

PERFORMANCE ANALYSIS OF PARALLEL POLLARD'S RHO FACTORING ALGORITHM

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ABSTRACT

Integer factorization is one of the vital algorithms discussed as a part of analysis of any black-box cipher suites where the cipher algorithm is based on number theory. The origin of the problem is from Discrete Logarithmic Problem which appears under the analysis of the cryptographic algorithms as seen by a cryptanalyst. The integer factorization algorithm poses a potential in computational science too, obtaining the factors of a very large number is challenging with a limited computing infrastructure. This paper analyses the Pollard's Rho heuristic with a varying input size to evaluate the performance under a multi-core environment and also to estimate the threshold for each computing infrastructure.

KEYWORDS

Pollard's Rho, Brent's Implementation, Monte-Carlo Algorithm, Integer Factorization, Discrete Log Problem.

1. INTRODUCTION

Cryptanalysis is vital study under cryptology that gives insight on strength of cryptographic algorithm; the base for such algorithm stands on the distinguishable properties like key size, number of rounds, chosen number and many more. Most of the encryption algorithm is based on Number theory aspects and strength of such algorithm purely depends on the size of the number. For instance, standard RSA algorithm implementation employs prime numbers which of 309 digits. Such algorithm gives a broad avenue for cryptanalysis study to be performed.

On the other hand from computation perspective, there is growing demand on the porting the legacy algorithm to state-of-the-art computation. This is to effectively use the optimal strength of the underlying computer architecture or computing infrastructure. A normal tendency of any algorithm is that they are either computation driven or data driven, for instance, a computation drive algorithm can be calculating a square root of number which is around 25 digits in size and data driven algorithm can be simple sorting algorithm sorting around 50,000 numbers. The former has to be addressed with multi-core architecture and latter has to be addressed by cluster computing methods.

Pollard's Rho Algorithm [6] is one such algorithm which would require computation driven solution that is well addressed under a multi-core architecture. As the number of digits in number

increases the more number of cores are required to factorize the number. The most important application of this is with Discrete Logarithmic Problem (DLP). In DLP, given two large prime numbers p and g , public key is calculated as follows –

$$y = g^x \text{ mod } p$$

Where x is kept private.

To calculate the x -

$$x = \log_g y \text{ mod } p$$

If a traditional technique such as the brute force is applied to find the value of x it would depend on the length of the prime factor. Hence a heuristic approach like the Pollard's Rho is applied to obtain the prime factors of x .

Earlier to the techniques of finding factor for a given number, the prominently employed algorithm was the Sieve of Eratosthenes [4]. Although the algorithm used an efficient data structure for performing the division by modulo N , it had serious flaws while operating with large numbers between 50 – 100 digits as memory key constraints for it. The algorithm is described below-

Algorithm: Sieve of Eratosthenes

Input: An integer n

Output: Returns an array of all prime numbers $\leq n$

1. $a[1] \leftarrow 0$
2. for $i \leftarrow 2$ to n do
3. $a[i] := 1$
4. $p \leftarrow 2$
5. while $p_2 < n$ do
6. $j \leftarrow p_2$
7. while $(j < n)$ do
8. $a[j] \leftarrow 0$
9. $j \leftarrow j+p$
10. repeat $p \leftarrow p+1$ until $a[p] = 1$
11. return(a)

The paper is organised in discuss more flavour of Pollard's Rho with respect to the required hardware implementation. The section -2 discusses in depth of the description of Pollard's Rho algorithm. The section -3 of the paper discusses about the design and implementation of the algorithm in multi-core environment. The section -4 discuss about the variety of results with the implementation.

2. POLLARD'S RHO ALGORITHM

Pollard's Rho algorithm [6,8] is a heuristic, i.e., the runningtime of this algorithm cannot be rigorously analysed. It is said to work very quickly when the number to be factorized has small factors, i.e., typically of the size of 10-12 digits. It is also very parallelizable [3]. This is, hence, our choice of algorithm for implementation. Algorithms such as trial division and sieve of Eratosthenes take a lot of time because they use the brute-force approach and are only suitable for finding factors of size 3-5 digits. The other integer factorization algorithms such as the General number field sieve and its open-source implementations (MSIEVE), compute factors of any size.

But these algorithms are preferred when it is known that the number to be factorized has large integers. This is because these algorithms take the same amount of time to find either large or small factors [1].

Description of Pollard's Rho Algorithm:

Algorithm: Pollard's Rho

Input: An integer n to be factorized, and a pseudo-random function f modulo n

Output: Factors of n

1. $i \leftarrow 1$
2. $x_1 \leftarrow f(0, n-1)$ while true
3. do $i \leftarrow i + 1$
4. $x_i \leftarrow f(0, n - 1)$
5. $d \leftarrow \gcd(|x_{i+1} - x_i|, n)$
6. if $d = 1$ or $d = n$ Output d

The greatest common divisor (GCD) in Line 5 of the Algorithm Pollard's Rho gives the greatest common divisor of two integers which have been passed as parameters. According to the birthday paradox [5], two numbers x and y (and hence $|x-y|$) are congruent modulo p with probability 0.5 after $(\theta \sqrt{p})$ numbers have been randomly chosen.

If p is a factor of n, then $p \leq \gcd(x - y, n) \leq n$. When the sequence of x_i start repeating after some iterations, this is detected using the Floyd's cycle detection algorithm and Pollard's Rho algorithm stops computation. This algorithm may or may not produce a factor of n. That is, either a correct factor or no factor is produced. This is why this algorithm falls under the class of Monte-Carlo methods.

The Floyd's detection algorithm [9] is terminating condition for the Pollard's Rho Algorithm, The description of the algorithm is given below-

Algorithm: Floyd's Cycle Detection

Input: A number $x_0 \in \{0 \dots p - 1\}$

Output: An index i such that $x_i = x_{2i}$ where $x_j = f(x_{j-1}) \quad \forall j \geq 1$

1. $i \leftarrow 1$ and $y_0 \leftarrow x_0$
2. Repeat
3. $x_i = f(x_{i-1})$ and $y_i = f(f(y_{i-1}))$
4. if $x_i = y_i$ then output i and stop
5. else $i \leftarrow i+1$

3. DESIGN AND IMPLEMENTATION

The pseudo-random number function used in Algorithm Pollard's Rho is of the form

$$x^2 + c$$

where 'c' is some integer other than -2 or 0.

Changing the c values in this function changes the numbers of iterations that the algorithm has to perform to produce a factor. For some c values, a factor may never be produced. The parallelization scheme performed is as follows.

1. Different 'c' values are used to create different random number functions.
2. The number n that is to be factorized and a random function is given to each CPU core available on the machine where this program is being run.
3. All the cores try to compute a factor, but because of the different c values, each core takes different amounts of time to compute a factor.
4. If one of the cores compute a factor, the computation on all the remaining cores are stopped.
5. The number n is divided by the computed factor to create another number n_0 .

In realistic application, input to the algorithm would deal with large integer number; for instance the size of integer number used is around 312 digits in RSA Algorithm[7]. This is beyond the storage capacity of any built-in datatypes of any programming language. Since the parallel implementation takes OpenMP as the programming paradigm, it is essential to find proper format /data structure to store such numbers. Hence GMP, an OpenSource version of GNU is employed to handle such large numbers. Now the size of the number may be restricted to available resources of the computing device. This number is given to all the cores with the old c values and the cores try to compute yet another factor of n. This process continues until all the prime factors of the given number are obtained.

However, implementations of Pollard's algorithm on CUDA also exist [2], but such implementations are dependent on the computing environment and cannot be distributed in cluster/grid environments.

4. RESULTS

The parallel version of Pollard's algorithm was run on one of the cores of a dual core machine, both cores of a dual core and four cores of quad core machine. The time taken to produce all factors in each case was measured (Table 4.1). The code was also run using only a single core, using 2 cores and then using all 4 cores of a single quad core machine measured the times taken to produce all factors in each case (Tables 4.2, 4.3 and 6.6). 5, 50 digit numbers, 5, 100 digit numbers and 5, 200 digit numbers.

To start with the five numbers of 50 digits are tested on the machines and the time required to compute the factors is captured in the table 4.1 –

Single Core	Dual Core	Quad Core
18.221s	11.162s	8.266s
24.605s	13.234s	9.800s
27.112s	15.334s	10.009s
21.499s	11.857s	8.459s
22.306s	13.706s	9.711s

Table 4.1: Time comparison of five 50 digit numbers on different cores machines

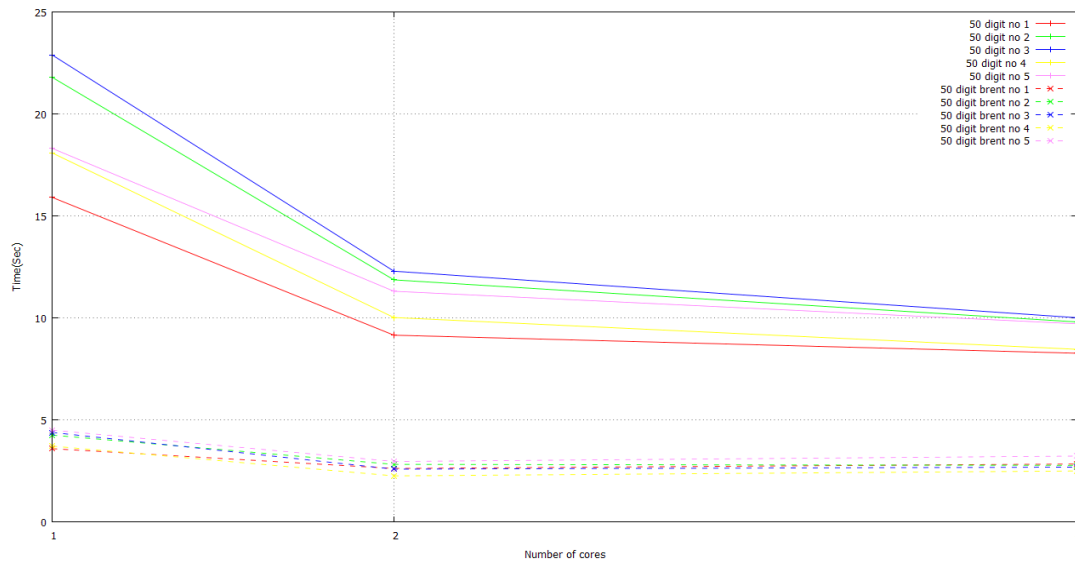


Figure 4.1: Graphical representation of 50 Digit numbers - Time Vs Cores

Testing of 100 digit numbers with different machine of variable cores, Table 4.2 captures the time taken to find factor along with graph in fig 4.2

Single Core	Dual Core	Quad Core
53.521s	28.343s	23.525s
48.313s	25.901s	22.703s
50.149s	26.824s	22.189s
58.799s	31.306s	25.408s
59.235s	31.234s	25.630s

Table 4.2: Time comparison of five 100 digit numbers on different cores machines

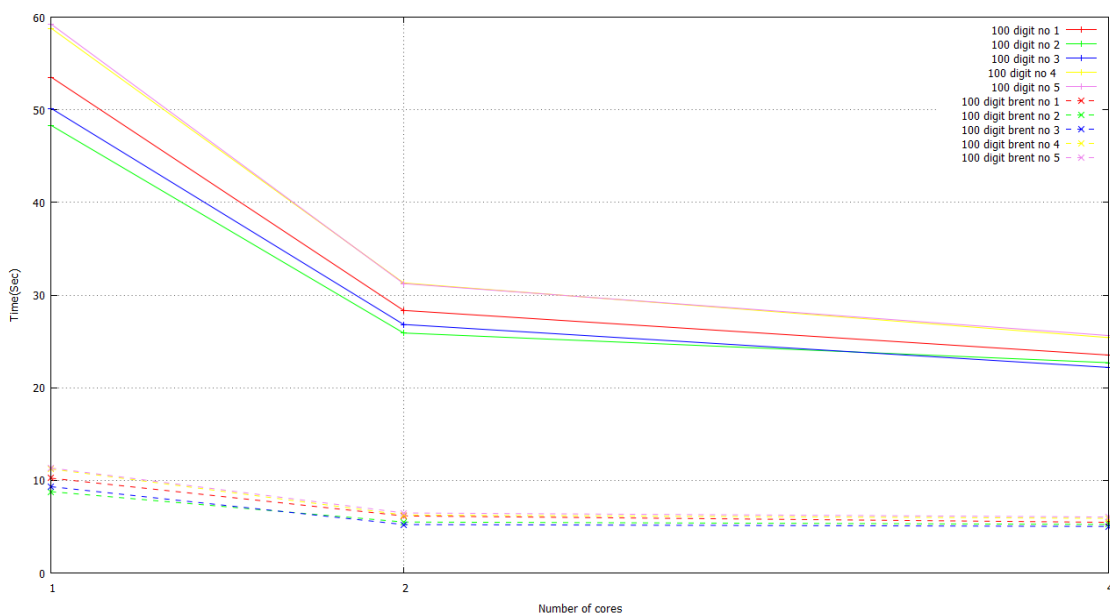


Figure 4.2: Graphical representation of 100 Digit numbers - Time Vs Cores

Further, the input number capacity is increased to 200 digits and the time required to find factor as per the our implementation is captured in table 4.3 with graphical representation in fig 4.3.

Single Core	Dual Core	Quad Core
137.006s	71.104s	52.327
136.315s	78.461s	64.412s
268.141s	139.403s	113.504
146.039s	93.481s	77.475
117.872s	74.880	63.116s

Table 4.3: Time comparison of five 200 digit numbers on different cores machines

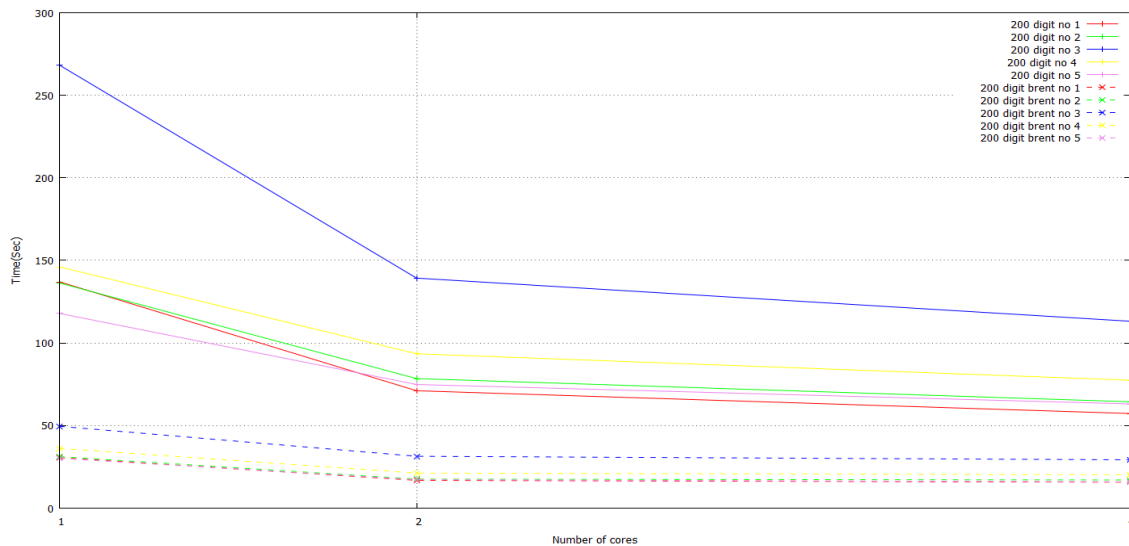


Figure 4.3: Graphical representation of five 200 Digit numbers - Time Vs Cores

To conclude this section of the paper we present an average time required by each machine to find factors when the number of digits is scaled. This is graphically captured in the fig 4.4.

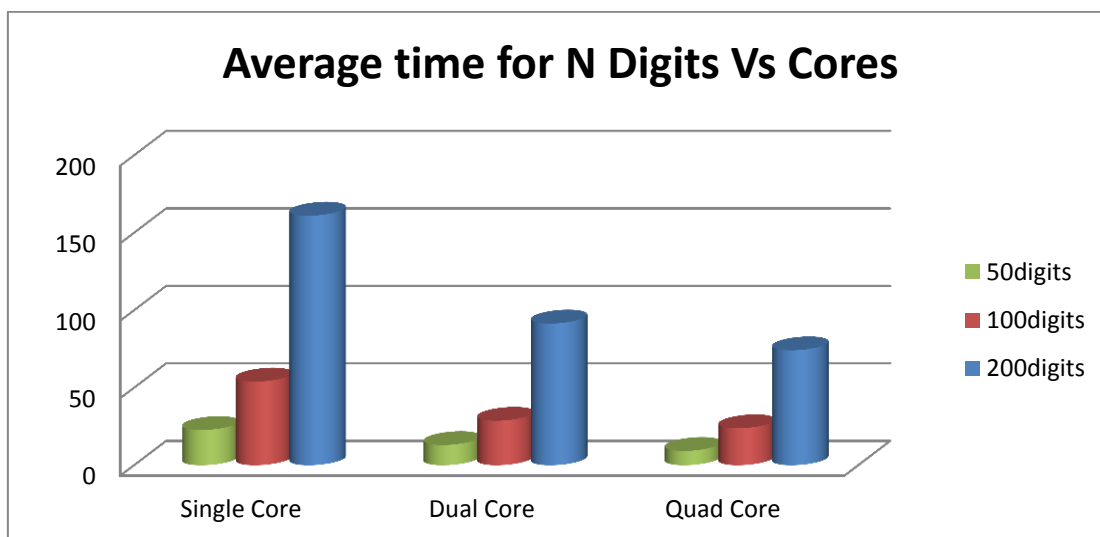


Figure 4.4: Graphical representation of Average Time Vs N-Digits Vs Cores

5. CONCLUSIONS

This paper presents one of the novel methods of parallelizing Pollard's Rho integer factorization and presents a coarse-grained parallelization in handling factorization computation and is based on the assumption and facts stated by Brent. This method of looking at parallel algorithm in general and Pollard's Rho algorithm in specific would give speedup of approximately three times when the number of cores are increased two folds. Every legacy algorithm has

unique way to parallelize and make them suit in the parallel environment. The art of parallelizing is not concrete and is dependent on the computing environment. Due to this aspect the comparison with other legacy algorithm in parallel version may mislead the results and the change the direction of research. Therefore there is immediate need to modify legacy algorithms to suit advanced computer architectures.

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