THE STUDY OF STABLE MARRIAGE PROBLEM WITH TIES AND INCOMPLETE BOUNDED LENGTH PREFERENCE LIST UNDER SOCIAL STABILITY

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ABSTRACT

We consider a variant of the Stable Marriage Problem where preference lists of man/woman may be incomplete, may contain ties and may have bounded length in presence of a notion of social stability. In real world matching applications like NRMP and Scottish medical matching scheme such restrictions can arise very frequently where set of agents (man/woman) is very large and providing a complete and strict order preference list is practically in-feasible. In presence of ties in preference lists, there exist three different notion of stability, weak stability, strong stability and super stability. The most common solution is to produce a weakly stable matching. It is a fact that in an instance of Stable Marriage problem with Ties and Incomplete list (SMTI), weakly stable matching can have different sizes. This motivates the problem of finding a maximum cardinality weakly socially stable matching.

In this paper, we find maximum size weakly socially stable matching for a special instance of Stable Marriage problem with Ties and Incomplete bounded length preference list under Social Stability. The motivation to consider this instance is the known fact, finding maximum size weakly socially stable matching in any larger instance of this problem is NP-hard.

KEYWORDS

The Stable Marriage Problem, Socially Stable Matching, Bipartite Matching, Stable Marriage Problem with Ties and Incomplete list, Two Sided Matching, Matching under Preference.

1. INTRODUCTION

The *Stable marriage problem* was first introduced by Gale and Shapley in 1962 [1]. The *classical* instance *I* of the stable marriage problem has a set of *n* men *U*, a set of *n* women *W* and *preference lists* of men over women and vice versa. Each preference list contains all members of opposite sex in a strict order. A man m_i and a woman w_j are called *acceptable* to each other in I instance *I* if m_i is in preference list of w_j and w_j is in preference list of m_i. Let α is the set of all *acceptable pairs* in the instance *I*. A *matching M* is a set of independent pairs (m_i, w_j) such that $m_i \in U$ and $w_j \in W$. If $(m_i, w_j) \in M$, we say that m_i is matched to w_j in *M* and vice versa and we denote $M(m_i) = w_j$ and $M(w_j) = m_i$.

A pair $(m_i, w_j) \notin M$ is called a *blocking pair* for matching *M* if both m_i and w_j prefer each other to their partners in *M*. A matching *M* is called a *stable matching* iff there is no blocking pair with respect to *M*. Gale and Shapley gave a deferred acceptance algorithm and proved that every

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instance I of the stable marriage problem admits a stable matching which can be found in polynomial time [1].

The largest and one of the best known applications of Hospitals Residents problem is National Resident Matching Program (NRMP) and Scottish medical matching scheme which match graduated medical students (residents) with their preferred hospitals on the basis of both side preference lists.

The research work in the field of The Stable Marriage Problem has a long history. As we have mentioned earlier, the first problem on stable marriage problem was introduced by Gale and Shapley in 1962. After that lots of variation on first problem came into the picture. Some major variations are Stable Marriage problem with Ties (SMT), Stable Marriage problem with Incomplete list (SMI), Stable Marriage problem with Ties and Incomplete list (SMTI) and Stable Marriage problem with Bounded length preference lists.

1.1 Stable Marriage Problem with Ties (SMT)

In Stable Marriage problem with Ties, each man can give a preference list over a set of women, where two or more women can hold the same place (*ties*) in the preference list and vice-versa. In SMT there are three notion of stability: weak stability, strong stability and super stability [2, 3]. A blocking pair $(m_i, w_j) \notin M$ with respect to a *weakly stable matching M* can be defined as follows: (a) m_i and w_j are acceptable to each other. (b) m_i strictly prefers w_j to $M(m_i)$ (partner of m_i in matching M) (c) w_j strictly prefers m_i to $M(w_j)$. For an instance I of weakly stable matching problem, a weakly stable matching M always exist and can be found in polynomial time [3].

A blocking pair $(m_i, w_j) \notin M$ with respect to a *strongly stable matching M* can be defined as follows: (a) m_i and w_j are acceptable to each other. (b) m_i strictly prefer w_j to $M(m_i)$ and w_j is indifferent between m_i and $M(w_j)$ and vice-versa.

A blocking pair $(m_i, w_j) \notin M$ with respect to a *super stable matching M* can be defined as follows: (a) m_i and w_j are acceptable to each other. (b) both m_i and w_j either strictly prefer each other to their partners M or indifferent between them. There could be an instance I that have neither super nor strongly stable matching but there is an algorithm which can find super and strong stable matching in I (if exist) in polynomial time [4]. Among these three stability notions, *weak stability* has received most attention in the literature [5-12].

1.2 Stable Marriage Problem with Incomplete Lists (SMI)

Stable Marriage with Incomplete list (SMI) is another variation of stable marriage problem in which number of men and women in an instance I need not be same. Each man and woman can give a preference list over a subset of opposite sex. For an instance I a pair (m_i, w_j) is called blocking pair with respect to a matching M if: (a) m_i and w_j are acceptable to each other (b) m_i is either unmatched in M or prefer w_j to $M(m_i)$ (c) w_j is either unmatched in M or prefer m_i to $M(w_j)$. A matching M is called stable if there is no blocking pair with respect to M.

In an instance I of SMI we can partition the set of men and women such that, one partition have those men and women which have partners in all stable matching and other partition have those men and women which are unmatched in all stable matching [13].

1.3 STABLE MARRIAGE PROBLEM WITH TIES AND INCOMPLETE LISTS (SMTI)

Stable Marriage with Ties and Incomplete list (SMTI) is an extension of classical stable marriage problem in which number of men and women in an instance *I* need not be same. Each man gives a preference list over a subset of women and vice-versa. Each preference list may contain ties (two or more men/women have same rank). A pair $(m_i, w_j) \notin M$ forms a blocking pair with respect to matching *M* if (a) Both m_i and w_j are acceptable to each other and (b) m_i is either unmatched or strictly prefers w_j to $M(m_i)$ and (c) w_j is either unmatched or strictly prefers m_i to $M(w_j)$. A matching *M* is called a weakly stable matching if there is no blocking pair with respect to *M*.

It is known that a weakly stable matching in an instance *I* of SMTI can have different sizes and finding maximum cardinality weakly stable matching is an NP-hard problem [6]. NP-hardness holds even if only one tie of size 2 occurs on men's preference list at the tail and women's preference list contain no ties [6].

1.4 STABLE MARRIAGE PROBLEM WITH BOUNDED LENGTH PREFERENCE LISTS

The idea behind bounded length preference list is, in case of large scale matching problems, the preference list of at-least one side of agent tend to be short. An example of large scale matching is Scottish medical matching scheme [14] where each student is required to rank only three hospitals in their preference list. This variation leads to a question, whether problem of finding maximum size stable matching becomes simpler? (For an instance, with one side or both sided bounded preference list).

Suppose (p, q)-MAX SMTI denotes such variation on MAX SMTI problem (finding maximum size matching in an instance of SMTI) where each man can give at-most p women in his preference list and each woman can give at-most q men in her preference list. Halldorsson et al. [7] showed that (4, 7)-MAX SMTI is NP-hard and not approximable within some $\delta > 1$ unless P = NP. A reduction from Minimum Vertex Cover to MAX SMTI, shows that later problem cannot be approximable within 21/19 unless P = NP [9]. Another study in [15] uses NP-hard restriction of minimum vertex cover of graph of minimum degree 3 in producing NP-hard result for (5, 5)-MAX SMTI. Irving et al. [16] shows that (3, 4)-MAX SMTI is NP-hard and not approximable within $\delta > 1$ unless P = NP.

1.5 THE HOSPITALS RESIDENTS PROBLEM

In the classical Hospitals Residents problem, agents are partitioned into the set of hospitals and set of residents. Each resident gives preference to a subset of hospitals in strict order and vice-versa. Each hospital h_j has a non-negative capacity c_j. A matching *M* is a set of resident hospital pairs such that each resident r_i is matched to at most one hospital and each hospital h_j is matched to at most c_j residents. In a matching *M*, $M(r_i)$ denotes hospital assigned to r_i and $M(h_j)$ denotes a set of residents assigned to h_j in *M*. A hospital h_j is *under-subscribed* if $M(h_j) < c_j$ and a resident r_i is *free* if he/she is not matched. A pair $(r_i, h_j) \notin M$ forms a blocking pair with respect to matching *M* if (a) either r_i is free or prefers h_j to $M(r_i)$ and (b) either h_j is under-subscribed or prefers r_i to one of its residents in *M*. A matching *M* is stable if there is no blocking pair with respect to matching *M*.

The research work in the field of hospital resident problem has a long history. After the seminal paper of Gale and Shapley [1] in 1962 lots of variation of this problem comes into the picture. Some of them are The Hospital Resident problem with ties [2] and Hospital Resident problem with couples [2].

2. RELATED WORK

Another variation of stable marriage problem is socially stable marriage problem. An instance I'' of socially stable marriage problem can be defined by (I, G) where I is an instance of classical stable marriage problem and $G = (U \cup W, A)$ is a social network graph. Here U and W are set of men and women respectively and A is set of man woman pair who knows each other in social network G. Set A is called the set of *acquainted pairs* which is the subset of all acceptable pairs ($A \subseteq \alpha$). A marriage M is called socially stable marriage if there is no socially blocking pair with respect to M. A socially blocking pair $(m_i, w_j) \notin M$ is defined as follows: (a) both m_i and w_j prefers each other to their partner in M and (b) m_i and w_j are connected in social network G.

In large scale matching like NRMP and Scottish medical matching scheme, social stability is a useful notion in which members of blocking pair block a matching M only if they know the existence of each other. Thus the notion of social stability allows us to increase the cardinality of matching without taking care of those pairs which are not socially connected in social network graph.

The work in this paper is motivated by the work of Irving et al. [16] where they study about stable marriage problem with ties and bounded length preference list. They show that if each man's list is of length at most two and women's lists are of unbounded length with ties, we can find a maximum size weakly stable matching in polynomial time.

Our work in this paper is also motivated by the work of Askaladis et al. [17] where they study about socially stable matching problem with bounded length preference list. They gave a $O(n^{3/2} \log n)$ time algorithm for (2, ∞) - MAX SMISS problem. Where (2, ∞)-MAX SMISS problem is to find a maximum size socially stable matching in an instance of stable marriage problem with incomplete list under social stability, where each man's list is of length at most two (without ties) and women's lists are of unbounded length (without ties).

3. OUR CONTRIBUTION

In an instance I of (2, ∞)-MAX SMISS problem if we include ties on both side preference lists, where the length of a tie could be arbitrary, this instance converts into an instance I' of (2, ∞)-MAX SMTISS. In this paper we will show that we can find maximum size weakly socially stable matching in instance I' in polynomial time. Due to presence of ties in both side preference lists there are three notion of stability: weak, strong and super. In this paper we are considering maximum size weakly stable matching in I' of (2, ∞)-MAX SMTISS.

As we mention earlier, Irving et al. [16] shows that (3, 4)-MAX SMTI is NP-hard and not approximable within $\delta > 1$ unless P = NP. It follows that the complexity status of (3, ∞)-MAX SMTI is also NP-hard. Similarly socially stable variation of (3, ∞)-MAX SMTI problem, "(3, ∞)-MAX Weakly SMTISS" is also NP-hard.

Given an instance I' of (2, ∞)-MAX Weakly SMTISS (Stable Marriage problem with Ties and Incomplete bounded length preference list under Social Stability), we present an algorithm that gives a maximum size weakly socially stable matching with time complexity $O(n^{3/2} \log n)$, where n is the total number of men and women in the instance I.

4. STABLE MARRIAGE PROBLEM WITH TIES AND INCOMPLETE BOUNDED LIST UNDER SOCIAL STABILITY (SMTISS)

An instance of Stable Marriage Problem with Ties and Incomplete bounded list under Social Stability (SMTISS) can be defined by (I, G) where I is the instance of SMTI and $G = (U \cup W, A)$, where A (the set of all acceptable pairs). A man m_i and a woman w_j are called *socially connected* to each other in graph G if $(m_i, w_j) \in A$. Each preference list is a partial order on a subset of opposite sex. A matching M is called *weakly socially stable* if there is no socially blocking pair. A pair $(m_i, w_j) \notin M$ is a socially blocking pair if (a) $(m_i, w_j) \in A$ and (b) m_i is either unmatched or strictly prefers w_j to his partner in M and (c) w_j is either unmatched or strictly prefers m_i to her partner in M. In general, for any instance I of SMTISS problem, one of the aim is to compute a maximum cardinality socially stable matching (weakly, strong, super etc). In an incomplete tied preference list, arbitrary breaking of ties need not always lead to a maximum weakly socially stable matching. The following example shows that if we break ties arbitrarily we can find weakly socially stable matching of different sizes.

Example: Men's preference lists Women's preference lists

Men's preference lists	Women's preference lists
m_1: ($\underline{w_1}$, w_2)	w_1: $\underline{m_1}$, m_2
m_2: w_1	w_2: m_1

In above example the underline shows a social connection in G. Here man m_1 has a social connection with woman w_1. Observe that if we break the tie of m_1 as $m_1 : \underline{w_1}, w_2$ then maximum weakly socially stable matching will be $\{(m_1, w_1)\}$ of size 1 and if we break tie of m_1 as $m_1 : w_2, \underline{w_1}$ then maximum weakly socially stable matching will be $\{(m_1, w_2), (m_2, w_1)\}$ of size 2. The above example motivates us to find maximum cardinality weakly socially stable matching in an instance I of SMTISS.

Observe that if we restrict the length of all ties equal to 1 in an instance I of SMTISS then it will reduce into an instance I'' of SMISS. Since it is known that finding a maximum cardinality socially stable matching in an instance of SMISS is NP-complete [17], finding a maximum cardinality weakly socially stable matching in an instance of SMTISS is also NP-complete. Askalidis et al. showed that the problem (2, ∞)-MAX SMISS ((2, ∞)-MAX SMTISS with ties length 1) is solvable in polynomial time [17], this result directed us to a more general version called (2, ∞)-MAX Weakly SMTISS problem where ties length could be two or more. It may seem that one can consider that if we break the ties arbitrary and apply (2, ∞)-Max SMISS algorithm then we can find maximum cardinality weakly socially stable matching for (2, ∞)-SMTISS instance, but this is not always true. We can verify this by above example.

4.1. ALGORITHM FOR (2, ∞)-MAX WEAKLY SMTISS

The objective of this problem is to find a maximum cardinality weakly socially stable matching in SMTI instance under social stability, where each man can give a preference list of length at most two and each woman can give unbounded length incomplete list, with or without ties of any length. We present an $O(n^{3/2} \log n)$ time algorithm for this problem. Similar to (2, ∞)-MAX

SMISS given in [17], this algorithm also completes in three phases. In phase 1 we delete all pairs which can never belong to any weakly socially stable matching. The intuition behind phase 1 is, if there is a man m_i who is socially connected to his first choice woman w_j then any man who is less preferable than m_i in w_j preference list, cannot match with w_j in any socially stable matching. If it happens, (m_i, w_j) will be blocking pair for resultant socially stable matching M.

In phase 2, first we build a graph from the reduced instance from phase 1 and weight each edge (m_i, w_j) by $rank(w_j, m_i)$, where $rank(w_j, m_i)$ is rank of man m_i in w_j's reduced preference list. Now we construct a minimum weight maximum matching M_G in graph. Finally, in phase 3 we settle those pairs which are matched in phase 2 but will be socially blocking pair for output matching M.

Lemma 4.1.1. (2, ∞)-MAX weakly SMTISS algorithm terminates.

Proof. We start phase 1 by unmarking all men. Now we mark those men who are unmarked and have a non-empty reduced list. When every man becomes either marked or having an empty reduced preference list, phase 1 will terminate. Since a man m_i can be marked at most twice during phase 1 and total number of men in instance (I, G) is finite, phase 1 will terminate. In phase 2 of algorithm we are finding a minimum weight maximum matching of the reduced instance, therefore phase 2 will also terminate. In phase 3, each iteration improves the choice of a man from his second choice woman to his first choice woman and no man obtains worse woman or becomes unmatched. Since total number of possible improvements for men is finite, therefore the total number of iterations is also finite and hence phase 3 will also terminate.

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/ Phase 1 /
Set all men to be unmarked;
while some man m_i is unmarked and m_i has a non-empty reduced list do
    Set m_i to be marked;
    if m_i's reduced list is not a tie of length 2 then
        w_j := woman in first position on m_i's reduced list;
        if (m_i,w_j) ∈ A then
            for each successor m_k of m_i on w_j's list do
                Set m_k to be unmarked ;
                Delete pair (m_k,w_j);

/ Phase 2 /
G = Build Graph();
M_G = Minimum weight maximum matching in G;

/ Phase 3 /
M = M_G;
while (there exists a man m_i who is assigned to his second choice woman w_k and his first choice woman w_j is free in M and (m_i,w_j) ∈ A) do
    M = M \ {(m_i, w_k)};
    M = M ∪ {(m_i, w_j)};
Return M;

Build Graph();
V = U ∪ W;
E = φ;
for each man m_i ∈ U do
    for each woman w_j on m_i reduced list do
        E = E ∪ (m_i,w_j);
        weight(m_i, w_j) = rank(w_j, m_i);
G = (V, E);
Return G;
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Figure 1. (2, ∞)-MAX Weakly SMTISS Algorithm

Lemma 4.1.2. Phase 1 of (2, ∞)-MAX Weakly SMTISS Algorithm never deletes a weakly socially stable pair.

Proof. Suppose (m_i, w_j) is a weakly socially stable pair which has been deleted during execution of phase 1 of algorithm 1 such that $(m_i, w_j) \in M$, where M is a weakly socially stable matching in (I, G). Suppose this is the first weakly stable pair deleted during phase 1. This deletion was done because of w_j being the first choice of some man m_r where $(m_r, w_j) \in A$, m_r*'s* reduced list was not a tie of length 2 and w_j prefers m_r to m_i. But in that case pair (m_r, w_j) becomes a social blocking pair with respect to matching M. This is a contradiction to the fact that M is a weakly socially stable matching.

Lemma 4.1.3. The matching returned by algorithm (2, ∞)-MAX weakly SMTISS is weakly socially stable in (I, G).

Proof. Suppose our algorithm outputs the matching M and this matching is not weakly socially stable in (I, G). It means there exist a pair (m_i, w_j) which is a socially blocking pair with respect to M. We can consider following four cases corresponding to a socially blocking pair.

Case (i) both m_i and w_j are unmatched in M:

We know once a man m_i is matched in M_G, he will never be unmatched in phase 3. Either m_i remains with his partner in M_G or he can improve his partner (if possible) during phase 3. Therefore, if a man m_i is unmatched in M, he was unmatched in M_G. Woman w_j can either be unmatched in M_G or during phase 3. In first case, suppose a woman w_j is unmatched in M_G, then we can increase the size of matching M_G by adding the edge (m_i, w_j), which contradicts the fact that matching M_G is a maximum matching. In second case, suppose a woman w_j becomes unmatched during phase 3, then it means that her partner in M_G, say m_{p1} , had a strict preference list of length 2 and got his first choice woman, say w_{q1} , where $(m_{p1}, w_{q1}) \in A$. Now again we have two cases for w_{q1}. In the first case, woman w_{q1} is unmatched in M_G. This leads to an augmenting path $\{(m_i, w_j), (m_{p1}, w_j), (m_{p1}, w_{q1})\}$ in M_G, where the first and the last edges are not in M_G. So we can increase the size of the matching M_G by one. This is a contradiction to the fact that M_G is maximum matching. In second case, suppose w_{q1} becomes unmatched during phase 3, then it means her partner in M_G, say m_{p2}, had a strict preference list of size 2 and got his first choice woman w_{q2}, where m_{p2} and w_{q2} had a social connection in (I, G). Again we can observe an augmenting path $\{(m_i, w_j), (m_{p1}, w_j), (m_{p1}, w_{q1}) (m_{p2}, w_{q1}), (m_{p2}, w_{q2})\}$ in M_G which contradicts the fact that M_G is maximum matching in (I, G). Similarly, if we keep on doing this operation, number of men is finite and since every man strictly improves his partner in M_G, there exist a finite number of women who can become unmatched after phase 3. Hence at some time there exists a man m_{pr}, who is matched with $w_{q_{r-1}}$ in M_G, and switched to his first choice w_{qr} and w_{qr} is unmatched in M_G. Here we can form an augmenting path $\{(m_i, w_j), (m_{p1}, w_j), (m_{p1}, w_{q1}), (m_{p2}, w_{q1}), (m_{p2}, w_{q2}), \ldots, (m_{pr}, w_{qr})\}$, which leads to a contradiction that M_G is a maximum matching.

Case (ii) m_i is unmatched in M and w_j prefers m_i to $M(w_j)$:

As explained before m_i is unmatched in M_G. Woman w_j is either matched to $M(w_j)$ in M_G or matched to some m_{p1} in M_G and after that matched to $M(w_j)$ in phase 3. In first case, if w_j is matched to $M(w_j)$ in M_G then it leads to contradiction that M_G is a minimum weight maximum matching. We can simply discard the edge $(M(w_j),w_j)$ from M_G and add edge (m_i,w_j), which reduces the weight of M_G without reducing its cardinality. In second case, w_j is matched to some m_{p1} in M_G, where $m_{p1} \neq m_i \neq M(w_j)$ and after that matched to $M(w_j)$ in phase 3. Now, after phase 3, w_j is not matched with m_{p1}, which means that m_{p1} got his first choice woman say w_{q1} in phase 3. Now woman w_{q1} is either free or matched to some man in M_G. In both cases, using similar arguments as in Case (i) we can construct an augmenting path and contradict that M_G is a maximum matching.

Case (iii) m_i is matched to $M(m_i)$ in M, m_i prefers w_j to $M(m_i)$ and w_j is unmatched in M:
We know that man m_i has a strict preference list of size 2. It follows that w_j is the first woman of m_i's preference list. Since (m_i, w_j) is an edge in social graph G, this satisfies the loop condition of phase 3 and m_i will be matched to w_j during phase 3. Therefore this case will never occur after execution of this algorithm.

Case (iv) m_i is matched with $w_k = M(m_i)$, and m_i prefers w_j to w_k and w_j is assigned to $m_r = M(w_j)$ and w_j prefers m_i to m_r:
We know that length of preference list of m_i is 2 and m_i is in social connection with w_j. m_i strictly prefers w_j to w_k, which means that w_j is first choice of m_i. Woman w_j strictly prefers m_i to m_r. Therefore the loop condition of phase 1 will be true and phase 1 will delete the pair (m_r, w_j). Hence this case will never occur in our algorithm.

Since by lemma 1 we know that phase 1 of algorithm never deletes a socially stable pair, in phase 2 we constructed a minimum weight maximum matching M_G from the reduced preference list by phase 1 using algorithm in [18]. During phase 3 we never decrease the size of M_G, which follows that resultant matching M after phase 3 is a maximum cardinality matching. Lemma 3 ensures that the matching produced by (2, ∞)-MAX weakly SMTISS algorithm is weakly socially stable. It follows that the algorithm produced a maximum weakly socially stable matching in instance (I, G). The running time complexity of the algorithm is dominated by phase 2 which constructs a minimum weight maximum matching in $G'(V, E')$ in time $O(\sqrt{|V|}\,|E'|\,log|V|)$ [18]. Suppose $|V| = n = n_1 + n_2$ is total number of men and women, then the cardinality of set of acceptable pairs is at most $2n_1 = O(n)$. It follows that the time complexity of (2, ∞)-MAX weakly SMTISS algorithm is $O(n^{3/2} \log n)$.

Theorem 4.1. For a given instance (I, G) of (2, ∞)-MAX Weakly SMTISS, Algorithm (2, ∞)-MAX weakly SMTISS produces a maximum size weakly socially stable matching in $O(n^{3/2} \log n)$ time, where n is the total number of men and women in I.

5. Conclusion and Future Work

In this paper we have presented an algorithm for an instance of stable marriage problem with ties and incomplete bounded length preference list under social stability, where each man can give at most 2 women in his preference list (with or without ties) and each woman can give unbounded length preference list (with or without ties). Length of ties in women preference list could be 2 or more. We have found that this instance can be solved in polynomial time and gave an algorithm having complexity $O(n^{3/2} \log n)$ time, where n is the total number of men and women in I. These instances are very common in real world scenario like NRMP and Scottish medical matching scheme where medical students can give small size preference list. It would be interesting to study about maximum size strongly stable matching and super stable matching in the scenario of social stability. It would also be interesting to find a parameterized algorithm for a larger instance. We are leaving these problems as open problems.

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