6587	0.0235
18 - 21.5	19.75	55	87	0.3411	0.3479	0.0067
21.5 - 25	23.25	25	32	0.1254	0.1370	0.0115
25 - 28.5	26.75	4	7	0.0274	0.0422	0.0148
28.5 - 32	30.25	1	3	0.0117	0.0106	0.0010
32 - 35.5	33.75	1	2	0.0078	0.0022	0.0055
35.5 - 39	37.25	1	1	0.0039	0	0.0039
Total		255				0.0791

Note:

Mean speed = 16.6206 kmph; m/s^2 = 0.8125; k = 14

K-S table value for 255 observations = 1.36 / (Number of observations)$^{0.5}$ = 0.0851

K-S observed = 0.0791

Since K-S observed is lesser than K-S table value, the null hypothesis that there is no significant difference between the observed data and the theoretical distribution was accepted for a significance level of 5%.

Table A.4a: *Fitting negative exponential for headways: KS Rao Road (pre-peak)*

Headway Range (Sec)	Mid Value (xi)	Observed Frequency (fi)	Lower Class Limit (t_L)	Total Observed Frequency $\geq t_L$	Theoretical Exponential prob of $h \geq t$	Cumulative Theoretical frequency	Theoretical frequency in each class (Fi)	fi^2/Fi
0-2	1	289	0	528	1	528	295.42	282.7195
2-4	3	156	2	239	0.4405	232.58	130.15	186.9842
4-6	5	48	4	83	0.194	102.43	57.29	40.2164
6-8	7	20	6	35	0.0855	45.14	25.29	15.8165
8-10	9	9	8	15	0.0376	19.85	11.09	7.3038
10-12	11	6	10	6	0.0166	8.76	8.76	4.1095
12-14	13		12	2	0.0073	3.85		
Total		528				941	528	537.1502

Note:
Mean time headway = 2.4393 s; $m/s^2 = 0.5633$.
Chi-square table value for 5 degrees of freedom = 11.1
Chi-square observed = 9.1502
Since the Chi-square value for observed data is lesser than the table value, the null hypothesis that there is no significant difference between the observed data and the theoretical distribution was accepted for a significance level of 5%.

Table A.4b: *Fitting Erlang distribution for headways: KS Rao Road (peak)*

Head-way Range (sec)	Mid Value (xi)	Frequency (fi)	Observed cumulative frequency (fi)	Relative cumulative frequency (fi/∑fi) (a)	Erlang probability (b)	K-S difference (a-b)
0-2	1	81	480	1	1	0
2-4	3	186	399	0.8312	0.8282	0.0029
4-6	5	111	213	0.4437	0.4592	0.0155
6-8	7	57	102	0.2125	0.2018	0.0106
8-10	9	34	45	0.0937	0.0775	0.0162
10-12	11	8	11	0.0229	0.0273	0.0044
12-14	13	3	3	0.0062	0.0090	0.0028
Total		480				0.0525

Note:
Mean time headway = 4.2208 s; $m/s^2 = 0.6738$; $k = 3$
K-S table value for 480 observations = 1.36 / (Number of observations)$^{0.5}$ = 0.06207
K-S observed = 0.0526
Since K-S observed is lesser than K-S table value, the null hypothesis that there is no significant difference between the observed data and the theoretical distribution was accepted for a significance level of 5%.