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# Analysis on the Adjacent Anchor Effect of Prestressed Tunnel Liner with Circular Anchored Tendons

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**Abstract:** In this paper, based on theory of elastic mechanics, the mechanical property of prestressed concrete lining structure was studied; theoretical formulas of normal displacement and inner force on the middle surface were deduced and the area of adjacent anchor effect was determined. Combing observed data in practical project, the theoretical result in the area of adjacent anchor effect and measured result of circumferential stress in prestressed concrete lining structure was compared. Results reveal that the theoretical results given by the formulas of adjacent anchor effect are in very agreement with the field test, the theoretical results being reasonable. The calculating method of the adjacent anchor effect in the prestressed tunnel liner with circular anchored tendons presented in the paper can be used in practical project.

***Keywords:*** *prestressed concrete liner, elastic theory, adjacent anchor effect, measured data*

## 1. Introduction

In recent years, prestressed concrete lining structure has been applied into many hydraulic tunnels where the internal water pressure is higher while the geological conditions of wall rock are poor, such as Xiaolangdi sediment tunnel lining, Tunnel passing through Yellow River and Dahuofang Reservoir in Liaoning[1]~[3]. During the anchor tension of prestressed tunnel liner with circular anchored tendons, the adjacent anchor effect is an inevitable phenomenon. At first, a new concept of the area of adjacent anchor effect is proposed. The area refers to the influence of tension stress (such as circumferential stress, axial stress, etc.) on both sides of the cable when the tension cable is stretched. Due to the tension of single anchor cable and tendon spacing are two important parameters of the design of prestressed concrete structures with circular anchored tendons, and the stress in any point of prestressed concrete lining is the results within a certain range of several anchor interaction. Therefore, it is necessary that understanding the adjacent anchor effect and the distribution rule of internal stress for determining the reasonable tension of single anchor cable and tendon spacing. For this reason, based on theory of elastic mechanics, the mechanical property of prestressed concrete lining structure and the calculating method of adjacent anchor effect were studied in this paper.

## 2. Internal stress distribution in the area of adjacent anchor effect

### 2.1. Basic Assumptions

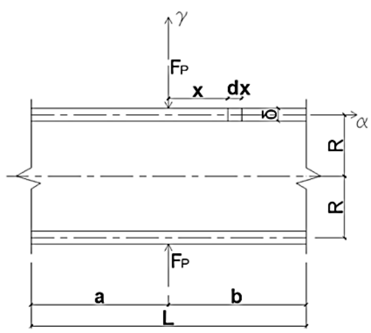
According to Theory of Elastic Mechanics, the basic assumptions are as follows:

(1) The liner concrete was seen as an isotropous elastomer during the study on mechanical property;
(2) Liner concrete conforms to plane cross-section assumption before and after the deformation; and surface curvatures of inside and outside surfaces of liner concrete on the same cross section are the same;
(3) After the stress, displacement and deformation of liner concrete are too small.

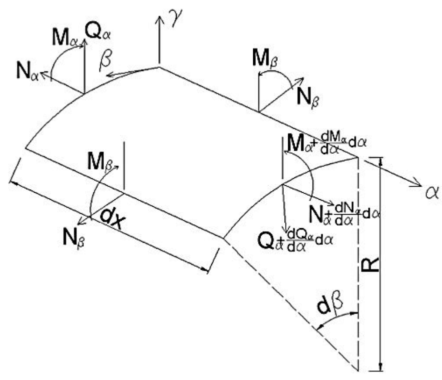
### 2.2. Theoretical Model of Elastic Mechanics

The prestressed concrete lining structure under the force of prestressed anchor cable is a cylindrical shell bearing the force distributed in axial symmetry. With single anchor cable, the deformation and internal force produced on the cylinder wall have a range of influence along the axis of cylinder which is called the area of adjacent anchor effect. A model of elastic mechanics was established to study the area of adjacent anchor effect when the tensile force of single anchor cable was certain.

The length of the prestressed tunnel liner with circular anchored tendons is *L*, the diameter of middle surface is R and the thickness of wall is $\delta$, all of which are shown as follows:




(a)




(b)

***Figure 1:*** *Geometrical Parameters and Stress State of Prestressed Concrete Lining Structure*

Circumferential force will be applied to the concrete liner once the prestressed anchor cable is pulled. To simplify the calculation, this circumferential force is seen as the equivalent to a evenly-distributed radial pressure $F_P$ along the per unit length and the direction pointing to the center of cylinder cross section is positive (Fig.1a). Width of location $x$ far away from the cross section while prestressed anchor cable is placed is $dx$, the micro-fission was the object of study (Fig. 1b). While the prestressed structure is under the force of $F_P$, an axial force $N_\alpha$ will be produced on the cross section where the micro-fission is perpendicular to $\alpha$-axis and circumferential force $N_\beta$ and circumferential moment $M_\beta$ will emerge on the cross section where the fission is perpendicular to $\beta$-axis. $N_\alpha$ and $N_\beta$ subjected to tension are positive; $M_\alpha$ and $M_\beta$ tensing outside of the cylinder are positive, pointing to the outside is positive.

Since the model of elastic mechanics established is about axisymmetric problem of cylindrical shell, according to cylindrical shell theory of elastic mechanics, the equation solving displacement and internal force is the following differential equation:

$$\frac{d^4w}{d\alpha^4}+\frac{E\delta}{R^2D}w=\frac{q_3}{D} \quad (1)$$

$w$ is the normal displacement on the middle surface; E is the elasticity modulus of cylinder wall material; $q_3$ is the normal load on the cylindric structure whose flexural rigidity is, $D=\frac{E\delta^3}{12(1-\mu^2)}$, where $\mu$ is Poisson's ratio of the material.

### 2.3. Calculating of internal stress in the area of adjacent anchor effect

According to cylindrical shell theory of elastic mechanics, the internal force of micro-fission (Fig.1b) meets the following conditions:

$$M_\alpha=-\lambda^2D\frac{d^2w}{d\xi^2},\quad Q_\alpha=-\lambda^3D\frac{d^3w}{d\xi^3},\quad N_\beta=\frac{E\delta}{R}w,\quad M_\beta=\mu M_\alpha \quad (2)$$

Here $\xi=\lambda\alpha$ and coefficient of elastic characteristics $\lambda=\sqrt[4]{\frac{E\delta}{4R^2D}}=\sqrt[4]{\frac{3(1-\mu^2)}{R^2\delta^2}}$; which clearly shows that both the axial and circumferential internal force are direct correlated with the normal displacement of middle surface. Therefore, this can be simplified just through solving the displacement $w$ in Equation (1).

As shown in Fig.1a, $F_P$ evenly distributes along the ring direction on the prestressed tunnel liner. Action point of $F_P$ was used as the origin of coordinates to set up a cylindrical coordinate system; $q_3=F_P$ was substituted into Equation (1) to solve this equation based on the symmetric structure, thus the displacement of any cross section $\alpha$ close to anchor cable was calculated.

$$w=-\frac{F_P}{8\lambda^3D}f_1(\xi) \quad (3)$$

Substitute Equation (3) into Equation (2) and intersection angle, $\theta\left(\theta=\frac{dw}{d\alpha}=\lambda\frac{dw}{d\xi}\right)$,

$$N_\beta=\frac{E\delta}{R}w=-\frac{F_PE\delta}{8\lambda^3DR}f_1(\xi), \quad (4)$$

$$\sigma_\beta=\frac{N_\beta}{\delta}=-\frac{F_PE}{8\lambda^3DR}f_1(\xi), \quad (5)$$

$$M_\alpha=-\lambda^2D\frac{d^2w}{d\xi^2}=-\frac{F_P}{4\lambda}f_3(\xi), \quad (6)$$

$$Q_\alpha=-\lambda^3D\frac{d^3w}{d\xi^3}=\frac{F_P}{2}f_4(\xi), \quad (7)$$

$$\theta=\frac{dw}{d\alpha}=\lambda\frac{dw}{d\xi}=\frac{F_P}{4\lambda^2D}f_2(\xi). \quad (8)$$

$\sigma_\beta$ is the circumferential stress and $f_1(\xi)=e^{-\xi}(\cos\xi+\sin\xi)$, $f_2(\xi)=e^{-\xi}\sin\xi$, $f_3(\xi)=e^{-\xi}(\cos\xi-\sin\xi)$, $f_4(\xi)=e^{-\xi}\cos\xi$. If $\xi=\lambda\alpha$ is large enough, these 4 special functions— $f_1(\xi)$ ~$f_4(\xi)$ will have increasingly smaller values, which implies that under the force of single anchor cable, the normal displacement and its internal force are local [5]. When ∟$\xi=\lambda\alpha>\pi$, the absolute value of each special function is smaller than 5% of its maximum absolute value, when, $\alpha>\pi/\lambda=\pi\sqrt[4]{\frac{R^2\delta^2}{3(1-\mu^2)}}=2\sqrt{R\delta}\sim2.5\sqrt{R\delta}$.

In other words, $2\sqrt{R\delta}$~$2.5\sqrt{R\delta}$farther away from the stressed cross section, the normal displacement and internal force is too small to consider.

## 3. Analysis on practical project of the area of adjacent anchor effect

### 3.1. Practical engineering project

With the data observed through a permanent observation apparatus in a China's prestressed concrete lining project, the influence scope of stress of single anchor cable on the cylindrical structure was calculated theoretically.

Un-bonded prestressed concrete liner is used in Xiaolangdi Desalting Tunnel project[8]; the highest design level during operating period is 122m; one segment of concrete pouring runs as long as 12m and the elasticity modulus of lining concrete is about 32.5GPa; the liner was designed as thick as 0.65m and its inner diameter is 6.50m; space between anchor cables is 0.50m and the cable consists of 8 un-bonded steel strands which are strong and loose and the tension of single anchor cable is designed at 1674kN. These steel strands wind in double rings along the tunnel; the anchorage slot is arrayed in two symmetric rows along the axial direction of tunnel at the bottom of liner and the intersection angle of center between two adjacent slots is 90°. One pouring segment AB in the observation apparatus is cut out to be used as the object of study. The layout of AB segment of prestressed anchor cable is displayed in Fig. 2.

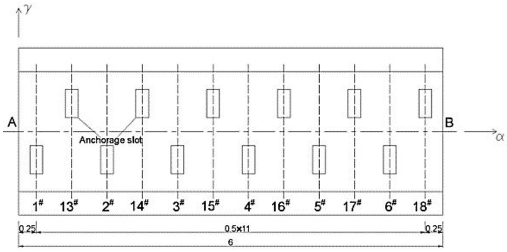



***Figure 2:*** *Layout of AB segment of prestressed anchor cable*

### 3.2. Verification on the area of adjacent anchor effect

Scope of influence of single anchor cable's stress was measured in site according to the record of stretch-draw of $3^{\#}$ anchor cables observed through the permanent observation apparatus. In order to verify the rationality of the theoretical calculation results, the finite element model was established with the engineering example, and the adjacent anchor effect generated by tension of the single anchor cable was calculated. After theoretical analysis, there is $\alpha = \pi/\lambda = \pi\sqrt[4]{\frac{R^2\delta^2}{3(1-\mu^2)}} = 2.41\sqrt{R\delta}$ ($\mu$=0.2), which reveals that the size of influence scope is only related to the diameter of middle surface-$R$ and thickness of cylinder wall-$\delta$. With the stretch-draw of $3^{\#}$ anchor cables, the theoretical value of circumferential force can be calculated through Equation (5) and the results are shown in Tab.1.Because of the anchorage slot, stress state of lining under semi-ring is complex and is not conducive to analyze the calculation results of adjacent anchor effect. The finite element results in Table 1 are obtained from the middle circumferential stress of the top of the lining. The results of the internal axial stress at the top of the lining during the tension of the $3^{\#}$ cable are shown in Figure 3.

***Table 1:*** *Theoretical value and measured value of circumferential stress on cross section with the stretch-draw of 3# anchor cables*

| **Tendon Number** | **1** | **13** | **2** | **14** |
|---|---|---|---|---|
| Measured results /MPa | -0.02 | -0.33 | -0.56 | -1.06 |
| Theoretical results /MPa | -0.21 | -0.40 | -0.70 | -1.02 |
| Finite element results /MPa | -0.24 | -0.47 | -0.88 | -1.06 |
| **Tendon Number** | **3** | **15** | **4** | **16** |
| Measured results /MPa | -1.37 | -0.99 | -0.59 | -0.37 |
| Theoretical results /MPa | -1.19 | -1.03 | -0.72 | -0.42 |
| Finite element results /MPa | -1.26 | -1.09 | -0.93 | -0.50 |
| **Tendon Number** | **5** | **17** | **6** | **18** |
| Measured results /MPa | -0.16 | -0.08 | -0.02 | 0.00 |
| Theoretical results /MPa | -0.19 | -0.05 | 0.03 | 0.05 |
| Finite element results /MPa | -0.18 | -0.04 | 0.04 | 0.06 |

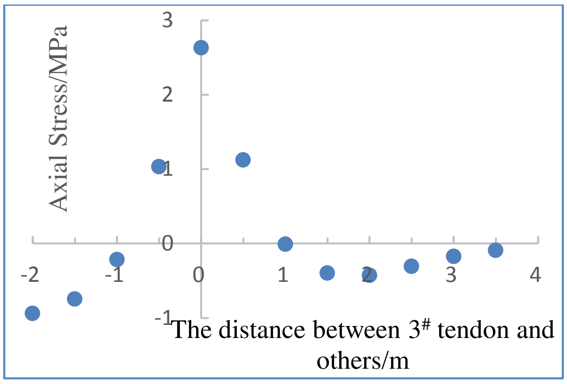



***Figure 3*** *The internal axial stress along the axial at the top of the lining during the tension of the 3# cable*

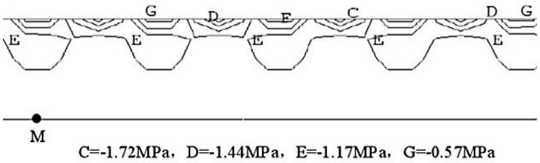
From Table 1, it can be seen that when stretching $3^{\#}$ anchor cable, the circumferential force produced on the cross section is strongest while the circumferential force on the cross section of adjacent cables will become smaller and smaller to 0 at the $6^{th}$ cable away from the $3^{\#}$ cable. Consequently, the area of adjacent anchor effect observed through the apparatus is about 3.3m. Result from theoretical calculation is 3.5m which is quite similar to the measured result—only 5% difference.

In the Figure 3, during stretching $3^{\#}$ anchor cable, the distribution law along axis of internal axial stress at the top of liner is the same as that of circumferential stress. But the maximum axial tensile stress in the area of adjacent anchor effect is 2.63 MPa, far exceeding the tensile strength of the concrete, which produced circumferential cracks inside the positions of anchor cable in the structure model test after tension. In order to avoid the circumferential cracks in the tension process, the loading method is taken step by step in Xiaolangdi desilting tunnel. The anchor tension of the permanent observation apparatus is loaded by 3 times and the order ranges from $13^{\#}$ to $18^{\#}$, and from $6^{\#}$ to $1^{\#}$. The first tensile load is 50% of the design load, and the second is increased from 50% to 77%, and the third is increased from 77% to 100%. It is also seen that at about 3.5m away from $3^{\#}$ cable, the axial stress of the section is about 0. It is verified that the area of adjacent anchor effect is reasonable once again.

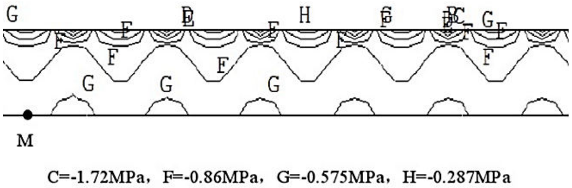
### 3.3. The relationship between the area of adjacent anchor effect and tendon spacing

In this paper, tendon spacing which is 50cm, 70cm, 100cm, 150cm and 300cm is selected to be analyzed by finite element method as studying object in the liner. And the axial stress distribution at the bottom of internal liner at the different tendon spacing is cut for typical section as shown in Figure 4:

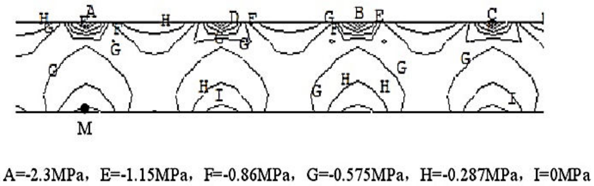
From Figure 4, it can be seen that the compressive stress in the tendon position increases with tendon spacing increasing, but will tend to a fixed value. Because the adjacent anchor effect is appeared when tendon spacing is small; when tendon spacing increases, adjacent anchor effect decreases gradually; adjacent anchor effect disappears when tendon spacing increases to a certain degree, axial compressive stress also tend to constant.



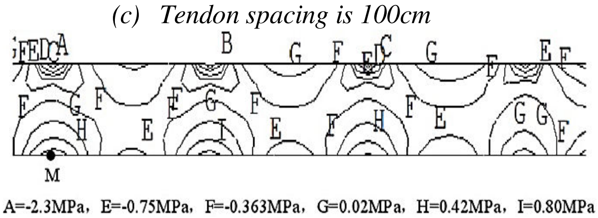

*(a) Tendon spacing is 50cm*



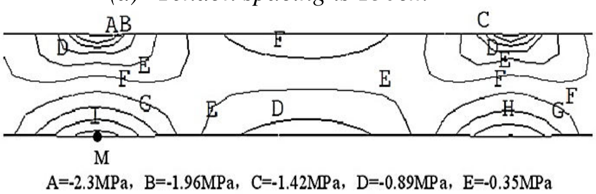

*(b) Tendon spacing is 70cm*




*(c) Tendon spacing is 100cm*




*(d) Tendon spacing is 150cm*




*(e) Tendon spacing is 300cm*

***Figure 4** The contour map of axial stress at different tendon spacing*

With tendon spacing increasing, axial compressive stress between two tendons is changed from compressive stress to tensile stress. But the variation is small. When tendon spacing is increased to 300cm, the axial tensile stress is only 0.19MPa, and far less than the tensile strength of concrete. When tendon spacing is 50cm, the axial stress of inside liner is uniform; when tendon spacing increases to 70cm, the position of tendon inside liner (Figure shown in point M) appears stress concentration; when tendon spacing increases to 100cm, it is appeared tensile stress at point M; when tendon spacing increases to 300cm, the axial stress at point M has been greater than the tensile strength of concrete, and concrete cracks.

## 4. Conclusions

(1) Theoretical analysis reveals that the formula of maximum scope of influence is $\alpha_{max}=\pi/\lambda$, and the theoretical result coincides with the measured result.
(2) When the area of adjacent anchor effect is large, the axial tensile stress which is at the anchor position inside lining is maximum and more than the tensile strength of concrete. The steps to tension should be taken.
(3) Through the analysis of finite element method, the relationship between tendon spacing and adjacent anchor effect is inverse.

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**References:**

[1] J.J. Zhou, “Analysis on the working behavior of prestressed concrete lining of River Hydropower Station”, Hubei Water Power, Vol.34, No.3, 13-15, 1999.
[2] Z.C. Sun, S.X. Zhong, “Key Points in Implementation of Bonded Circumferential Pre-stressing of Yellow River Crossing Tunnel on South-to-North Water Diversion Project”, Tunnel Construction, Vol.34, No.1, 73-77, 2014
[3] C.Y Wei, W.Z. Luan, “Study on prestressed concrete lining of water conveyance tunnel in Dahuofang”, Water Resources & Hydropower of Northeast China, Vol.5, 28-29+43, 2012
[4] Ministry of Water Resources of the People's Republic of China, *DL/T5195-2004 Specification for Design of Hydraulic Tunnel*, Beijing: China Electric Power Press, 2004.
[5] Z.L. Xu, *Mechanics of Elasticity*, vol. 4th edition, Beijing: Higher Education Press, 1982.
[6] J.F. Kang, Y.M. Hu, “Model Test on Prestressed Tunnel Liners of Xiao Langdi Sediment Tunnel”, Civil Engineering Journal, Vol.36, No.6, 80-84, 2003.
[7] Z.B. Qu, *Analysis and research of prestressed observational data integration for Xiao langdi sediment tunnel*, Zheng zhou: Yellow River Water Conservancy Publishing House, 2009.
[8] X.S. Lin, F.S. Shen and Y. Zhang, *Study and Practice on the Post-Tensioned Sediment Tunnel Liner of Xiao Langdi Project*, Huanghe Water Press, Zhengzhou, 1999.
[9] X.R. Yu, J.F. Kang and X.P. Li, *Construction Technique of Concrete Tunnel Liner Post-Tensioned with Double-Looped Unbonded Tendons*, China Water Conservancy and Hydropower Press, Beijing, 2000.
[10] HETE′NYI, “A General Solution for The Bending of Beams On An Elastic Foundation of Arbitrary Continuity”, J Appl Phys, Vol.21, 42-44, 1977.
[11] Z.H. Jin, *Design and Research on Unbonded Prestressed Concrete of Pressure Tunnels*, Nanjing: Hohai University, 2006