ADAPTIVE DESIGN OF CONTROLLER AND SYNCHRONIZER FOR LU-XIAO CHAOTIC SYSTEM WITH UNKNOWN PARAMETERS

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ABSTRACT

This paper establishes new results for the adaptive design of controller and synchronizer for the Lu-Xiao chaotic system (2012) with unknown parameters. First part of this paper involves the design of adaptive controller for the Lu-Xiao chaotic system to stabilize to its unstable equilibrium at the origin. The adaptive controller design is carried out using Lyapunov stability theory and adaptive control theory. The second part of this paper involves the design of adaptive synchronizer for identical Lu-Xiao chaotic systems with unknown parameters. The adaptive synchronizer design is carried out using Lyapunov stability theory and adaptive control theory. Numerical simulations using MATLAB have been shown to depict and validate the adaptive design of controller and synchronizer for the Lu-Xiao chaotic system with unknown parameters.

KEYWORDS

Adaptive Control, Adaptive Design, Adaptive Synchronization, Chaos, Lu-Xiao System.

1. INTRODUCTION

Since the observation of chaos phenomenon in weather models by Lorenz ([1], 1963), chaos theory has been received great attention in the nonlinear systems literature. Chaos theory finds applications in many areas in science and engineering such as physical systems [2], chemical systems [3], ecology [4], biology [5], secure communications [6-7], robotics [8], etc.

Control and synchronization of chaotic systems are important research problems with potential applications in many fields. By the control of a chaotic system, we mean the problem of finding a state feedback control law to stabilize a chaotic system around its unstable equilibrium [9-10].
By the synchronization of chaotic systems, we mean the problem of finding a control law attached to the slave system so as to synchronize the state trajectories of a pair of chaotic systems known as *master* and *slave* systems. In the last few decades, there has been a great interest for the synchronization of chaotic and hyperchaotic systems due to their applications.

There are many methods studied in the literature for chaos synchronization such as PC method [11], active control method [12-14], adaptive control method [15-16], time-delay feedback method [17], sampled-data feedback method [18], backstepping method [19-21], sliding mode control method [22-23], etc.

In this paper, we derive new results for the adaptive design of stabilizing Lu-Xiao chaotic system ([24], 2012) to its unstable equilibrium at the origin. We also derive new results for the adaptive design of synchronizing identical Lu-Xiao chaotic systems with unknown parameters. Numerical simulations using MATLAB have been shown to illustrate our results for the Lu-Xiao chaotic system with unknown parameters.

2. ANALYSIS OF LU-XIAO CHAOTIC SYSTEM

Lu-Xiao chaotic system ([24], 2012) is a novel chaotic system described by the dynamics

$$\begin{aligned} \dot{x}_1 &= a(x_2 - x_1) + x_2 x_3 \\ \dot{x}_2 &= -b x_1 x_3 + c x_1 \\ \dot{x}_3 &= d x_1 x_2 - \varepsilon x_3 \end{aligned} \tag{1}$$

where $x \in R^3$ is the state and a, b, c, d, ε are constant, positive parameters of the system. Lu and Xiao observed chaotic behaviour in the system (1) when the parameter values are

$$a = 20,\ b = 5,\ c = 40,\ d = 4 \text{ and } \varepsilon = 3 \tag{2}$$

The strange, double-scroll, chaotic attractor of the Lu-Xiao system is shown in Figure 1.

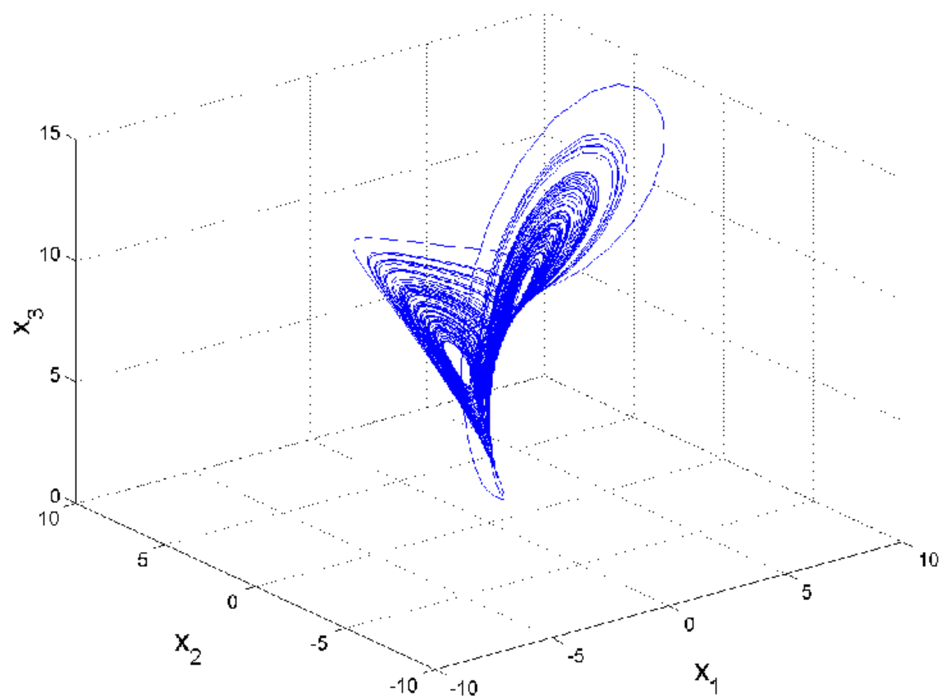

Figure 1. Strange Double-Scroll Attractor of the Lu-Xiao Chaotic System

The linearization matrix of the Lu-Xiao system (1) at the equilibrium point $E_0 = (0,0,0)$ is given by

$$A = \begin{bmatrix} -a & a & 0 \\ c & 0 & 0 \\ 0 & 0 & -\varepsilon \end{bmatrix}$$

which has the eigenvalues

$$\lambda_1 = -\varepsilon, \;\; \lambda_2 = \frac{1}{2}\left(-a - \sqrt{a^2 + 4ac}\right) \text{ and } \lambda_3 = \frac{1}{2}\left(-a + \sqrt{a^2 + 4ac}\right).$$

Since λ_3 is a positive eigenvalues of A, it follows from Lyapunov's first stability theorem [25] that the Lu-Xiao chaotic system (1) is unstable at the equilibrium point $E_0 = (0,0,0)$.

3. Adaptive Control of the Lu-Xiao Chaotic System

This section describes an adaptive design of a globally stabilizing feedback controller for the Lu-Xiao chaotic system with unknown parameters. The design is carried out using adaptive control theory and Lyapunov stability theory.

We start the design procedure by considering a controlled Lu-Xiao chaotic system given by

$$\begin{aligned} \dot{x}_1 &= a(x_2 - x_1) + x_2 x_3 + u_1 \\ \dot{x}_2 &= -b x_1 x_3 + c x_1 + u_2 \\ \dot{x}_3 &= d x_1 x_2 - \varepsilon x_3 + u_3 \end{aligned} \tag{3}$$

where u_1, u_2, u_3 are adaptive controllers to be found using the states x_1, x_2, x_3 and estimates $\alpha(t),\ \beta(t),\ \gamma(t),\ \delta(t),\ \eta(t)$ of the unknown parameters a, b, c, d, ε of the system, respectively.
Our design goal is to ensure that the controlled system (3) globally converges to the origin asymptotically for all values of the initial state $x(0) \in R^3$ and all initial values of the parameter estimates $\alpha(0), \beta(0), \gamma(0), \delta(0), \eta(0) \in R.$

For this purpose, we consider the adaptive controller given by

$$\begin{aligned} u_1(t) &= -\alpha(t)(x_2 - x_1) - x_2 x_3 - k_1 x_1 \\ u_2(t) &= \beta(t) x_1 x_3 - \gamma(t) x_1 - k_2 x_2 \\ u_3(t) &= -\delta(t) x_1 x_2 + \eta(t) x_3 - k_3 x_3 \end{aligned} \tag{4}$$

where k_1, k_2, k_3 are positive gains.

Lu-Xiao dynamics (3), we get

$$\begin{aligned}\dot{x}_1 &= (a-\alpha)(x_2 - x_1) - k_1 x_1 \\ \dot{x}_2 &= -(b-\beta)x_1 x_3 + (c-\gamma)x_1 - k_2 x_2 \\ \dot{x}_3 &= (d-\delta)x_1 x_2 - (\varepsilon - \eta)x_3 - k_3 x_3\end{aligned} \tag{5}$$

We define the parameter estimation errors as

$$\begin{aligned}e_a(t) &= a - \alpha(t) \\ e_b(t) &= b - \beta(t) \\ e_c(t) &= c - \gamma(t) \\ e_d(t) &= d - \delta(t) \\ e_\varepsilon(t) &= \varepsilon - \eta(t)\end{aligned} \tag{6}$$

Using (6), the closed-loop dynamics (5) can be simplified to obtain the following:

$$\begin{aligned}\dot{x}_1 &= e_a(x_2 - x_1) - k_1 x_1 \\ \dot{x}_2 &= -e_b x_1 x_3 + e_c x_1 - k_2 x_2 \\ \dot{x}_3 &= e_d x_1 x_2 - e_\varepsilon x_3 - k_3 x_3\end{aligned} \tag{7}$$

We adopt the Lyapunov approach for deriving an update law for the parameter estimates.

We consider the quadratic Lyapunov function given by

$$V(x_1, x_2, x_3, e_a, e_b, e_c, e_d, e_\varepsilon) = \frac{1}{2}\left(x_1^2 + x_2^2 + x_3^2 + e_a^2 + e_b^2 + e_c^2 + e_d^2 + e_\varepsilon^2\right), \tag{8}$$

which is a positive definite function on R^8.
A simple calculation from the equations (6) yields

$$\begin{aligned}\dot{e}_a &= -\dot{\alpha} \\ \dot{e}_b &= -\dot{\beta} \\ \dot{e}_c &= -\dot{\gamma} \\ \dot{e}_d &= -\dot{\delta} \\ \dot{e}_\varepsilon &= -\dot{\eta}\end{aligned} \tag{9}$$

By differentiating the Lyapunov function V along the trajectories of (7) and using (9), we get

$$\begin{aligned}\dot{V} = -k_1 x_1^2 - k_2 x_2^2 - k_3 x_3^2 &+ e_a\left[x_1(x_2 - x_1) - \dot{\alpha}\right] + e_b\left[-x_1 x_2 x_3 - \dot{\beta}\right] \\ &+ e_c\left[x_1 x_2 - \dot{\gamma}\right] + e_d\left[x_1 x_2 x_3 - \dot{\delta}\right] + e_\varepsilon\left[-x_3^2 - \dot{\eta}\right]\end{aligned} \tag{10}$$

In view of the equation (10), the parameter estimates are updated by the following law:

$$\begin{aligned}
\dot{\alpha} &= x_1(x_2 - x_1) + k_4 e_a \\
\dot{\beta} &= -x_1 x_2 x_3 + k_6 e_b \\
\dot{\gamma} &= x_1 x_2 + k_6 e_c \\
\dot{\delta} &= x_1 x_2 x_3 + k_7 e_d \\
\dot{\eta} &= -x_3^2 + k_8 e_\varepsilon
\end{aligned} \tag{11}$$

where k_4, k_5, k_6, k_7 and k_8 are positive constants.

Next, we establish the following result for the adaptive control of Lu-Xiao chaotic system.

Theorem 1. *The controlled Lu-Xiao chaotic system (3) having unknown system parameters* a, b, c, d, ε *is globally and exponentially stabilized for all initial conditions* $x(0) \in R^3$ *and all initial values of the parameter estimates* $\alpha(t), \beta(t), \gamma(t), \delta(t), \eta(t)$ *for the unknown parameters* a, b, c, d, ε, *respectively, by the adaptive control law (4) and the parameter update law (11), where the gains* k_i, $(i = 1, \ldots, 8)$ *are positive constants. Also, the parameter estimation errors* $e_a, e_b, e_c, e_d, e_\varepsilon$ *converge to zero exponentially with time.*

Proof. Substituting the parameter update law (11) into (10), we get

$$\dot{V} = -k_1\, x_1^2 - k_2\, x_2^2 - k_3\, x_3^2 - k_4\, x_4^2 - k_5 e_a^2 - k_6 e_b^2 - k_7 e_c^2 - k_8 e_d^2 \tag{12}$$

which is a negative definite function on R^8. By the direct method of Lyapunov [25], it follows that $x_1(t), x_2(t), x_3(t), e_a(t), e_b(t), e_c(t), e_d(t), e_\varepsilon(t)$ are globally exponentially stable.

Numerical Results:

For numerical simulations, we use the classical fourth-order Runge-Kutta method (MATLAB) with the step-size $h = 10^{-8}$ to solve the Lu-Xiao system (3) with the adaptive control law (4) and the parameter update law (11). The parameters of the Lu-Xiao system (3) are taken as

$$a = 20,\ b = 5,\ c = 40,\ d = 4 \ \text{and}\ \varepsilon = 3$$

For the adaptive and update laws, we take $k_i = 5, \quad (i = 1, 2, \ldots, 8)$.
Suppose that the initial values of the parameter estimates are

$$\alpha(0) = 9, \quad \beta(0) = 3, \quad \gamma(0) = 4, \quad \delta(0) = 6, \quad \eta(0) = 8$$

The initial state of the controlled Lu-Xiao system (3) is taken as

$$x_1(0) = 14, \quad x_2(0) = 26, \quad x_3(0) = 7$$

The numerical simulations for the adaptive control of the Lu-Xiao chaotic system are depicted in Figures 2-5.

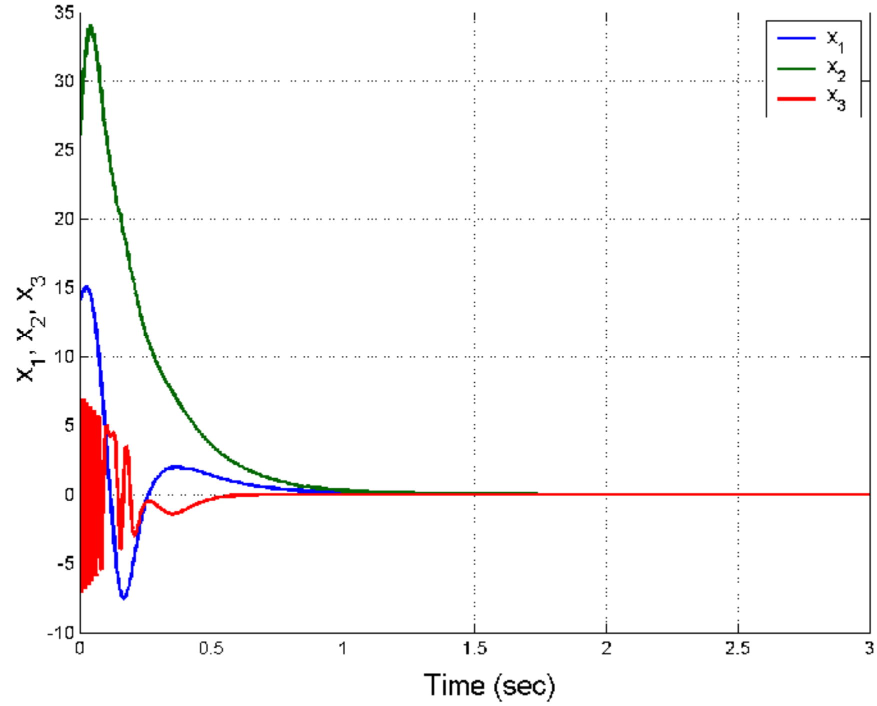

Figure 2. Time Responses of the Controlled Lu-Xiao System

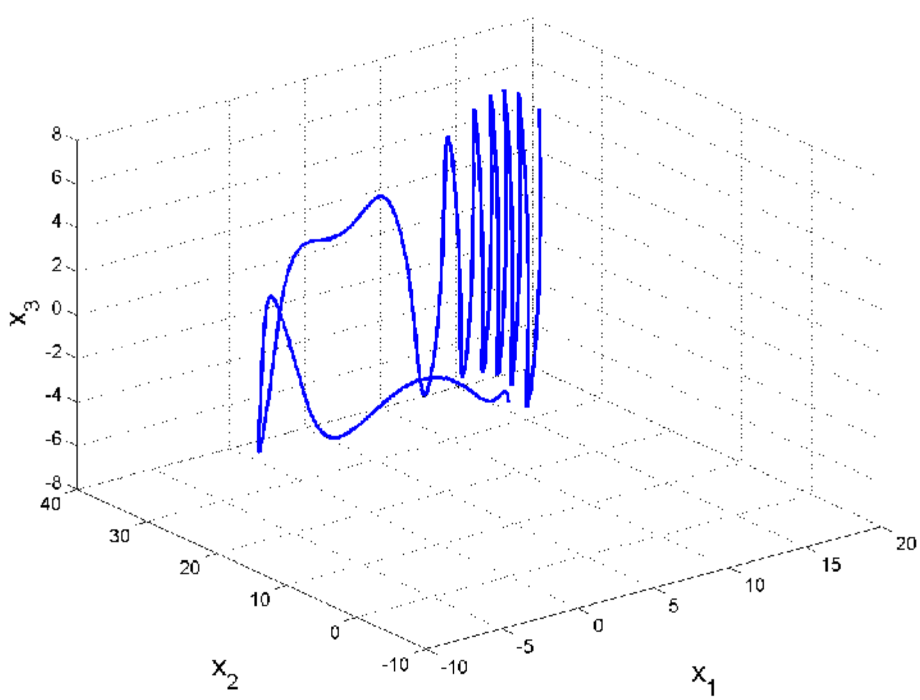

Figure 3. State Orbit of the Controlled Lu-Xiao System

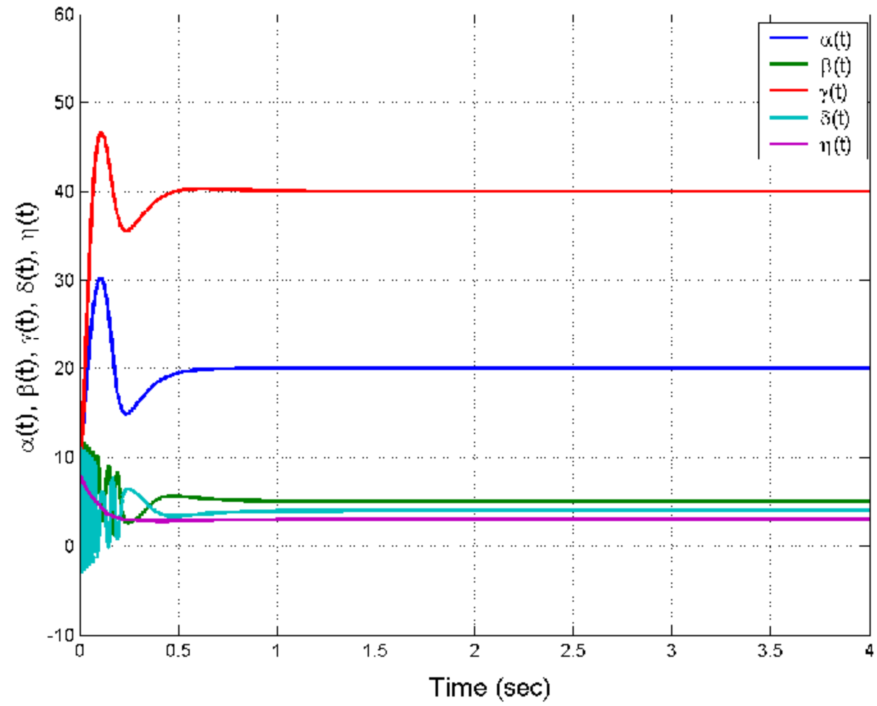

Figure 4. Time-History of the Parameter Estimates $\alpha(t), \beta(t), \gamma(t), \delta(t), \eta(t)$

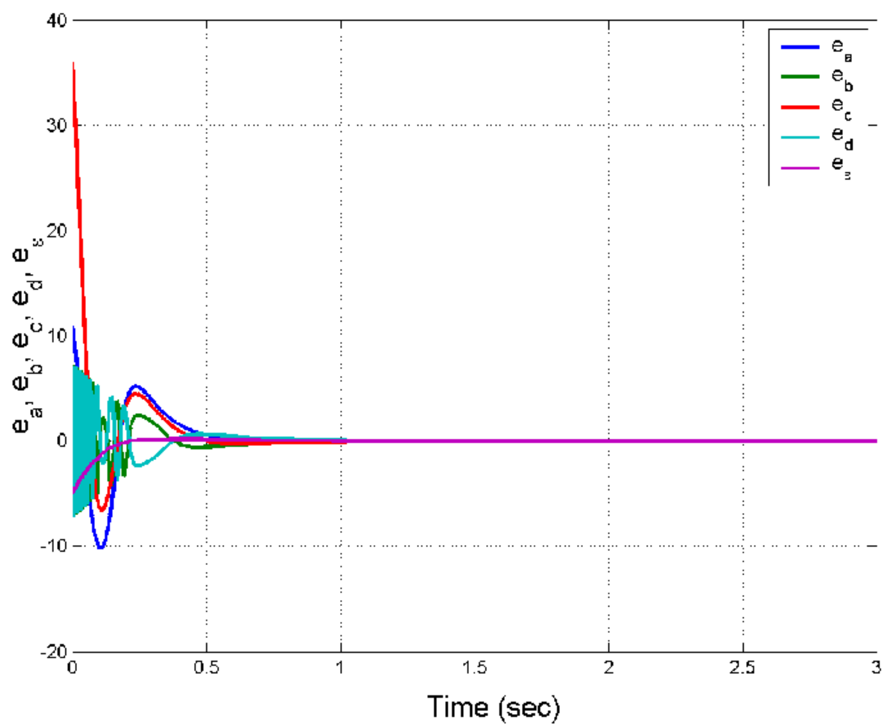

Figure 5. Time-History of the Parameter Estimation Errors $e_a(t), e_b(t), e_c(t), e_d(t), e_\varepsilon(t)$

4. ADAPTIVE SYNCHRONIZATION OF IDENTICAL LU-XIAO SYSTEMS

This section describes an adaptive design of global chaos synchronization of identical Lu-Xiao chaotic systems with unknown parameters. The design is carried out using adaptive control theory and Lyapunov stability theory.

As the master system, we take the Lu-Xiao dynamics described by

$$\begin{aligned} \dot{x}_1 &= a(x_2 - x_1) + x_2 x_3 \\ \dot{x}_2 &= -b x_1 x_3 + c x_1 \\ \dot{x}_3 &= d x_1 x_2 - \varepsilon x_3 \end{aligned} \tag{13}$$

where $x \in R^3$ is the state and a, b, c, d are unknown system parameters.
As the slave system, we take the controlled Lu-Xiao dynamics described by

$$\begin{aligned} \dot{y}_1 &= a(y_2 - y_1) + y_2 y_3 + u_1 \\ \dot{y}_2 &= -b y_1 y_3 + c y_1 + u_2 \\ \dot{y}_3 &= d y_1 y_2 - \varepsilon y_3 + u_3 \end{aligned} \tag{14}$$

where $y \in R^3$ is the state and u_1, u_2, u_3 are adaptive controllers to be found using the states x_1, x_2, x_3 and estimates $\alpha(t),\ \beta(t),\ \gamma(t),\ \delta(t),\ \eta(t)$ of the unknown parameters a, b, c, d, ε of the system, respectively.

The synchronization error is defined by

$$e_i = y_i - x_i, \quad (i = 1, 2, 3) \tag{15}$$

A simple calculation results in the error dynamics

$$\begin{aligned} \dot{e}_1 &= a(e_2 - e_1) + y_2 y_3 - x_2 x_3 + u_1 \\ \dot{e}_2 &= c e_1 - b(y_1 y_3 - x_1 x_3) + u_2 \\ \dot{e}_3 &= -\varepsilon e_3 + d(y_1 y_2 - x_1 x_2) + u_3 \end{aligned} \tag{16}$$

Our design goal is to synchronize the Lu-Xiao chaotic systems (13) and (14) for all values of the initial state $x(0) \in R^3$ and all initial values of the parameter estimates. So, we take

$$\begin{aligned} u_1(t) &= -\alpha(t)(e_2 - e_1) - y_2 y_3 + x_2 x_3 - k_1 e_1 \\ u_2(t) &= -\gamma(t) e_1 + \beta(t)(y_1 y_3 - x_1 x_3) - k_2 e_2 \\ u_3(t) &= \eta(t) e_3 - \delta(t)(y_1 y_2 - x_1 x_2) - k_3 e_3 \end{aligned} \tag{17}$$

where $\alpha, \beta, \gamma, \delta, \eta$ are estimates of the parameters a, b, c, d, ε respectively, and k_1, k_2, k_3 are positive constants.

By substituting the control law (17) into (16), we get the closed-loop error dynamics as

$$\begin{aligned}\dot{e}_1 &= (a-\alpha)(e_2-e_1)-k_1e_1\\ \dot{e}_2 &= (c-\gamma)e_1-(b-\beta)(y_1y_3-x_1x_3)-k_2e_2\\ \dot{e}_3 &= -(\varepsilon-\eta)e_3+(d-\delta)(y_1y_2-x_1x_2)-k_3e_3\end{aligned} \tag{18}$$

We define parameter estimation errors as

$$e_a = a-\alpha(t),\ e_b = b-\beta(t),\ e_c = c-\gamma(t),\ e_d = d-\delta(t),\ e_\varepsilon = \varepsilon-\eta(t) \tag{19}$$

If we substitute (19) into (18), then the error dynamics simplifies to

$$\begin{aligned}\dot{e}_1 &= e_a(e_2-e_1)-k_1e_1\\ \dot{e}_2 &= e_ce_1-e_b(y_1y_3-x_1x_3)-k_2e_2\\ \dot{e}_3 &= -e_\varepsilon e_3+e_d(y_1y_2-x_1x_2)-k_3e_3\end{aligned} \tag{20}$$

We adopt the Lyapunov approach for finding the update law for the estimates of the parameters.

We consider the quadratic Lyapunov function defined by

$$V(e_1,e_2,e_3,e_a,e_b,e_c,e_d,e_\varepsilon) = \frac{1}{2}\left(e_1^2+e_2^2+e_3^2+e_a^2+e_b^2+e_c^2+e_d^2+e_\varepsilon^2\right), \tag{21}$$

which is a positive definite function on R^8.

A simple calculation from the equations (20) yields

$$\dot{e}_a = -\dot{\alpha},\ \ \dot{e}_b = -\dot{\beta},\ \ \dot{e}_c = -\dot{\gamma},\ \ \dot{e}_d = -\dot{\delta},\ \ \dot{e}_\varepsilon = -\dot{\eta} \tag{22}$$

By differentiating V along the trajectories of (22) and using (20), we get

$$\begin{aligned}\dot{V} = &-k_1x_1^2-k_2x_2^2-k_3x_3^2+e_a\left[e_1(e_2-e_1)-\dot{\alpha}\right]+e_b\left[-e_2(y_1y_3-x_2x_3)-\dot{\beta}\right]\\ &+e_c\left[e_1e_2-\dot{\gamma}\right]+e_d\left[e_3(y_1y_2-x_1x_2)-\dot{\delta}\right]+e_\varepsilon\left[-e_3^2-\dot{\eta}\right]\end{aligned} \tag{23}$$

In view of equation (23), the parameter estimates are updated by the following law:

$$\begin{aligned}\dot{\alpha} &= e_1(e_2-e_1)+k_4e_a\\ \dot{\beta} &= -e_2(y_1y_3-x_1x_3)+k_6e_b\\ \dot{\gamma} &= e_1e_2+k_6e_c\\ \dot{\delta} &= e_3(y_1y_2-x_1x_2)+k_7e_d\\ \dot{\eta} &= -e_3^2+k_8e_\varepsilon\end{aligned} \tag{24}$$

where k_4,k_5,k_6,k_7,k_8 are positive constants.

Theorem 2. *The identical Lu-Xiao systems (13) and (14) with unknown parameters* a, b, c, d, ε *are globally and exponentially stabilized for all initial conditions* $x(0), y(0) \in R^3$ *and all initial values of the parameter estimates* $\alpha(t), \beta(t), \gamma(t), \delta(t), \eta(t)$ *for the unknown parameters* $a, b, c, d, \varepsilon,$ *respectively, by the adaptive control law (17) and the parameter update law (24), where the gains* $k_i, \ (i = 1, \ldots, 8)$ *are positive constants. Also, the parameter estimation errors* $e_a, e_b, e_c, e_d, e_\varepsilon$ *converge to zero exponentially with time.*

Proof. Substituting (24) into (23), we get

$$\dot{V} = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 - k_4 e_4^2 - k_5 e_a^2 - k_6 e_b^2 - k_7 e_c^2 - k_8 e_d^2 \tag{25}$$

From (25), we find that $\dot{V}$ is a negative definite function on R^8.

Thus, by Lyapunov stability theory [25], it is immediate that the synchronization error and the parameter error decay to zero exponentially with time for all initial conditions.

Numerical Results:

For numerical simulations, we use the classical fourth-order Runge-Kutta method (MATLAB) with the step-size $h = 10^{-8}$ to solve the Lu-Xiao systems (13) and (14) with the adaptive control law (17) and the parameter update law (24).

The parameters of the Lu-Xiao system are taken as

$$a = 20, \ b = 5, \ c = 40, \ d = 4 \ \text{and} \ \varepsilon = 3$$

For the adaptive and update laws, we take $k_i = 5, \ \ (i = 1, 2, \ldots, 8).$

Suppose that the initial values of the parameter estimates are

$$\alpha(0) = 12, \ \ \beta(0) = 20, \ \ \gamma(0) = 4, \ \ \delta(0) = 16, \ \ \ \eta(0) = 5$$

Suppose that the initial values of the master system (13) are

$$x_1(0) = 7, \ \ x_2(0) = -5, \ \ x_3(0) = 26$$

Suppose that the initial values of the slave system (14) are

$$y_1(0) = -24, \ \ y_2(0) = 18, \ \ y_3(0) = 5$$

Figure 6 shows the adaptive chaos synchronization of the identical Lu-Xiao systems.

Figure 7 shows the time-history of the synchronization error $e_1, e_2, e_3.$

Figure 8 shows the time-history of the parameter estimates $\alpha(t), \beta(t), \gamma(t), \delta(t), \eta(t).$

Figure 9 shows the time-history of the parameter estimation errors $e_a, e_b, e_c, e_d, e_\varepsilon.$

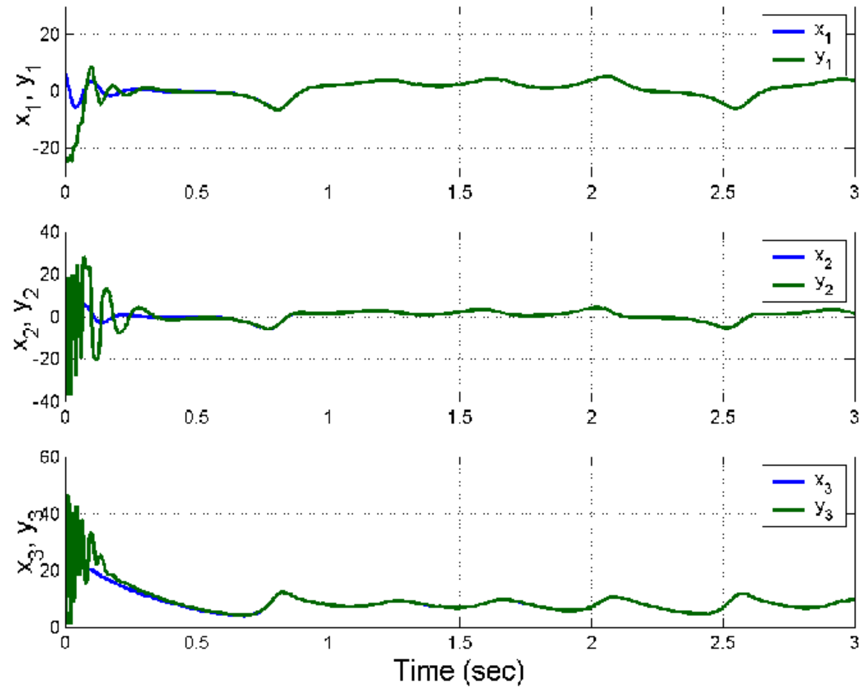

Figure 6. Adaptive Synchronization of the Lu-Xiao Systems

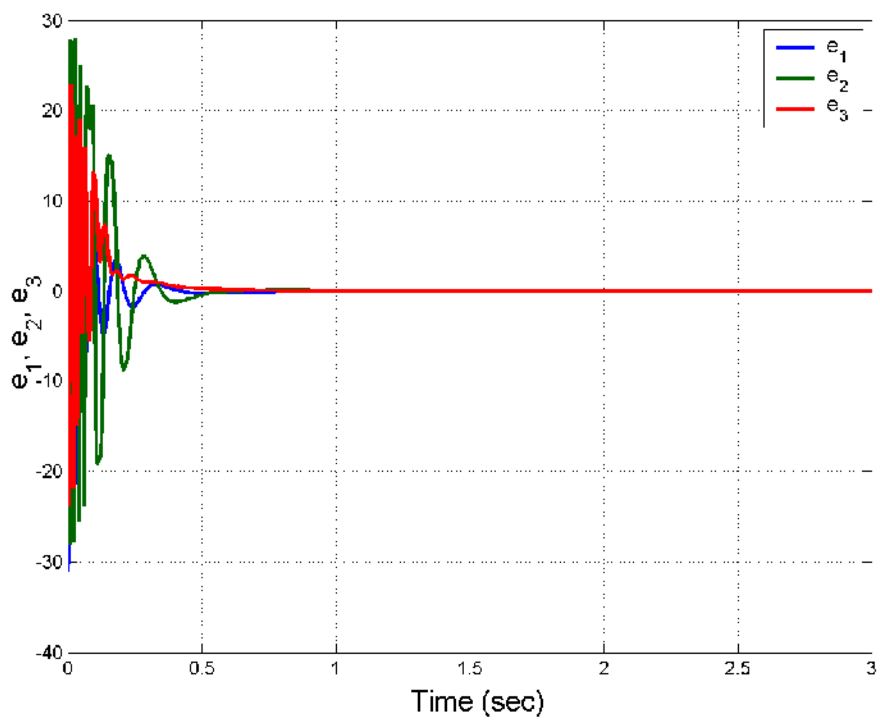

Figure 7. Time-History of the Synchronization Errors e_1, e_2, e_3

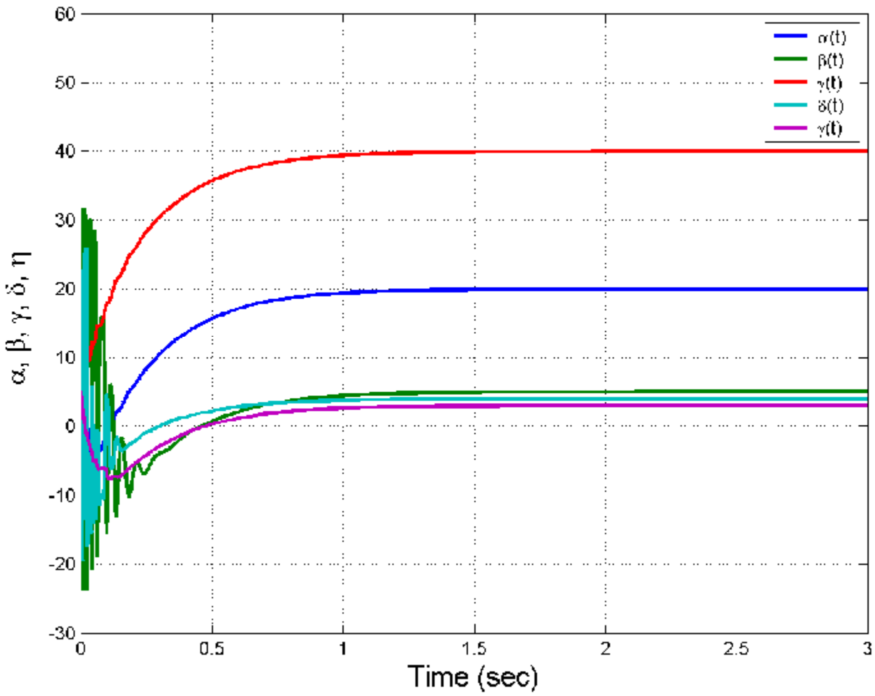

Figure 8. Time-History of the Parameter Estimates $\alpha(t), \beta(t), \gamma(t), \delta(t), \eta(t)$

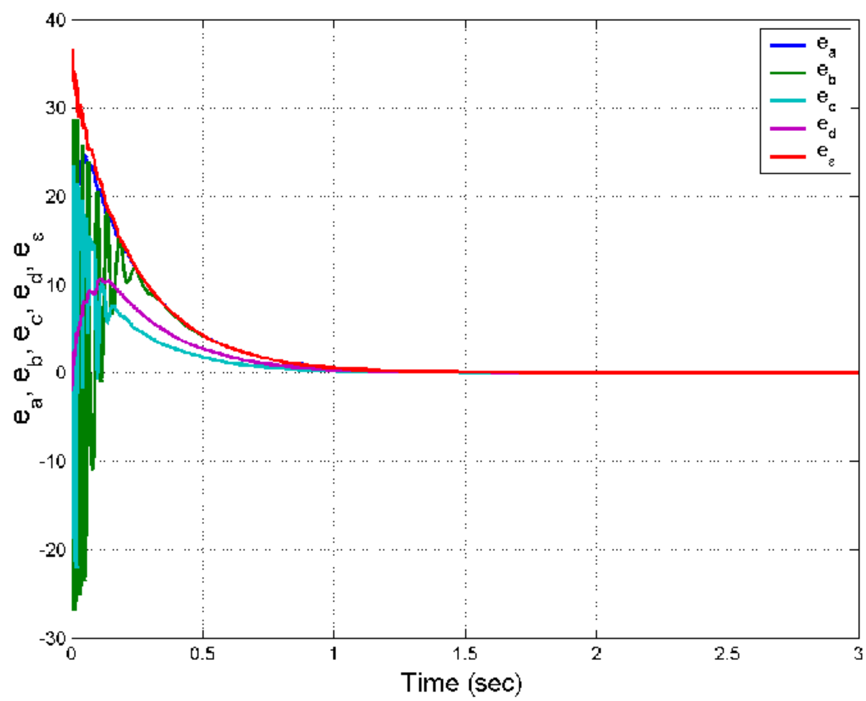

Figure 9. Time-History of the Parameter Estimation Errors $e_a, e_b, e_c, e_d, e_\varepsilon$

5. CONCLUSIONS

In this paper, we found new results for the adaptive design of controller and synchronizer for the Lu-Xiao chaotic system (2012) with unknown system parameters. The main theorems of this paper have been proved via adaptive control theory and Lyapunov stability theory. Numerical simulations using MATLAB were shown to depict and demonstrate the proposed adaptive control and synchronization schemes for the Lu-Xiao chaotic system.

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