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GPR Deconvolution Method Based on Formation Spectrum Compensation Using the Fractional Fourier Transform

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Abstract: On the premise that the system is a linear time-varying model, this paper describes a ground-penetrating radar (GPR) deconvolution technology based on a formation spectrum compensation factor The formation spectrum correction factor in the fractional Fourier transform domain is determined using an optimal filtering algorithm. This verifies the original records of the spectrum using the correction factor in the transform domain, and improves the resolution of the GPR data. The algorithm proposed in this paper is validated using a simulation model data. Its performance is then compared with that of a conventional GPR deconvolution algorithm. The results verify that the proposed algorithm is more reasonable and effective than the traditional method.

Keywords: *Linear time-varying model, Ground penetrating radar (GPR), Deconvolution ,Formation spectrum compensation, Fractional Fourier transform (FrFT) ,Optimal filtering*

1. Introduction

Deconvolution is commonly used to improve the resolution of ground penetrating radar (GPR). Under typical processes such as least-squares deconvolution and predictive deconvolution, the basic radar wavelet is compressed to improve the temporal resolution of the GPR data. These deconvolution methods assume that the reflection coefficient series is an uncorrelated white noise sequence and that the radar wavelet is a minimum phase wavelet [1-3]. Moran used a Wiener filtering deconvolution method that broadens the wavelet and enhances the GPR resolution [4]. Su proposed a deconvolution method based on formation spectrum correction [5]. In this method, the formation is considered as a system. According to the characteristics of the formation system response model, one or more formation spectrum correction factors were obtained in the original GPR data. The spectra of the original records were then corrected using these correction factors. This approach reduces noise and improves the resolution. The above deconvolution methods were built on the basis of linear time-invariant systems, and certain restrictions and requirements on the radar wavelet and reflection sequence. Specifically, the wavelet must be of minimum phase, and the reflection must be a white noise sequence. The wavelet characteristics are estimated using the auto correlation of echo records and linear filtering. However, in practice, the reflection sequence is not white and the wavelet is not minimum phase. Thus, this kind of deconvolution method is not always effective. There are some nonlinear methods that do not require the wavelet to be minimum phase, but their algorithms tend to be complex, and they are only effective in a certain range [6-10]. The GPR deconvolution method presented in this paper uses a fractional Fourier transform to compensate the formation spectrum, and is applicable to linear time-varying systems. The echo signal can be considered as the linear integral form of the wavelets and formation. Under the proposed method, there is no constraint on whether the wavelet is minimum phase, only that the wavelet is invariant in the process of transmission. A compensation factor is obtained in the fractional Fourier transform domain. The formation records are corrected in this domain to improve the GPR resolution.

2. The foundation of formation correction:

According to the foundations of formation correction, the formation is considered as a system. Depending on the characteristics of the formation system response model, one or more formation spectrum correction factors are obtained in the GPR original data records. The original Fourier transform domain records are then subjected to spectrum correction using these correction factors [5].The formation response model is shown in Figure 1, where T and R are the transmitting and receiving antennas of the GPR, respectively. $x_i(t)$ is the radar signal received from the reflection at interface *i*. After intercepting the information of $x_i(t)$, this reflection wavelet of the interface is retained for use in determining the formation correction factor. $x_i(t)$ arrives at interface *j* across a formation thickness of *h* m. The formation of thickness *h* that is penetrated by the radar wave is seen as a formation response system. The corresponding radar reflection signal wavelet from interface j is, $x_j(t)$. This obeys the following relationship

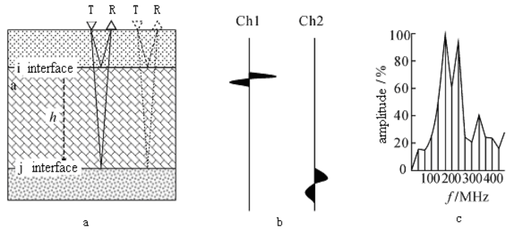

Figure 1. *Foundation of the formation spectrum correction method (a) Formation system response model, (b) Interface response wavelet intercepted, (c) Formation correction factor spectrum diagram*

$$x_j(t) = q_h(t) * x_i(t) \tag{1}$$

Where, * denotes the convolution operation, and $q_h(t)$ represents the absorption and attenuation influence of reflection signal $x_i(t)$ resulting from the formation of thickness h. The inverse operator $p_h = q_h^{-1} = \{p_h(t)\}$ can then be used to correct the influence of the reflection signal $x_i(t)$ resulting from the formation of thickness h. This can be written as:

$$x_i(t) = p_h(t) * x_j(t) \tag{2}$$

The formation correction factor p_h is determined in the frequency domain. Applying a Fourier transform to both sides of equation (2), we obtain the following expression for the inverse operator, $P_h(f)$:

$$P_h(f) = X_i(f) / X_j(f) = \frac{X_i(f)\bar{X}_j(f)}{\left|X_j(f)\right|^2 + \sigma^2} \tag{3}$$

Where, $X_i(f)$, $X_j(f)$ are the Fourier transforms of $x_i(t)$, $x_j(t)$, respectively, $P_h(f)$ is the Fourier transform of $p_h(t)$, σ is the noise term, and $P_h(f)$ is the desired formation spectrum correction factor. To correct the spectra of the original records using $P_h(f)$, we must apply a Hilbert transform to $P_h(f)$. The spectrum correction factor can then be written as:

$$P_h(f) = U_{ph}(f) + iV_{ph}(f) \tag{4}$$

The amplitude spectrum of the correction factor is given by:

$$A_{ph}(f) = \left|P_h(f)\right| = \sqrt{U_{ph}^2(f) + V_{ph}^2(f)} \tag{5}$$

In practical applications, the corrected GPR data cannot produce a time shift. Hence, the phase spectrum of the formation spectrum correction factor must be zero [6]. Then:

$$P_h(f) = A_{ph}(f) \tag{6}$$

Assuming that the spectrum of the *k*th original GPR data point $x_k(t)$ is:

$$X_k(f) = U_k(f) + iV_k(f) \tag{7}$$

The application of the formation spectrum correction factor gives:

$$Y_k(f) = A_{pk}(f) X_k(f) \tag{8}$$

Formation spectrum correction is achieved in the frequency domain through the above derivation. Finally, the corrected GPR time domain data are obtained through an inverse Fourier transform. In the proposed algorithm, the following equation replaces the convolution formation response model of equation (1) [11]:

$$x_j(t) = \int_{-\infty}^{+\infty} q_h(t,t') x_i(t') \mathrm{d}t' \tag{9}$$

In equation (9), $q_h(t,t')$ represents the absorption and attenuation influence of reflection signal $x_i(t)$ resulting from the formation of thickness *h*. This is a time-varying expression. The relationship between radar reflection signal wavelets $x_j(t)$ and $x_i(t)$ has an integral form. This linear time-varying system represents media information more accurately than the conventional convolution mode. For such a model, the method of determining the formation correction factor uses fractional Fourier transform optimal filtering.

3. The basic theory of fractional Fourier transforms:

The fractional Fourier transform of order p of the function $x(t)$ defined for $0 < |p| < 2$ through its integral kernel [12]:

$$\{F^p x\}(t_p) = \int_{-\infty}^{+\infty} K_p(t_p, t) x(t) dt \tag{10}$$

This is a linear integration in which $K_p(t_p,t) = K_\alpha \exp[\mathrm{j}\pi(t_p{}^2\cot\alpha\text{-}2t_p t\csc\alpha + t^2\cot\alpha)]$ is the kernel function of the fractional Fourier transform, $\alpha = {}^{p\pi}\!/_2$, $p \neq 2n$, n is an integer and $K_p = \exp[-j(\pi\,\mathrm{sgn}(p)/4 - p/2)] / |\sin(p)|^{0.5}$.The kernel $K_p(t_p,t)$ is defined separately for $p = 0$ and $p = \pm 2$ as $K_0(t_p,t) = \delta(t_p - t)$ and $K_{\pm 2}(t_p,t) = \delta(t_p + t)$. The definition can easily be extended outside the interval $[-2,2]$ by noting that $F^{4l+p}x(t_p) = F^p x(t_p)$ for any integer. Because $\alpha = {}^{p\pi}\!/_2$ only appears in the parameters of trigonometric functions, there is a four-phase (or 2π) cycle in the definitions including p (or α) in equation (10). Thus, we must consider $p \in (-2,2]$ (or $\alpha \in (-\pi,\pi]$) in this study. Equation (10) indicates that the zero-order transform of the function is equal to itself. The ± 2-order transform of the function is equal to $x(-t)$. This definition ensures that $K_p(t_p,t)$ is continuous for all values of p.

The most important properties of the FrFT are
1) Unitarity: $(F^p)^{-1} = F^{-p} = (F^p)^*$, where $(\cdot)^*$ denotes
Hermitian conjugation;
2) Index additivity: $F^{p_1}F^{p_2} = F^{p_2}F^{p_1} = F^{p_1+p_2}$;
3) Reduction to the ordinary Fourier transform when $p=1$.

4. Obtaining a formation correction factor based on the fractional Fourier transform:

It is assumed that the autocorrelation and cross-correlation functions of $x_i(t)$, $x_j(t)$ are known as a priori knowledge. The correction operator can be expressed as:

$$\hat{x}_i(t) = \int_{-\infty}^{+\infty} g(t,t')x_j(t')\mathrm{d}t' \quad (11)$$

In formula (11), $g(t,t')$ is the desired formation correction factor. Because it is assumed that the autocorrelation and cross-correlation functions are known, $\hat{x}_i(t)$ is the estimated value of $x_i(t)$.The function expressions are mean-square-integrable, non-stationary signals. Their mean square error can be expressed as:

$$\sigma_e{}^2 = E[\|x_i(t) - \hat{x}_i(t)\|^2] \quad (12)$$

Where, $\|\cdot\|$ is the L_2-space norm, defined as:

$$\|x\|^2 = \int_{-\infty}^{+\infty} x_i(t)x_i^*(t)\mathrm{d}t \quad (13)$$

The minimum optimal estimation operator in equation (12) should satisfy:

$$g_{\mathrm{opt}}(t,t') = \arg\min_g \sigma_e^2 \quad (14)$$

Where, $g_{\mathrm{opt}}(t,t')$ is the desired formation estimation operator. For the optimal filtering of fractional Fourier transforms, the estimation operator is expressed as a multiplication filter in the p-order fractional Fourier domain. It is written as:

$$\hat{x}_i = F^{-p}[g \cdot F^p[x_j(t)]] \quad (15)$$

Where, $F^p(\cdot)$ represents the fractional Fourier transform whose angle is, $p\pi/2$, and g is the multiplication filter in the fractional Fourier domain. According to the definition of the estimation operator in formula (15), the optimal filter is given by the following formula:

$$g = \arg\min_g \sigma_e^2 \quad (16)$$

Where, σ_e^2 is defined according to equation (12) and $\hat{x}_i$ is defined according to equation (15). In equation (16), the cost function J is defined as the mean-square error in equation (12). According the properties of the FrFT, the FrFT is a unitary transform and the norm remains unchanged, then the mean-square error in the p-order FrFT domain is equal to J. Namely [12-15]:

$$J = \sigma_e^2 = E[\|x_i(t) - \hat{x}_i(t)\|^2] = E[\|X_p - \hat{X}_p\|^2] \quad (17)$$

Where, $\hat{X}_p = g \cdot F^p[x_j(t)]$, $X_p = F^p[x_i(t)]$, and g is the estimation operator. Because $\hat{X}_p$ is varying, J can change with, g, and the minimum of J is related to the value of g. Let, $g = g_0 + a\delta g_0$, $a = a_{re} + ia_{im}$ is a complex scalar parameter, g_0 is a optimization operator, and δg_0 is an arbitrary distracter. Then

$$\hat{X}_p(t_p,a) = (g_0(t_p) + (a_{re} + ia_{im})\delta g_0) \cdot F^p[x_j(t)] \quad (18)$$

$$J(a) = E[\int_{-\infty}^{+\infty} (X_p(t_p) - \hat{X}_p(t_p,a)) \cdot (X_p(t_p) - \hat{X}_p(t_p,a))^* dt] \quad (19)$$

When, $\left.\frac{\partial J(a)}{\partial a_{re}}\right|_{a=0}$ $\left.\frac{\partial J(a)}{\partial a_{im}}\right|_{a=0}$, that is $g = g_{\mathrm{opt}}$, J attains its minimum. The optimal estimation operator g_{opt} is [13]:

$$g_{\mathrm{opt}} = \frac{R_{X_{ip}X_{jp}}(t_p,t_p)}{R_{X_{jp}X_{jp}}(t_p,t_p)} \quad (20)$$

The correlation function can be obtained as follows:

$$R_{X_{ip}X_{jp}}(t_p,t_p) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} K_p(t_p,t)K_p(t_p,t') \cdot R_{x_ix_j}(t,t')\mathrm{d}t'\mathrm{d}t \quad (21)$$

$$R_{X_{jp}X_{jp}}(t_p,t_p) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} K_p(t_p,t)K_p(t_p,t') \cdot R_{x_jx_j}(t,t')\mathrm{d}t'\mathrm{d}t \quad (22)$$

Equation (20) gives the optimal operator in the p-order FrFT domain. To find the optimal value, equation (20) is inserted into the mean-square error of equation (17). This completes the investigation of the optimal order p in FrFT domain.

$$\sigma_{e,0}^2 = E[\int_{-\infty}^{+\infty} (X_p(t_p) - \hat{X}_p(t_p.0)) \cdot (X_p(t_p) - \hat{X}_p(t_p.0))^* \mathrm{d}u]$$
$$= \int_{-\infty}^{+\infty} \begin{matrix} (R_{X_pX_p}(t_p,t_p) - 2\mathrm{Re}(g_{\mathrm{opt}}^*(t_p)R_{X_pY_p}(t_p,t_p)) \\ + g_{\mathrm{opt}}(t_p)g_{\mathrm{opt}}^*(t_p)R_{Y_pY_p}(t_p,t_p))\mathrm{d}t_p \end{matrix} \quad (23)$$

The p value is determined within the range $(-2,2]$ using the formula (23). Some p values can be found in the FrFT domain. This value minimizes the mean square error, giving the optimal filter value g_{opt}.The original record of the kth GPR trace is assumed to be $x_k(t)$. Thus, the kth corrected signal y_k according to the formation correction factor g_{opt} is:

$$y_k = F^{-p}[g_{opt} \cdot F^p[x_k(t)]]. \quad (24)$$

5. Model test:

The GPR model was produced using finite-difference time-domain theory. The relative parameters of the model were as follows: dimensions of 9 m × 5 m, with a distance between the thin layer and the upper surface of 5 m, a thin layer of 0.2 m thickness, antenna center frequency of 200 MHz, transmitting and receiving antenna distance of 1 m, and a point distance of 0.1 m. Medium A was assumed to have a relative permittivity of 5, conductivity of 0.0005 s/m, and relative magnetic permeability of 1. Medium B was assumed to have a relative permittivity of 30, conductivity of 0.0001 s/m, and relative magnetic permeability of 1. Figure 2 shows a GPR profile of the model. The thin layer of 0.2 m thickness is difficult to see in this section. According to the theoretical analysis, the two-way travel time to the first reflecting surface should be 75 ns. The electromagnetic wave transmits to the second reflecting surface, and the two-way travel time to the thin layer should be 82 ns. Figure 3 shows the results using the formation correction factor proposed in this paper. The two reflecting surfaces can be clearly seen in the time domain. Enlarging this figure shows that the time to reach the first reflecting surface is close to the theoretical value. It can also be seen that the energy increases significantly after the formation has been corrected. Figure 4 shows the corrected results given by the conventional convolution model. Though two reflecting surfaces can be seen, the time taken by the electromagnetic wave is not as close to the theoretical value. Figure 5b shows the wave spectrum diagram produced by applying the formation correction method described in this paper. The high-frequency energy in the spectrum of Figure 5a is lacking prior to correction, and quickly decays to zero at around 200MHz. This high-frequency energy is greatly improved in the spectrum of the corrected signal (Figure 5b), and the decay is somewhat slower. The bandwidth has also been improved and the frequency gains are correctly compensated.

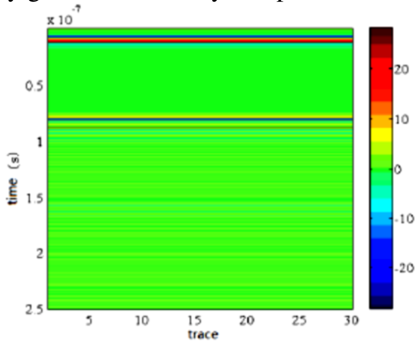

***Figure 2.** Model profile of a numerical simulation of GPR*

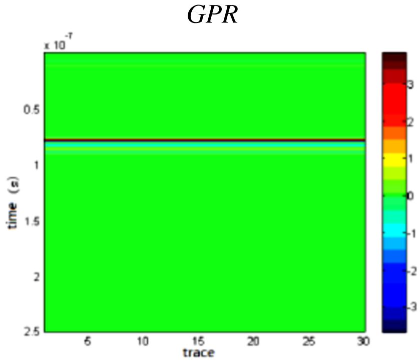

***Figure 3.** Profile of GPR data corrected using the proposed method*

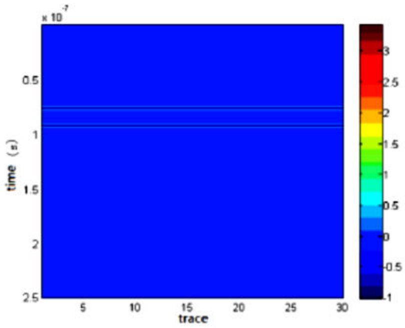

***Figure 4.** Profile of GPR data after convolution model correction*

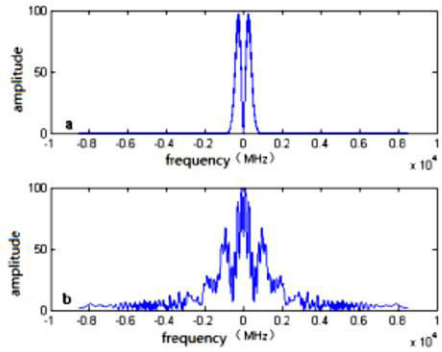

***Figure 5.** Data spectrogram of GPR before and after correction(a) Before correction, (b) After correction*

6. Conclusions:

In this paper, the GPR formation response signal was represented by a time-varying linear system. Fractional Fourier transform theory was used to compensate the formation spectrum. The correction factor was obtained in the fractional Fourier transform domain, and the GPR signal was then corrected in this domain. The theoretical and experimental results showed that the time-varying linear model conformed to the transmitting mechanism when electromagnetic waves were transmitted through the formation. It was also shown that the formation correction factor produces a valid formation signal. The corrected signal spectra and the high-frequency energy have been greatly improved. This method can improve the resolution of GPR signals.

7. Acknowledgements:

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