MATCHING STRUCTURE AND THE EVOLUTION OF COOPERATION IN THE PRISONER'S DILEMMA

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ABSTRACT

With the help of both an experiment and analytical techniques, Axelrod and Hamilton [1] showed that cooperation can evolve in a Prisoner's Dilemma game when pairs of individuals interact repeatedly. They also demonstrated that, when pairing of individual is not completely random, cooperating behaviour can evolve in a world initially dominated by defectors. This result leads us to address the following question: Since non-random pairing is a powerful mechanism for the promotion of cooperation in a repeated Prisoner's Dilemma game, can this mechanism also promote the evolution of cooperation in a non-repeated version of the game? Computer simulations are used to study the relation between non-random pairing and the maintenance of cooperative behaviour under evolutionary dynamics. We conclude that non-random pairing can secure cooperation also when the possibility of repeated interaction among the same pairs of individuals is ruled out.

KEYWORDS

Prisoner's dilemma, cooperation, evolution, non-random pairing

1. INTRODUCTION

In their influential contributions to the evolution of cooperation Axelrod and Hamilton [1] showed that cooperation is the most likely evolutionary outcome of a Prisoner's Dilemma game when pairs of individuals interact repeatedly. Their model goes roughly as follows: Suppose that pairs of individuals interact repeatedly over time and that each member of a pair has the opportunity to provide a benefit to the other at a cost to himself by cooperating. Now consider a population of Tit-for-Tatters who cooperate on the first interaction and keep on cooperating only as long as their partner cooperates. Axelrod and Hamilton showed that Tit-for-Tatters can resist invasion by defectors who never cooperate as long as the long-run benefit of mutual cooperation is greater than the shortrun benefit that a defector gets by exploiting a co-operator. This widely cited result has inspired much theoretical work on the problem of cooperation.

As discussed by Axelrod and Hamilton [1], a population of Tit-for-Tatters is not, however, the only one that is evolutionary stable. In fact, a population where all are defectors is also evolutionary stable. If (almost) all players in a population are defectors, a co-operator will have no one to cooperate with. Therefore, a player cannot do any better than playing defect. The long-run benefit associated with sustained cooperation becomes irrelevant. This raises the problem

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concerning initiation of cooperation from a previous asocial state. How could an evolutionary trend towards cooperative behaviour have started in the first place?

In order to study this question more closely Axelrod and Hamilton [1] introduced the concept of segregation (or clustering as they name it). Segregated interaction means that the probability for a Tit-for-Tatter to meet another Tit-for-Tatter is higher than the proportion of Tit-for-Tatters in the population. They demonstrated that if there are few Tit-for-Tatters in the population, and if the long-run benefit of cooperation is big, only a small amount of segregation is needed in order to secure Tit-for-Tatters a higher expected payoff than defectors. An evolutionary trend towards universal cooperation can then start.

The results established by Axelrod and Hamilton are generated within a setup where pairs of individuals interact repeatedly over time, and where everybody is able to remember the action taken by each member of the population in previous interactions. However, in many human social environments, Axelrod and Hamilton's conditions favouring cooperation can be questioned. Individuals do not always interact repeatedly over long periods of time, and in large groups it can be difficult to remember the action taken by a potential exchange partner in previously interactions. This leads us to the main question of this paper: Since segregation is a powerful mechanism for the promotion of cooperation when pairs of individuals meet repeatedly, can segregation also promote the evolution of cooperation when individuals are matched to play a one-shot Prisoner's Dilemma game? If so, how much segregation is needed, and how does cooperative behaviour evolve over time depending on the degree of segregation?

The idea that cooperative behaviour can survive under evolutionary competition with defecting behaviour, when interaction is not random, goes back at least to Hamilton [2] and Eshel [3], and has received attention more recently by Wilson and Dugatkin [4], Bowles [5], Nowak [6] and Mengel [7] among others. Although these papers differ in how the problem of cooperation is formulated, they reach qualitatively the same result: Non-random pairing means that cooperating players are more likely to meet other cooperating players, while defectors are more likely to meet other defectors. This gives rise to cooperative and non-cooperative clusters, with individuals in cooperative clusters earning higher payoffs on average than individuals in non-cooperative clusters.

The work most closely related to ours is Boyd and Richerson [8]. As the present paper, their model is designed to be as close as possible to Axelrod and Hamilton [1]. But while Axelrod and Hamilton's paper deals with repeated interaction among pairs of individuals, Boyd and Richerson [8] extend the analysis to groups of people repeatedly interacting in an *n* person Prisoner's Dilemma. They show that when group size rises, cooperation becomes a much less likely evolutionary outcome. However, when they allow for some segregated interaction, coupled with the possibility of repeated interaction, cooperation evolves in situations in which neither factor alone would cause cooperation. In this paper we model segregation in exactly the same way as Boyd and Richerson [8], but we but do not allow for repeated interaction among the same pairs of individuals. Instead we assume that players are repeatedly matched to play a one-shot interaction Prisoners Dilemma.

After addressing the problem of cooperation within the framework of the Prisoner's Dilemma in the next section, we briefly review the model by Axelrod and Hamilton [1] in Section 3. In Section 4 we use the same set-up as Axelrod and Hamilton to analyse the evolution of cooperation when the possibility of repeated interaction and reciprocity is ruled out. An index of segregation is defined that captures the idea that co-operators are more likely to meet co-operators than are defectors. Section 5 gives a description of the algorithm used in the simulation of the non-repeated and non-random version of the Axelrod and Hamilton model. Section 6 presents the results from the simulation. Section 7 concludes the paper.

2. THE PROBLEM OF COOPERATION

Consider a large population of players who interact in pairs with available actions and payoffs describing a Prisoner's Dilemma game. We have the following payoff matrix, where $a > b > c > d$.If both players cooperate, they both receive a payoff of b. If both defect, they both receive payoffs of c. If one cooperates and the other defects, the co-operator receives a payoff of d, while the defector does very well with a payoff of a.

	Cooperate	Defect
Cooperate	b , b	d , a
Defect	a , d	c , c

Figure 1. The Prisoner's Dilemma

Assume further that individuals in the larger population are either (perhaps due to cultural experiences, perhaps due to genes) co-operators (C) or defectors (D) in a single period Prisoner's Dilemma. Let p denote the proportion of the population that are co-operators and $(1-p)$ the proportion of defectors. If the members of the population are randomly paired, the expected payoffs are given by

(1) $V(C) = pb + (1-p)d$
(2) $V(D) = pa + (1-p)c$

where $V(C)$ and $V(D)$ are the expected payoff for a co-operator and a defector respectively. Equation (1) says that with probability p a co-operator is paired with another co-operator producing a payoff b, and with probability $(1-p)$ is paired with a defector producing a payoff d. Equation (2) has a similar interpretation:With probability p a defector is paired with a co-operator producing a payoff a, and with probability $(1-p)$ is paired with a another defector producing a payoff c.

Assume now the following simple evolutionary dynamics: At any time, the growth rate of the proportion of co-operators (p) is positive or negative, depending on whether the expected payoff for co-operators is higher or lower than the expected payoff for defectors. The population distribution p will be unchanging, producing an equilibrium, if

(3) $V(C) = V(D)$

It is easy to see from (1) and (2) that the only evolutionary stable equilibrium is $p = 0$, where all members of the population defects. This result follows from the fact that $a > b$ and $c > d$, which gives $V(C) < V(D)$ for all $p \in (0,1)$. Co-operators cooperate irrespective of the type of player whom they meet. Defectors take advantage of such indiscriminate cooperative behaviour and get a higher expected payoff compared to co-operators. Defectors increase in numbers and in the long run take over the whole population. This result motivated Axelrod and Hamilton to examine more closely conditions, not captured in the situation just studied, that can lead to the evolution of cooperation when co-operators and defectors meet to play the Prisoner's Dilemma.

3. THE AXELROD AND HAMILTON MODEL

The main idea in the work by Axelrod and Hamilton [1] can be formalized in the following way, where we build on Bowles [9]. Let us allow the Prisoner's Dilemma game introduced above to be repeated with an unknown number of rounds. More precisely, after each round there is a probability β that another round will be played. Hence, the expected number of rounds is $1/(1-\beta)$. Assume also that the population consists of two types of players; unconditional defectors and conditional co-operators. The unconditional defectors always defect, while the conditional co-operators are endowed with the Tit-for-Tat strategy. The Tit-for-Tat strategy dictate co-operators to cooperate on the first round and on all subsequent rounds do what the partner did on the previous round. The fraction of the population adopting Tit-fot-Tat is p, while the remaining is adopting unconditional Defect. The expected payoff for co-operators adopting Tit-for-Tat and defectors respectively are then

$$(4) \qquad V(C) = p\left(\frac{b}{1-\beta}\right) + (1-p)\left(d + \frac{c\beta}{1-\beta}\right)$$

$$(5) \qquad V(D) = p\left(a + \frac{c\beta}{1-\beta}\right) + (1-p)\left(\frac{c}{1-\beta}\right)$$

Equation (4) says that when two Tit-for-Tatters meet, they will both cooperate on the first interaction and then continue to do so until the interaction is terminated, giving an expected payoff of $b/(1-\beta)$. When a Tit-for-Tatter meets a defector, the former gets d on the first interaction while the defector gets a.Then both will defect until the game terminates, the expected number of iterations after the first round being $(1/(1-\beta)) - 1 = \beta/(1-\beta)$. Equation (5) has a similar interpretation.

According to (3) the condition for equilibrium is $V(C) = V(D)$, giving

$$(6) \qquad p = \frac{c-d}{\frac{b-c\beta}{1-\beta} - a + c - d} \equiv p^*$$

Since $c > d$, $p^* \in (0,1)$ if

$$(7) \qquad \beta > \frac{a-b}{a-c} \equiv \beta^*$$

Where $\beta^* \in (0,1)$ since $b > c$. If (7) holds, we have an interior unstable evolutionary equilibrium. That is, $V(C) = V(D)$ for $p^* \in (0,1)$. This situation can be explained as follows: Suppose that the initial proportion of co-operators is lower than p^*. Rare co-operators are likely to be paired with defectors, producing a low payoff for co-operators (i.e. $V(C) < V(D)$). The proportion of co-operators then decreases until $p = 0$. If, however, the initial proportion of co-operators is higher than p^*, co-operators are likely to be paired with other co-operators, producing a high payoff for co-operators (i.e. $V(C) > V(D)$). The proportion of co-operators then increases until $p = 1$. Hence, we can draw the following conclusion: *In a population where defecting behaviour is not too common* $(p > p^*)$, *the cooperating Tit-for-Tat strategy leads to universal cooperation if pairs of individual are likely to interact many times* $(\beta > \beta^*)$.

However, even if β is high (higher than β^*) we still need a certain proportion of Tit-for-Tatter in order to start a process where Tit-for-Tatters increase in numbers. This illustrates that the model fails to answer what many consider as the most fundamental problem related to the evolution of cooperation: How could cooperation ever have started from a previous asocial state where (almost) all are defectors? To solve this puzzle Axelrod and Hamilton introduce the concept of segregation (or clustering as they name it). When there is some segregated interaction, Tit-for-Tatters are more likely paired with each other than chance alone would dictate. If the long-run benefit of cooperation is big, even a small amount of segregation can cause the expected payoff of Tit-for-Tatters to exceed the expected payoff of defectors. An evolutionary trend towards universal cooperation can then get started.

4. NON-RANDOM MATCHING

As noted, a main result in the work by Axelrod and Hamilton is that segregation can be very effective for the evolution of cooperation in a repeated Prisoner's Dilemma game. But what about the non-repeated version of the game? Can segregation also promote the evolution of cooperation when the players meet to play a series of one-shot Prisoner's Dilemma games, that is when $\beta = 0$? It is immediately clear that complete segregation of co-operators and defectors within a large population secure cooperation. Complete segregation means that co-operators always meet co-operators, and defectors always meet defectors. Co-operators get a payoff of b, while defectors get c. Since $b > c$ co-operating behaviour will spread, and in the long run take over the whole population.

The case where co-operators and defectors are only partly segregated can be modelled by using the following formulation, adopted from Boyd and Richerson [8]. Let $r \in (0,1)$ be a measure of the degree of segregation in the population. If p is the proportion of co-operators, the probability that a co-operator meets another co-operator is now $r + (1 - r)p$, while the probability that a defector meets another defector is $r + (1 - r)(1 - p)$. When $r \in (0,1)$, this corresponds to a situation with non-random pairing. If $r = 1$, we have complete segregation, implying that co-operators never interact with defectors. If $r = 0$, we are back to the situation with random matching, analysed in Section 2.

In the simulation we are interested in analysing the situation where the segregation parameter (r) lies between the two extreme cases, $r = 0$ and $r = 1$, giving rise to a situation with non-random pairing. In particular we are interesting in finding out how small r can be in order to support an evolutionary stable proportionof co-operators. In addition to r, it is clear that the expected payoffs are influenced also by the proportion co-operators (p) and defectors ($1 - p$) in the population. In the simulation we therefore have to vary both the segregation parameter and the initial proportion of co-operators and defectors in the population. This makes it possible to study how different combinations of r and p affect the evolution of co-operators and defectors.

5. THE SIMULATION

As a simulation model, we use an agent-based simulation approach in which an agent represents a player with a predefined strategy. The basic activity of the agent is to play a series of one-shot Prisoner's Dilemma (PD) games. Each agent is identified using a unique label. The label C is used to identify the agents choosing the cooperative strategy, while the label D is used for those choosing the defective strategy. Each agent's label can be viewed as a mapping from one state of the game to a new state in the next round, and the simulation experiments searches for the ability of an agent to survive the evolution process.

The simulation of the game is outlined in Algorithm 1, and its components are described as follows. Initially, a population of agents is generated. A user defined parameter will determine the percentage of agents playing the cooperative strategy against those playing the defective strategy.

The payoff of all agents is set to 0. The next step of the algorithm proceeds by pairing off agents to play one game of PD. This step can be viewed as a matching process.

To begin with, a random number *random* is drawn uniformly on the interval $(0,1)$. Thereafter, an $agent_k$ is drawn randomly from the set of unmatched agents. If $agent_k$ is assigned the label C, then the matching scheme will select a randomly unmatched agent with the label C provided the following inequality $(random < r + (1 - r)p_C)$ holds, otherwise the matching mate of $agent_k$ will be a randomly chosen unmatched agent with the label D. The value of p_C represents the proportion of agents playing the cooperative strategy. On the other hand, if $agent_k$ is assigned the label D, then its matching mate will be chosen with the label Dprovided the inequality $(random < r + (1 - r)(1 - p_C))$ holds, otherwise the matching mate of $agent_k$ will be selected with the label C. If, by chance, the matching scheme is unable to locate the matching mate with the required label, the $agent_k$ will be left unmatched. At the endof each tournament, the agents of the current population P_t are transformed into a new population P_{t+1} that engages in a new round of PD based on each agent's payoff. In the simulation we use the same payoff parameters as Axelrod and Hamilton [1]. These are shown in the payoff matrix in Figure 2.

	Cooperate	Defect
Cooperate	3 , 3	0 , 5
Defect	5 , 0	1 , 1

Figure 2 Numbers used in the simulation

The payoff received will determine whether an agent is removed from the game or allowed to continue. It is assumed that the size of the entire population stays fixed during the whole simulation process. All the unmatched agents from P_t will automatically be allowed to be part of the new population P_{t+1}.The agents that were engaged in the one-shot PD game are ranked according to their payoff from best to worse (i.e. sorting agents to decreasing payoff values) and those with the highest payoff will be allowed to proceed to the next round and multiplies by cloning a duplicate agent with similar strategy. Each agent resets its payoff to 0 before starting a new round of the PD game. The simulation process is assumed to have reached a stabilization of its convergence when all the agents have similar strategy.

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Algorithm:
$t \leftarrow 0$;
Initialize random population of agents in $P(t)$;
Set payoff to each agent in $P(t)$ to 0;
While (not stop) do
    Choose the mating pairs of Agents in $P(t)$;
    Compute the payoff of each agent in $P(t)$;
    Select the agents for inclusion in $P(t + 1)$;
    $t \leftarrow t + 1$;
    Cloning a duplicate agent for each selected agent in $P(t)$ with a payoff $> 0$;
    Set the payoff of each Agent in $P(t)$ to 0;
end
```

6. RESULTS

The simulation model has a number of user-defined parameters such as the segregation parameter, and the starting initial conditions (i.e., percentages of co-operators and defectors). We perform several simulations using instances defined by the 4-duplen, p_C, p_D, r, where n denotes the number of agents, p_C denotes the percentage of co-operators, p_D denotes the percentage of defectors, and r the segregation parameter. We set the number of agents to 1000. In order to obtain a more fair understanding of the simulation process, we vary the parameters r from 0.1 to 0.9 with a step size of 0.1, and p_C from 10% to 90% with a step size of 10. This strategy produces 81 different pairs of r and p_C. Because of the stochastic nature of the simulation process, we let each simulation do 100 independent runs, each run with a different random seed. In this way every result we present is averaged over 100 runs. The simulation process ends when the population of agents converges to either 100% co-operatorsor100% defectors, or a maximum of 106 generations have been performed.

As a benchmark we first conduct an experiment using $p_C = 90, p_D = 10$ and setting the segregation parameter r to 0. Figure 3 shows one typical run of the simulation experiment. The course of the percentage function suggests an interesting feature which is the existence of two phases. The first phase starts with a steady decline of agents with the cooperative strategy over the first generations before it flattens off as we mount the plateau, marking the start of the second phase. The plateau spans a region where the percentage of *C*'s and *D*'s fluctuates around50%. The plateau is rather short and becomes less pronounced as the number of generation increases. Then the percentage of *C*'s start to decrease before finally it jumps to 0%. This example illustrates how agents tend to evolve strategies that increasingly defect in the absence of the segregation. This is not surprising. The agents evolve in a random environment, and therefore the agents that manage to survive the simulation process are those willing to always defect.

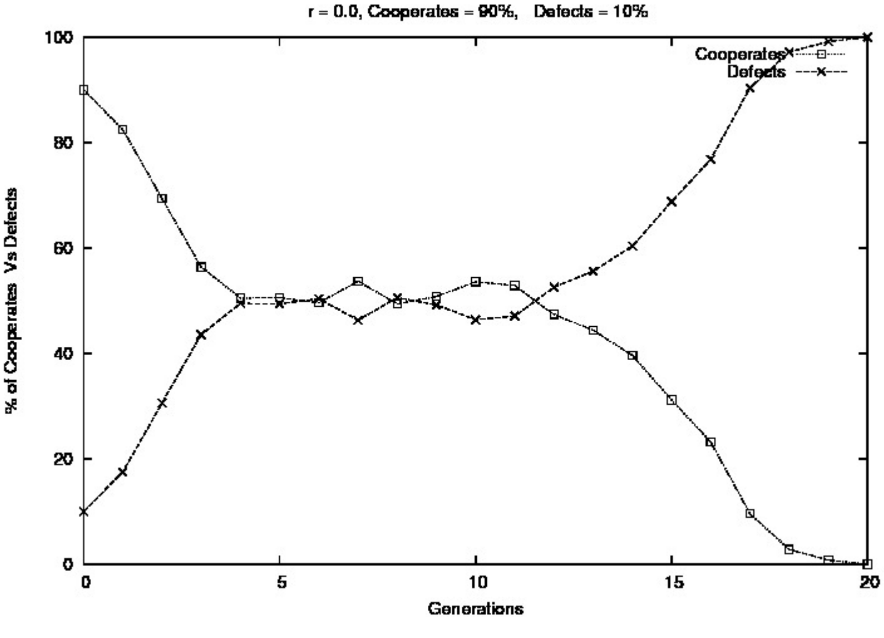

Figure 3. The evolutionary process with $r = 0$

Table 1 gives the results of the simulations with different values of r and p_C. A quick look at this table reveals the existence of three different regions. The first region, where the values of p_C and

r are low (upper left corner), shows that the result of the simulation converges globally to D's with a success ratio equal to 1. This region starts in the classes where $p_C = 10$ and $r = 0.4$, $p_C = 20$ and $r = 0.3$, and finally $p_C = 30$ and $r = 0.2$.The second region lies in the right corner where $r = 0.5$. In this region, the simulation converges to C's with a success ratio equals to 1 regardless of the starting percentage of co-operators. Finally, a third region which lies between the two other regions where, for every pair of p_C and r, the result of the simulation process includes a mixture of C's and D's. This region is marked with numbers in bold, where the numbers shows the final percentages of C's and D's.

p_C ↓		Segregation: r →								
		0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
10	C	0	0	0	0	1	1	1	1	1
	D	1	1	1	1	0	0	0	0	0
20	C	0	0	0	**0.86**	1	1	1	1	1
	D	1	1	1	**0.14**	0	0	0	0	0
30	C	0	0	**0.70**	**0.85**	1	1	1	1	1
	D	1	1	**0.30**	**0.15**	0	0	0	0	0
40	C	0	**0.65**	**0.72**	**0.86**	1	1	1	1	1
	D	1	**0.35**	**0.28**	**0.14**	0	0	0	0	0
50	C	**0.53**	**0.62**	**0.70**	**0.85**	1	1	1	1	1
	D	**0.47**	**0.38**	**0.30**	**0.15**	0	0	0	0	0
60	C	**0.56**	**0.64**	**0.71**	**0.84**	1	1	1	1	1
	D	**0.44**	**0.36**	**0.29**	**0.16**	0	0	0	0	0
70	C	**0.55**	**0.61**	**0.72**	**0.84**	1	1	1	1	1
	D	**0.45**	**0.39**	**0.28**	**0.16**	0	0	0	0	0
80	C	**0.56**	**0.62**	**0.69**	**0.85**	1	1	1	1	1
	D	**0.44**	**0.38**	**0.31**	**0.15**	0	0	0	0	0
90	C	**0.53**	**0.63**	**0.67**	**0.82**	1	1	1	1	1
	D	**0.47**	**0.64**	**0.33**	**0.18**	0	0	0	0	0

Table 1. Convergence ratios for co-operators and defectors

Figure 4-6 show the course of the simulation process regarding of the percentage of C's and D's in the three regions. Figure 4 shows a plot of the simulation process representing a case in the region where the convergence results always are in favour of the agents choosing the defect strategy. The result shows a rapid rise of D's before it reaches 100% at about the sixteenth generation. Choosing the values of r and p_C in this region prevent agents with the cooperative strategy to develop, leading to a random working environment where the agents with the defect strategy proliferate.

Figure 5 shows a plot of the simulation process representing a case in the phase transition with $r = 0.3$ and $p_C = 50\%$, where the convergence results always in a mix population of C's and D's. Notice the rapid increase in the percentage of C's and the rapid decline in the percentage of D' during the first generations. Both strategies reach a peak value at about 400 generations and periodically fluctuate between a low and a high percentage range and remain there indefinitely.

Finally, figure 6 shows a plot representing a case in the third region characterised by a convergence resulting always in favour of the agents choosing the cooperative strategy. The plot shows an upward trend in the percentage of C's and the possibility of having the chance to develop due to the right choice of the segregation parameter. Accordingly, in subsequent generations, the population of agents become increasingly dominated by C's.

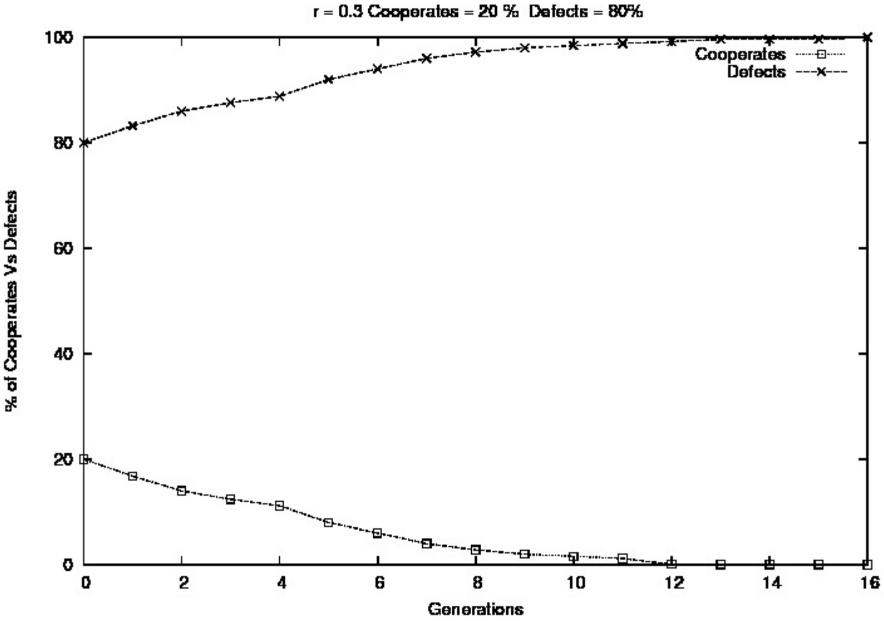

Figure 4. The evolutionary process with $r = 0.3$ and $p_C = 80\%$

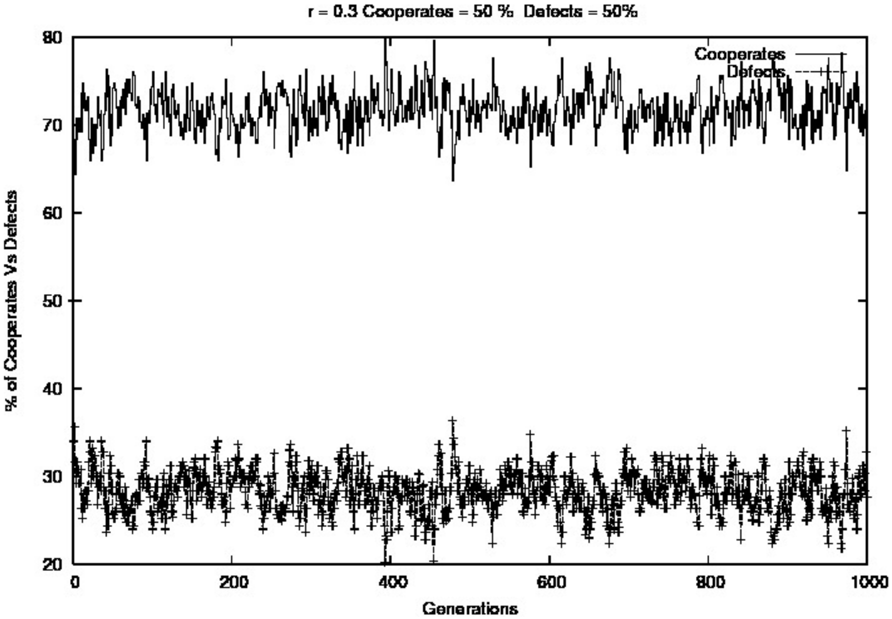

Figure 5. The evolutionary process with $r = 0.3$ and $p_C = 50\%$

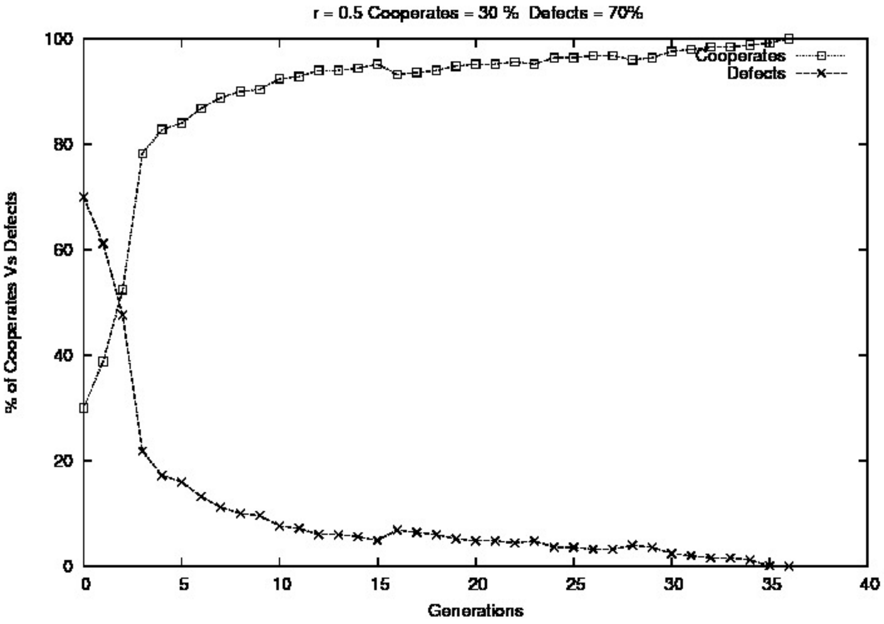

Figure 6. The evolutionary process with $r = 0.5$ and $p_C = 30\%$

Figure 7shows the impact of the segregation parameter on the complexity of the simulation. By keeping the percentage of co-operators constant while varying the segregation parameter, there exists a crossover point at which the number of generations needed to attain convergence increases. This turning point occurs at $r = 0.5$ and represents the problem instances that are harder to solve as the number of players increases.

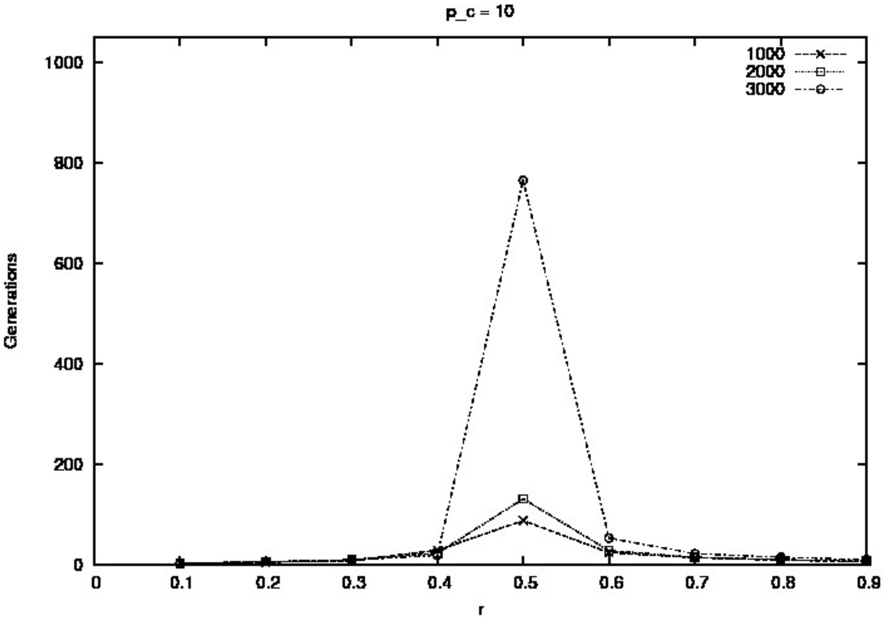

Figure 7. Scale-up test

7. CONCLUSION

In their influential contribution to the evolution of cooperation Axelrod and Hamilton introduced the concept of segregation (or clustering) to show how cooperation can evolve in a world initially dominated by defectors. The idea that segregation allows cooperative strategies to increase when rare in a Prisoner's Dilemma game is highly intuitive. Segregation means that co-operators clump together in a concentrated group which means that they are partly protected from defectors defecting behaviour. This leads to higher payoffs to co-operators compared to defectors, who tend to be surrounded by other defectors. The main part of Axelrod and Hamilton's paper deals, however, with the case of random pairing. In fact, most game-theoretic treatments of the problem of cooperation adopt the assumption of random pairing. This is somewhat strange since social interaction is hardly ever random. As discussed by Sober and Wilson [10] and Bowles [5], among others, non-random interaction constitutes an important aspect of our social architecture. In most societies there is a strong tendency that the population is structured in more or less homogeneous groups. Members of these groups interact more frequent with each other than with members of the society at large. Hence, since non-random pairing plays an important role in most social interaction, it should be taken into consideration when the evolution of behaviour and norms are analysed. The aim of this paper has been to contribute to this. We have shown that segregated interaction is a powerful mechanism for the evolution of cooperation in a Prisoner's Dilemma game, where co-operators interact with defectors. This conclusion holds even if we drop the possibility of repeated interaction and reciprocity, which was essential for the results generated in Axelrod and Hamilton's influential paper.

REFERENCES

[1] Axelrod, R., A.W. Hamilton, (1981)"The evolution of cooperation", Science, Vol.211, pp1390-1396.

[2] Hamilton, W.D. (1964) "The Genetic Evolution of Social Behaviour", Journal of Theoretical Biology, Vol.7, pp 1-52.

[3] Eshel, I. (1972)"On the Neighbourhood Effect and the Evolution of Altruistic Traits", Theoretical Population Biology, Vol. 3, pp 258-277.

[4] Wilson, D.S. and L.A. Dugatkin (1997)"Group Selection and Assortative Interactions", The American Naturalist, Vol. 149, No. 2, pp 336-351.

[5] Bowles, S. (2001) "Individual Interactions, Group Conflicts, and the Evolution of Preferences", in S.N. Durlauf and H.P. Young (eds.). Social Dynamics, Cambridge MA, The MIT Press.

[6] Nowak, M. (2006) "Five Rules for the Evolution of Cooperation", Science, Vol. 314, pp 1560-1563.

[7] Mengel, F. (2008) "Matching Structure and the Cultural Transmission of Social Norms", Journal of Economic Behaviour Organization, Vol. 67, pp 608-623.

[8] Boyd, R., P. Richerson (1988)"The evolution of Reciprocity in Sizable Groups", Journal of Theoretical Biology, Vol. 132, pp 337−356.

[9] Bowles, S. (2004) Microeconomics. Behaviour, Institutions, and Evolution, Princeton, Princeton University Press.

[10] Sober, E. and D.S. Wilson (1998) Unto Others: The Evolution and Psychology of Unselfish Behaviour, Cambridge MA, Harvard University.