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Fuzzy Mathematical Programming Approaches to Design Cell Formation in Cellular Manufacturing Systems

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Abstract

Mathematical programming approaches based on fuzzy set theory are proposed for cell formation design in cellular manufacturing systems (CMS). The objective is to find part families and machine groups to minimize the fuzzy total production cost consisting of processing and maintenance costs subject to fuzzy machine time availability. Two cases are used to illustrate the applications. Results indicate that the cell formations obtained by the fuzzy approaches yield less total production costs compared to the non-fuzzy (crisp) approaches.

Key words: Fuzzy mathematical programming; cell formation; cellular manufacturing

1. Introduction

Many techniques for the design of cell formation in cellular manufacturing systems (CMS) have been based on the certainty principle. However, in reality manufacturing systems involve uncertainty due to fuzziness which is associated with imprecision and/or linguistic information. For example, the data on available machine hours and processing times may be imprecise, while the information on product size and operator skill may be expressed in linguistic terms. Problems involving such information can be handled by fuzzy set theory. This paper deals with the application of fuzzy set theory in the development of mathematical programming approaches to design cell formations in cellular manufacturing systems. Specifically a cell formation design for fuzzy machine time availability and fuzzy total production cost is proposed.

2. Literature Survey

In general various mathematical programming techniques, such as linear programming, integer programming, dynamic programming, nonlinear integer programming, mixed integer linear programming, and multiple objective programming have been proposed to design cell formation. Some examples are Chen (2001), Tsai and Lee (2006), and Wei and Gaither (2007). However, these papers did not consider the fuzzy data in the manufacturing environment. This paper proposes a fuzzy mathematical programming approach to design cell formation in cellular manufacturing systems.

3. A Cell Formation Design Model

Consider a manufacturing system that has several types of machines. Each machine can perform several types of operations. Machine time availability on each machine and the total production cost consisting of processing and maintenance costs cannot be stated precisely (fuzzy). Each part has a different sequence of operations with different processing times and each machine has the capability to run different operations with different operating costs per unit time. The objective of this model is to find part families and machine groups to minimize the total production cost.

The following notations are used in the proposed model.

i - index of part, i = 1,…,n
j - index of machine, j = 1,…,m
k - index of group, k = 1,...,q
l - index of operation, l = 1,…,r
c_{lj} - processing cost for operation l on machine j
f_j - annual maintenance cost of machine j
q_i - annual demand for part i.
$\tilde{T}_j$ - fuzzy annual time available on machine j
p_{il} - processing time for operation *l* of part i
G_i - set of machines needed to process part i
s_j - maximum tolerance for annual time available on machine j
B_k - maximum number of parts in cell k
U_k - maximum number of machines in cell k

$$x_{ijk} = \begin{cases} 1, \text{ if part i on machine j belongs to group k} \\ 0, \text{ otherwise} \end{cases}$$

$$x_{ik} = \begin{cases} 1, \text{ if part i belongs to group k} \\ 0, \text{ otherwise} \end{cases}$$

$$y_{jk} = \begin{cases} 1, \text{ if machine j belongs to group k} \\ 0, \text{ otherwise} \end{cases}$$

The objective of this approach is to minimize the overall system cost which consists of processing and maintenance costs as stated in the following equation.

$$\text{Minimize } \sum_i \sum_l \sum_j \sum_k P_{il} C_{lj} q_i x_{ijk} + \sum_j \sum_k f_j y_{jk} \tag{1}$$

Subject to:

- Annual machine time availability

$$\sum_i \sum_k \sum_l P_{il} q_i x_{ijk} \le \tilde{T}_j \quad , \quad \forall j \tag{2}$$

- Each part is allocated to only one cell

$$\sum_k x_{ik} = 1 \quad , \quad \forall i \tag{3}$$

- Maximum number of parts in a cell

$$\sum_i x_{ik} \le B_k \quad , \quad \forall k \tag{4}$$

- Part i is assigned to cell k, if machine is available to process at least one operation of part i.

$$\sum_j x_{ijk} - x_{ik} \ge 0 \quad , \quad \forall j \in G_i \quad , \quad \forall (i,k) \tag{5}$$

- Each machine belongs to only one cell

$$\sum_k y_{jk} = 1 \quad , \quad \forall j \tag{6}$$

- Maximum number of machines in a cell

$$\sum_j y_{jk} \le U_k \quad , \quad \forall k \tag{7}$$

- Each machine has to be used

$$\sum_j y_{jk} \le 1 \quad , \quad \forall k \tag{8}$$

- Prohibit assigning part i to machine j unless cell k is formed

$$\sum_i x_{ijk} \le (n-1) y_{jk} \quad , \quad \forall j \in G_i \quad , \quad \forall k \tag{9}$$

- Integrality of the decision variables

$$x_{ijk} \ , x_{ik} \ , y_{jk} \in (0,1) \qquad \forall (i,j,k) \tag{10}$$

3.1. Machine Time Availability in Non-increasing Linear Fuzzy Numbers

If annual time availability on a machine (Equation 2) is stated in interval value as $\tilde{T}_j \in \left[T_j \ , T_j + s_j\right], \ \forall j$, then Equation 2 can be stated as

$$H_j(x) = \sum_i \sum_k \sum_l P_{il} q_i x_{ijk} \le \left(T_j + \theta \, s_j\right) \quad \forall j, \ \theta \in \left[0 \, , 1\right] \tag{11}$$

The above model belongs to fuzzy mathematical programming. To solve the model, Verdegay's Approach (1982) is applied.

If the membership function of the machine time availability is represented as in Figure 1, then it can be mathematically stated as follows.

$$\mu_{T_j}(x) = \begin{cases} 1, & \text{if } H_j(x) < T_j \\ 1 - \left[H_j(x) - T_j\right] / s_j \ , & \text{if } T_j \le H_j(x) \le \left(T_j + s_j\right) \\ 0, & \text{if } H_j(x) > \left(T_j + s_j\right) \end{cases} \tag{12}$$

This membership function indicates that:

1. If time availability on machine j is less than Tj, then the corresponding constraint is absolutely satisfied, $\mu_{Tj}(x) = 1$.
2. If time availability on machine j is between T_j and $T_j + s_j$, then the membership function is monotonically decreasing.
3. If time availability on machine j is greater than $T_j + s_j$, then the corresponding constraint is absolutely violated, $\mu_{Tj}(x) = 0$.

Then the fuzzy linear programming model can be formulated as follows:

Minimize the objective function given by Equation (1).

Subject to

- The time availability on machines

$$H_j(x) = \sum_i \sum_k \sum_l P_{il} q_i x_{ijk} \le \left[T_j + (1-\alpha) s_j\right] \quad \forall j,\ \alpha \in [0, 1] \quad (13)$$

where α = the α level cut.

- Plus constraints represented by Equations (3) – (10)

3.2. Total Production Cost in Non-increasing Linear Fuzzy Numbers

Let the total production cost be between Z° and Z°+ s_o, where s_o is the tolerance value. The objective function can be stated as follows:

$$Z^0 \le Z(x, y) \le Z^0 + s_0 \quad (14)$$

If the membership function of the objective function is assumed to be a non-increasing continuous linear membership function as given in Figure 2, then it can be mathematically stated as follows

$$\mu_0(x, y) = \begin{cases} 1, & if\ Z(x, y) < Z^0 \\ 1 - \dfrac{Z(x, y) - Z^0}{s_0}, & if\ Z^0 \le Z(x, y) \le Z^0 + s_0 \\ 0, & if\ Z(x, y) > Z^0 + s_0 \end{cases} \quad (15)$$

This membership function indicates that:

1. If the total production cost is less than Z°, then the objective is satisfied, $\mu_o(x, y) = 1$.

2. If the total production cost is between Z° and Z° $+s_o$, then the membership function is monotonically decreasing.

3. If the total production cost is greater than Z° $+s_o$, then the objective is absolutely violated, $\mu_o(x, y) = 0$.

4. Fuzzy Mathematical Programming Approaches for Cell Formation in CMS

If the machine time availability is fuzzy (Figure 1) with the membership function of Equation (12) and the total cost is fuzzy (Figure 2) with the membership function of Equation (15), then the model becomes a fuzzy mathematical programming problem. Two approaches proposed by Zimmermann (1976) and Chanas (1983) are applied to solve the problem.

4.1. Zimmermann's Approach

The problem can be stated as

$$Maximize\ \ \alpha$$

Subject to

$$\mu_0(x, y) = 1 - \frac{Z(x, y) - Z^0}{s_0} \ge \alpha$$

$$\mu_{T_j}(x) = 1 - \left(g_j(x) - T_j\right)/ s_j \ge \alpha$$

Plus constraint Equations 3-10

Or

Subject to

$$Maximize\ \ \alpha$$

$$Z(x, y) \le Z^0 + (1-\alpha) s_0$$

$$g_j(x) \le T_j + (1-\alpha) s_j, \quad \forall j$$

Plus constraint Equations (3) – (10)

4.2. Chanas' Approach

In Zimmermann's approach the values of Z° and s_o are assumed to be known beforehand. In Chanas' approach, those values are calculated. According to Chanas (1983) the goal Z° and its tolerance value s_o are very difficult to determine beforehand due to the lack of information about the fuzzy feasible region. Thus in order to determine the values of Z° and s_o, the following model should be solved first.

$$Minimize \quad Z^0(x, y) = \sum_i \sum_l \sum_j \sum_k P_{il} C_{lj} q_i x_{ijk} + \sum_j \sum_k f_j y_{jk}$$

Subject to

$$g_j(x) = \sum_i \sum_k \sum_l P_{il} q_i x_{ijk} \le \tilde{T}_j, \qquad \forall j$$

Plus constraint Equations 3-10

Or

$$Minimize \quad Z^0(x,y) = \sum_i \sum_l \sum_j \sum_k P_{il} C_{lj} q_i x_{ijk} + \sum_j \sum_k f_j y_{jk}$$

Subject to

$$g_j(x) = \sum_i \sum_k \sum_l P_{il} q_i x_{ijk} \le T_j + \theta\, s_j, \quad \theta \in (0,1), \qquad \forall j$$

Plus constraint Equations (3) – (10)

Suppose the solution of this model is given by Z*(θ), x*(θ), and y*(θ). Based on these values, the decision maker can decide the value of Z° and its corresponding s_0. Thus the membership function of objective can be stated as follows.

$$\mu_0(x^*(\theta), y^*(\theta)) = \begin{cases} 1, & if\ Z^*(x,y) < Z^0 \\ 1 - \dfrac{Z^*(x,y) - Z^0}{s_0}, & if\ Z^0 \le Z^*(x,y) \le Z^0 + s_0 \\ 0, & if\ Z^*(x,y) > Z^0 + s_0 \end{cases} \tag{16}$$

The final optimal solution to the model $x^*(\theta^*),\ y^*(\theta^*),\ and\ z^*(\theta^*)$ will exist at

$$\max \mu_D(\theta) = \max \left\{ \min \left[\mu_0(\theta), \mu_{T_j}(\theta) \right] \right\}$$

5. Manufacturing Management Cases

Consider 8 parts each of which needs a different number of operations. There are 5 machines each of which can perform a number of operations. The processing time and annual demand for each part are given in Table 1.

The processing cost, annual machine time availability and its tolerance, and annual maintenance cost for each machine are given in Table 2.

5.1. Case 1

Consider the case where the number of cells to be formed is 3, and the maximum number of parts and machines in a cell are 3 and 3 respectively.

<u>Zimmermann's Approach</u>

Assume that the total production is between 600,000 and 650,000 units. By applying Equation (15), the membership function of the total production cost can be defined as follows.

$$\mu_0(x,y) = \begin{cases} 1, & if\ Z(x,y) < 600000 \\ 1 - \dfrac{Z(x,y) - 60000}{50000}, & if\ 600000 \le Z(x,y) \le 650000 \\ 0, & if\ Z(x,y) > 650000 \end{cases} \tag{17}$$

By applying Equation (1), the membership function of time availability on each machine can be determined as follows.

- Machine 1:

$$\mu_{T_1}(x) = \begin{cases} 1, & if\ \Sigma_{i=1}^{8}\Sigma_{l=1}^{5} P_{il} q_i x_{ijk} \le 5000 \\ 1 - \left[\Sigma_{i=1}^{8}\Sigma_{l=1}^{5} P_{il} q_i x_{ijk} - 5000\right] / 500, & if\ 5000 \le \Sigma_{i=1}^{8}\Sigma_{l=1}^{5} P_{il} q_i x_{ijk} \le 5500 \\ 0, & if\ \Sigma_{i=1}^{8}\Sigma_{l=1}^{5} P_{il} q_i x_{ijk} \ge 5500 \end{cases}$$

- Machine 2:

$$\mu_{T_2}(x) = \begin{cases} 1, & if\ \Sigma_{i=1}^{8}\Sigma_{l=1}^{5} P_{il} q_i x_{ijk} \le 6000 \\ 1 - \left[\Sigma_{i=1}^{8}\Sigma_{l=1}^{5} P_{il} q_i x_{ijk} - 6000\right] / 600, & if\ 6000 \le \Sigma_{i=1}^{8}\Sigma_{l=1}^{5} P_{il} q_i x_{ijk} \le 6600 \\ 0, & if\ \Sigma_{i=1}^{8}\Sigma_{l=1}^{5} P_{il} q_i x_{ijk} \ge 6600 \end{cases}$$

- Machine 3:

$$\mu_{T_3}(x) = \begin{cases} 1, & if\ \Sigma_{i=1}^{8}\Sigma_{l=1}^{5} P_{il} q_i x_{ijk} \le 8000 \\ 1 - \left[\Sigma_{i=1}^{8}\Sigma_{l=1}^{5} P_{il} q_i x_{ijk} - 8000\right] / 800, & if\ 8000 \le \Sigma_{i=1}^{8}\Sigma_{l=1}^{5} P_{il} q_i x_{ijk} \le 8800 \\ 0, & if\ \Sigma_{i=1}^{8}\Sigma_{l=1}^{5} P_{il} q_i x_{ijk} \ge 8800 \end{cases}$$

- Machine 4:

$$\mu_{T_4}(x) = \begin{cases} 1, & if\ \Sigma_{i=1}^{8}\Sigma_{l=1}^{5} P_{il} q_i x_{ijk} \le 6000 \\ 1 - \left[\Sigma_{i=1}^{8}\Sigma_{l=1}^{5} P_{il} q_i x_{ijk} - 6000\right] / 600, & if\ 6000 \le \Sigma_{i=1}^{8}\Sigma_{l=1}^{5} P_{il} q_i x_{ijk} \le 6600 \\ 0, & if\ \Sigma_{i=1}^{8}\Sigma_{l=1}^{5} P_{il} q_i x_{ijk} \ge 6600 \end{cases}$$

- Machine 5:

$$\mu_{T_5}(x) = \begin{cases} 1, & if\ \Sigma_{i=1}^{8}\Sigma_{l=1}^{5} P_{il} q_i x_{ijk} \le 8000 \\ 1 - \left[\Sigma_{i=1}^{8}\Sigma_{l=1}^{5} P_{il} q_i x_{ijk} - 8000\right] / 800, & if\ 8000 \le \Sigma_{i=1}^{8}\Sigma_{l=1}^{5} P_{il} q_i x_{ijk} \le 8800 \\ 0, & if\ \Sigma_{i=1}^{8}\Sigma_{l=1}^{5} P_{il} q_i x_{ijk} \ge 8800 \end{cases}$$

Thus the formulation of the problem is as follows.

Maximize α

Subject to

$$Z(x,y) = \sum_{i=1}^{8}\sum_{t=1}^{5}\sum_{j=1}^{5}\sum_{k=1}^{3} P_{il} C_{jl} q_i x_{ijk} + \sum_{j=1}^{5}\sum_{k=1}^{3} f_j y_{jk} \le 600000 + (1-\alpha)50000$$

$$g_1(x) = \sum_{i=1}^{8}\sum_{k=1}^{3}\sum_{l=1}^{5} P_{il} q_i x_{i1k} \le 5000 + (1-\alpha)500, \qquad \alpha \in (0,1)$$

$$g_2(x) = \sum_{i=1}^{8}\sum_{k=1}^{3}\sum_{l=1}^{5} P_{il} q_i x_{i2k} \le 6000 + (1-\alpha)600, \qquad \alpha \in (0,1)$$

$$g_3(x) = \sum_{i=1}^{8}\sum_{k=1}^{3}\sum_{l=1}^{5} P_{il} q_i x_{i3k} \le 8000 + (1-\alpha)800, \qquad \alpha \in (0,1)$$

$$g_4(x) = \sum_{i=1}^{8}\sum_{k=1}^{3}\sum_{l=1}^{5} P_{il} q_i x_{i4k} \le 6000 + (1-\alpha)600, \qquad \alpha \in (0,1)$$

$$g_5(x) = \sum_{i=1}^{8}\sum_{k=1}^{3}\sum_{l=1}^{5} P_{il} q_i x_{i5k} \le 8000 + (1-\alpha)800, \qquad \alpha \in (0,1)$$

Plus constraint Equations (3) – (10)

By using the LINGO software, we obtain the following cell formation.

α		Cell No.1	Cell No.2	Cell No.3
0.28	Parts	P1, P2, P6	P3, P4, P7	P5, P8
	Machines	M2, M5	M1, M3	M4
	Total Production Cost	$636,000		

<u>Chanas' Approach</u>

In order to obtain the values of Z^n and s_0, the following problem is solved first.

$$Minimize \quad Z^0(x,y) = \sum_i \sum_l \sum_j \sum_k P_{il} C_{lj} q_i x_{ijk} + \sum_j \sum_k f_i y_{jk}$$

Subject to

$$g_1(x) = \sum_{i=1}^{8} \sum_{k=1}^{3} \sum_{t=1}^{5} P_{il} q_i x_{i1k} \leq 5000 + 500\theta, \qquad \theta \in (0\,,1)$$

$$g_2(x) = \sum_{i=1}^{8} \sum_{k=1}^{3} \sum_{t=1}^{5} P_{il} q_i x_{i2k} \leq 6000 + 600\theta, \qquad \theta \in (0\,,1)$$

$$g_3(x) = \sum_{i=1}^{8} \sum_{k=1}^{3} \sum_{t=1}^{5} P_{il} q_i x_{i3k} \leq 8000 + 800\theta, \qquad \theta \in (0\,,1)$$

$$g_4(x) = \sum_{i=1}^{8} \sum_{k=1}^{3} \sum_{t=1}^{5} P_{il} q_i x_{i4k} \leq 6000 + 600\theta, \qquad \theta \in (0\,,1)$$

$$g_5(x) = \sum_{i=1}^{8} \sum_{k=1}^{3} \sum_{t=1}^{5} P_{il} q_i x_{i5k} \leq 8000 + 800\theta, \qquad \theta \in (0\,,1)$$

Plus constraint Equations (3) – (10)

By using the LINGO software, the total production costs (Z^0) for different values of θ are obtained as follows.

θ	Z0	θ	Z0
0	639000	0.6	636000
0.1	639000	0.7	636000
0.2	639000	0.8	636000
0.3	639000	0.9	636000
0.4	639000	1	627000
0.5	636000		

From the above results, suppose the decision maker decides that the total production cost should be between $625,000 and $635,000. Then we obtain the membership function of total production cost as follows.

$$\mu_0[Z^*(\theta)] = \begin{cases} 1, & if \ Z^*(\theta) < 625000 \\ 1 - \dfrac{Z^*(\theta) - 625000}{635000 - 625000}, & if \ 625000 \leq Z^*(\theta) \leq 635000 \\ 0, & if \ Z^*(\theta) > 635000 \end{cases}$$

Thus the problem can be stated as

Maximize θ

Subject to:

$$Z(x,y) = \Sigma_{i=1}^{8} \Sigma_{l=1}^{5} \Sigma_{j=1}^{5} \Sigma_{k=1}^{3} P_{il} C_{jl} q_i x_{ijk} + \Sigma_{j=1}^{5} \Sigma_{k=1}^{3} f_j y_{jk} \leq 625000 - 10000\theta, \ \theta \in (0,1)$$

Plus $g_1(x)$-$g_5(x)$ and constraint Equations (3) – (10)

By using the LINGO software, we obtain the following cell formation.

θ		Cell No.1	Cell No.2	Cell No.3
1.0	Parts	P2, P4, P5	P1, P7	P3, P6, P8
	Machines	M2, M4	M5	M1, M3
	Total Production Cost	$635,000		

5.2. Case 2

Consider the case where the number of cells to be formed is 2 and the maximum number of parts and machines in a cell are 5 and 3 respectively.

<u>Zimmermann's Approach</u>

By using the LINGO software, we obtain the following cell formation.

θ		Cell No.1	Cell No.2
0.28	Parts	P1, P3, P4, P6, P7	P2, P5, P8
	Machines	M1, M2, M3	M4, M5
	Total Production Cost	$636,000	

<u>Chanas' Approach</u>

By using the LINGO software, the total production costs (Z^0) for different values of θ are obtained as follows.

θ	Z^0	θ	Z^0
0	639000	0.6	636000
0.1	639000	0.7	636000
0.2	639000	0.8	636000
0.3	639000	0.9	636000
0.4	639000	1	615000
0.5	639000		

From the above results, suppose the decision maker decides that the total production cost should be between

$ 615000 and $ 635000. Then we obtain the membership function of total production cost as follows:

$$\mu_0(x,y) = \begin{cases} 1, & if\ Z(x,y) < 615000 \\ 1 - \dfrac{Z(x,y) - 615000}{635000 - 615000}, & if\ 615000 \le Z(x,y) \le 635000 \\ 0, & if\ Z(x,y) > 635000 \end{cases}$$

By using the LINGO software, we obtain the following cell formation.

θ		Cell No.1	Cell No.2
1.0	Parts	P1, P2, P6, P7	P2, P3, P4, P8
	Machines	M2, M4, M5	M1, M3
	Total Production Cost	$635,000	

6. Discussion

Different cell formation procedures for various cases of fuzzy information using fuzzy mathematical programming have been demonstrated. Cell formation performances obtained for different situations for case 1 (with three cells), and case 2 (with two cells) are summarized in Tables 3 and 4, respectively. From both tables we see that the cell formation obtained by implementing fuzzy set approaches yield less total production cost than that of non-fuzzy (crisp) approaches. In addition, fuzzy mathematical programming approaches for cell formation can treat constraints naturally and easily. However, the problem is NP-hard and can be very time consuming for fairly large problems.

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Part No.	Operation					Annual Demand
	1	2	3	4	5	
1	1.0		2.0	4.0	4.0	400
2		2.0	2.0		4.0	300
3	3.0		1.0		4.0	400
4	2.0	4.0	3.0		3.0	400
5		2.0	5.0	3.0		300
6			1.0	2.0	4.0	200
7		2.0	3.0		4.0	400
8	5.0	2.0		4.0	2.0	200

Table 1 Processing Times and Annual Demands

Machine	Operation					Machine Time Availability / Year	Tolerance	Maintenance Cost / Year
	1	2	3	4	5			
1	20	15	25	40	20	5000	500	6000
2	30	30	40	20	25	6000	600	8000
3	25	10	30	40	20	8000	800	7000
4	40	25	10	20	40	6000	600	8000
5	50	25	30	25	40	8000	800	6000

Table 2 Operating Costs and Machine Times Availability in Fuzzy Numbers

Cases	P/M	Cell No. 1	Cell No. 2	Cell No. 3	TPC ($)
Non-Fuzzy	P	P3, P7	P1, P6, P8	P2, P4, P5	646500
	M	M3	M2, M5	M1, M4	
F1 : with Nonincreasing linear fuzzy numbers	P	P5, P6, P7	P1, P2, P8	P3, P4	627000
	M	M4, M5	M1, M2	M3	
F2: using Zimmermann's Approach	P	P1, P2, P6	P3, P4, P7	P5, P8	636000
	M	M2, M5	M1, M3	M4	
F2: using Chanas' Approach	P	P2, P4, P5	P1, P7	P3, P6, P8	635000
	M	M2, M4	M5	M1, M3	
F1: Fuzzy Machining Hours Availability					
F2: Fuzzy TPC and Fuzzy Machining Hours Availability					
TPC= Total Production Cost in dollars; P= Parts; M= Machines					

Table 3 Cell Formation for Different Situations with Three Cells (Case 1)

Cases	P/M	Cell No. 1	Cell No. 2	TPC ($)
Non-Fuzzy	P	P1, P5, P6	P2, P3, P4, P7, P8	639000
	M	M2, M4	M1, M3, M5	
F1 : with Nonincreasing linear fuzzy numbers	P	P1, P2, P6, P8	P3, P4, P5, P7	615000
	M	M1, M2	M3, M4, M5	
F2: using Zimmermann's Approach	P	P1, P3, P4, P6, P7	P2, P5, P8	636000
	M	M1, M2, M3	M4, M5	
F2: using Chanas' Approach	P	P1, P2, P6, P7	P2, P3, P4, P8	635000
	M	M2, M4, M5	M1, M3	
F1: Fuzzy Machining Hours Availability				
F2: Fuzzy TPC and Fuzzy Machining Hours Availability				
TPC= Total Production Cost in dollars; P= Parts; M= Machines				

Table 4: Cell Formation for Different Situations with Two Cells (Case 2)

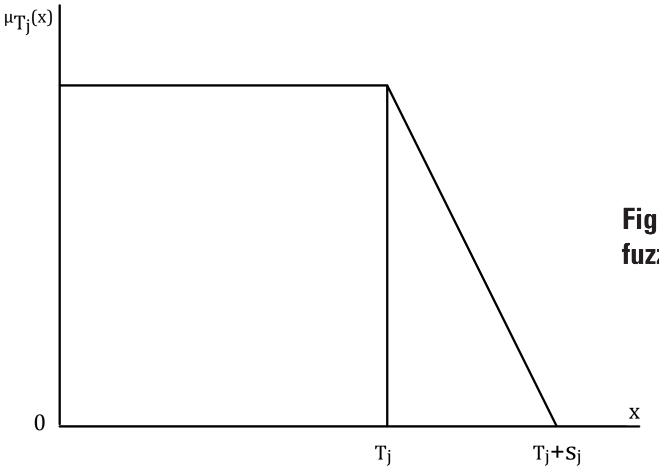

Figure 1 Membership function of fuzzy machine time availability

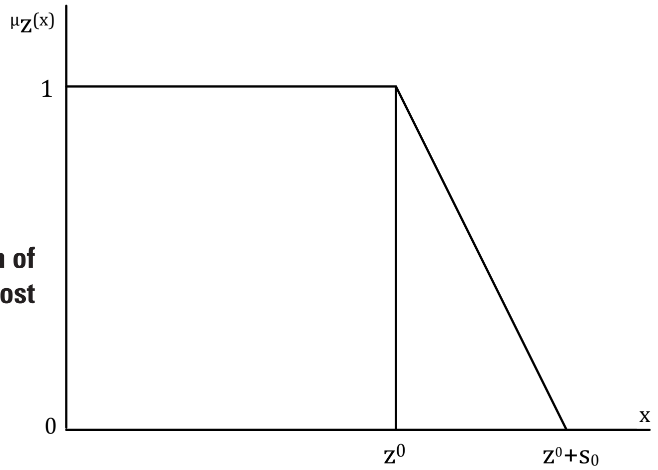

Figure 2 Membership function of fuzzy total production cost