ADAPTIVE BACKSTEPPING CONTROLLER AND SYNCHRONIZER DESIGN FOR ARNEODO CHAOTIC SYSTEM WITH UNKNOWN PARAMETERS

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ABSTRACT

In this paper, we apply backstepping control method to derive new results for the adaptive controller and synchronizer design for the Arneodo chaotic system (1980), when the system parameters are unknown. First, we design an adaptive backstepping controller to stabilize the Arneodo system to its unstable equilibrium at the origin. Next, we design an adaptive backstepping controller to achieve global chaos synchronization of the identical Arneodo chaotic systems with unknown parameters. MATLAB simulations have been detailed to illustrate the proposed adaptive backstepping controller and synchronizer design for Arneodo chaotic system with unknown parameters.

KEYWORDS

Backstepping Control, Adaptive Control, Adaptive Synchronization, Chaos, Arneodo System.

1. INTRODUCTION

Chaos theory is the qualitative study of unstable aperiodic behaviour in deterministic nonlinear dynamical systems. A chaotic system is popularly known as nonlinear dynamical system, which is very sensitive to initial conditions. In 1963, Lorenz discovered that a very small difference in the initial conditions led to large changes in his deterministic weather model [1]. Other classical 3-dimensional chaotic systems include Rössler system [2], Newton-Leipnik system [3], Chen system [4] and Lü system [5].

The problem of controlling a chaotic system aims to find a state feedback control law to stabilize the chaotic system around its unstable equilibrium points [6-7]. We use active control method [8-9], when the system parameters are known and we use adaptive control method [10-12], when the system parameters are unknown.

The problem of synchronizing chaotic systems aims to find a state feedback control law to synchronize a pair of coupled chaotic systems known as master-slave systems or drive-response systems.

The seminal paper on synchronization of chaotic systems was published by Pecora and Carroll in 1990 [13]. Afterwards, chaos synchronization has found applications in many fields such as physics [14-15], chemistry [16], ecology [17], biology [18], cardiology [19], neural networks [20], robotics [21-22], secure communications [23-24].

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Some commonly used methods for addressing chaos synchronization problem are active control method [25-30], adaptive control method [31-35], sampled-data feedback method [36], time-delay feedback method [37], sliding mode control method [38-44], backstepping control method [45-48], etc.

In this paper, we derive new results for the adaptive backstepping controller and adaptive backstepping synchronizer for the Arneodo chaotic system ([47], 1980) with unknown parameters. The stability results have been established using Lyapunov stability theory. MATLAB simulations have been detailed for the adaptive controllers derived in the paper.

2. SYSTEM DESCRIPTION

The Arneodo system ([47], 1980) is given by the 3-dimensional dynamics

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ \dot{x}_3 &= ax_1 - bx_2 - x_3 - x_1^2 \end{aligned} \tag{1}$$

where x_1, x_2, x_3 are the states and a, b are constant, positive parameters of the system. Arneodo system (1) undergoes *chaotic* behaviour when the parameter values are

$$a = 7.5 \text{ and } b = 3.8 \tag{2}$$

Figure 1 depicts the strange chaotic attractor of the Arneodo chaotic system.

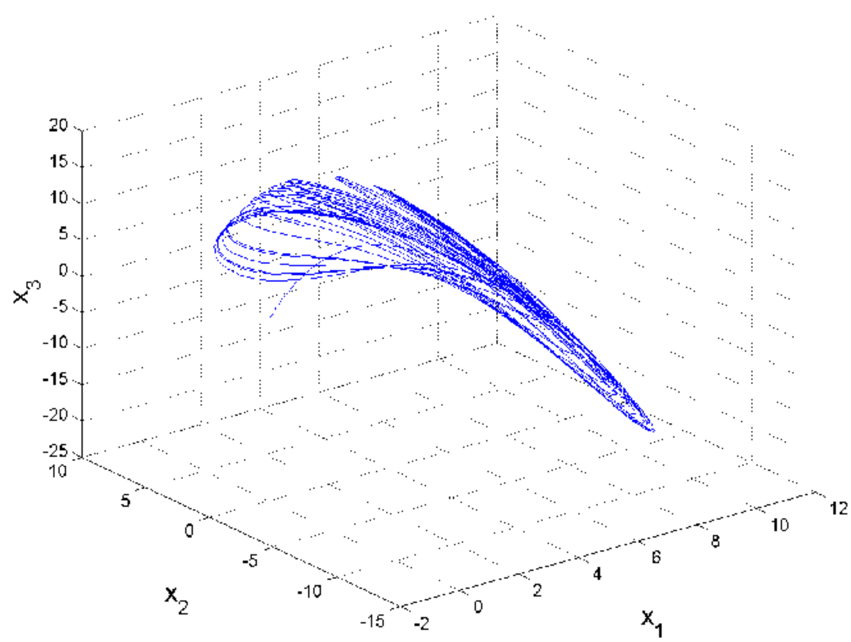

Figure 1. Strange Attractor of the Arneodo System

When the parameter values are taken as in (2) for Arneodo system (1), the system linearization matrix at the origin is given by

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ a & -b & -1 \end{bmatrix}$$

which has the characteristic polynomial

$$p(\lambda) = \det(\lambda I - A) = \lambda^3 + \lambda^2 + b\lambda - a$$

Since $a, b > 0,$ it is immediate that the coefficients of $p(\lambda)$ are not all positive. Hence, by Routh-Hurwitz criterion, the matrix A has an unstable eigenvalue. Thus, it is immediate that the Arneodo system (1) is unstable at the origin.

3. Adaptive Backstepping Control of the Arneodo Chaotic System

3.1 Main Results

In this section, we design an adaptive backstepping controller for globally stabilizing the Arneodo system (1980) with unknown parameters.

Thus, we consider the controlled Arneodo system

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ \dot{x}_3 &= ax_1 - bx_2 - x_3 - x_1^2 + u \end{aligned} \tag{3}$$

where u is a backstepping controller to be designed using the states x_1, x_2, x_3 and estimates $\hat{a}(t), \hat{b}(t)$ of the unknown parameters a, b of the system.

The parameter estimation errors are defined by

$$\begin{aligned} e_a(t) &= a - \hat{a}(t) \\ e_b(t) &= b - \hat{b}(t) \end{aligned} \tag{4}$$

Note that

$$\begin{aligned} \dot{e}_a(t) &= -\dot{\hat{a}}(t) \\ \dot{e}_b(t) &= -\dot{\hat{b}}(t) \end{aligned} \tag{5}$$

The main result for the adaptive backstepping controller design for the Arneodo system (3) is described by the following theorem.

Theorem 1. *The Arneodo chaotic system (3) with unknown parameters and is globally and exponentially stabilized for all values of* $x(0) \in R^3$ *by the backstepping controller*

$$u(t) = -(\hat{a}+3)x_1 - (5-\hat{b})x_2 - 2x_3 + x_1^2 \qquad (6)$$

where $\hat{a}(t), \hat{b}(t)$ *are estimates of the unknown parameters* a, b *and the parameter update law is given by*

$$\begin{aligned} \dot{\hat{a}} &= (2x_1 + 2x_2 + x_3)x_1 + k_a e_a \\ \dot{\hat{b}} &= -(2x_1 + 2x_2 + x_3)x_2 + k_b e_b \end{aligned} \qquad (7)$$

with gains $k_a, k_b > 0.$

Proof. We establish the main result using Lyapunov stability theory [48].

First, we define a Lyapunov function candidate

$$V_1 = \frac{1}{2} z_1^2, \qquad (8)$$

where $z_1 = x_1.$

Differentiating V_1 along the solutions of the Arneodo system (3), we get

$$\dot{V}_1 = z_1 \dot{z}_1 = x_1 x_2 = -z_1^2 + z_1(x_1 + x_2) \qquad (9)$$

Secondly, we choose the second Lyapunov function candidate as

$$V_2 = V_1 + \frac{1}{2} z_2^2 = \frac{1}{2}\left(z_1^2 + z_2^2\right) \qquad (10)$$

where $z_2 = x_1 + x_2.$

Differentiating V_2 along the solutions of the Arneodo system (3), we get

$$\dot{V}_2 = -z_1^2 - z_2^2 + z_2(2x_1 + 2x_2 + x_3) \qquad (11)$$

Finally, for the systems (3) and (5), we consider the Lyapunov function candidate

$$V = V_2 + \frac{1}{2} z_3^2 + \frac{1}{2} e_a^2 + \frac{1}{2} e_b^2 = \frac{1}{2}\left(z_1^2 + z_2^2 + z_3^2 + e_a^2 + e_b^2\right) \qquad (12)$$

where $z_3 = 2x_1 + 2x_2 + x_3.$

Differentiating V along the solutions of the systems (3) and (5), we obtain

$$\dot{V} = -z_1^2 - z_2^2 - z_3^2 + z_3\left[(3+a)x_1 + (5-b)x_2 + 2x_3 - x_1^2 + u\right] - e_a\dot{\hat{a}} - e_b\dot{\hat{b}} \tag{13}$$

Substituting the backstepping controller (6) into (13), we obtain

$$\dot{V} = -z_1^2 - z_2^2 - z_3^2 + z_3\left(e_a x_1 - e_b x_2\right) - e_a\dot{\hat{a}} - e_b\dot{\hat{b}}$$

i.e.

$$\dot{V} = -z_1^2 - z_2^2 - z_3^2 + e_a\left(z_3 x_1 - \dot{\hat{a}}\right) + e_b\left(-z_3 x_2 - \dot{\hat{b}}\right) \tag{14}$$

Substituting the parameter update law (7) into (14), we get

$$\dot{V} = -z_1^2 - z_2^2 - z_3^2 - k_a e_a^2 - k_b e_b^2 \tag{15}$$

which is a negative definite function.

Hence, by Lyapunov stability theory [48], the proof is complete.

3.2 Numerical Results

For numerical simulations, we have applied the fourth order Runge-Kutta method (MATLAB) with the step-size $h = 10^{-8}$ to solve the Arneodo system (3) with the adaptive backstepping control law (6) and the parameter update law (7).

The parameters of the Arneodo system (3) are taken as in the chaotic case, *i.e.*

$$a = 7.5, \quad b = 3.8$$

For the adaptive and update laws, we take $k_a = 6$ and $k_b = 6$.

Suppose that the initial values of the estimated parameters are

$$\hat{a}(0) = 15, \quad \hat{b}(0) = 6$$

The initial state of the controlled Arneodo system (3) is taken as

$$x_1(0) = 25, \quad x_2(0) = -16, \quad x_3(0) = 30$$

When the adaptive control law (6) and the parameter update law (7) are used, the state trajectories of the controlled Arneodo system converge exponentially to the equilibrium at the origin as shown in Figure 2.

The time-history of the parameter estimates $\hat{a}(t), \hat{b}(t)$ is shown in Figure 3.

The time-history of the parameter estimation errors $e_a(t), e_b(t)$ is shown in Figure 4.

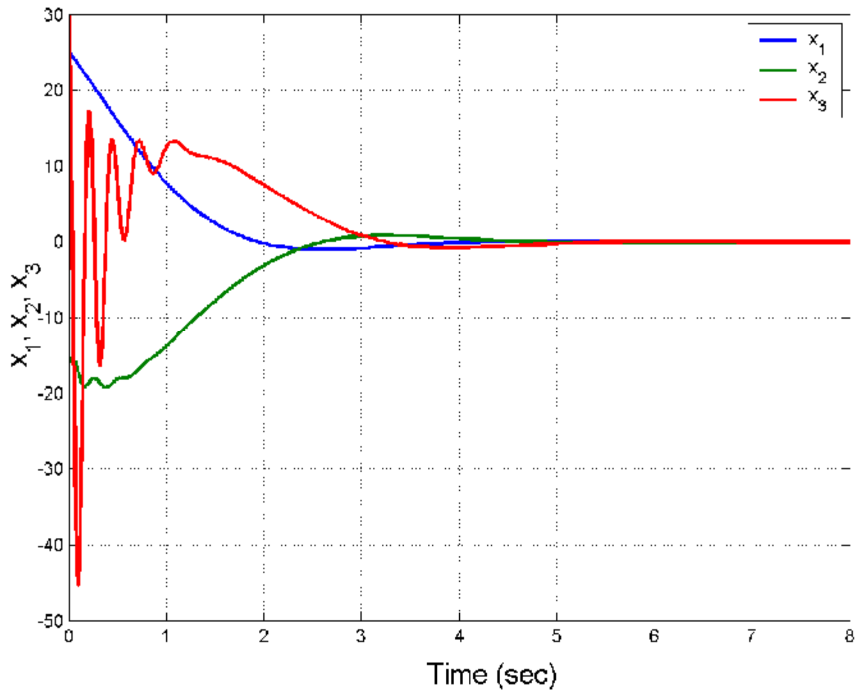

Figure 2. Time Responses of the Controlled Arneodo Chaotic System

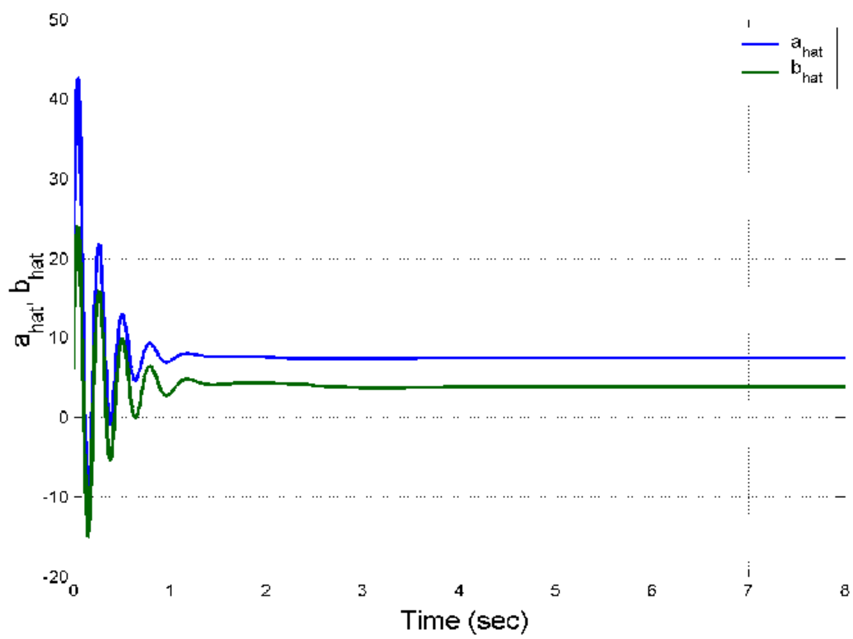

Figure 3. Time-History of the Parameter Estimates $\hat{a}(t)$, $\hat{b}(t)$

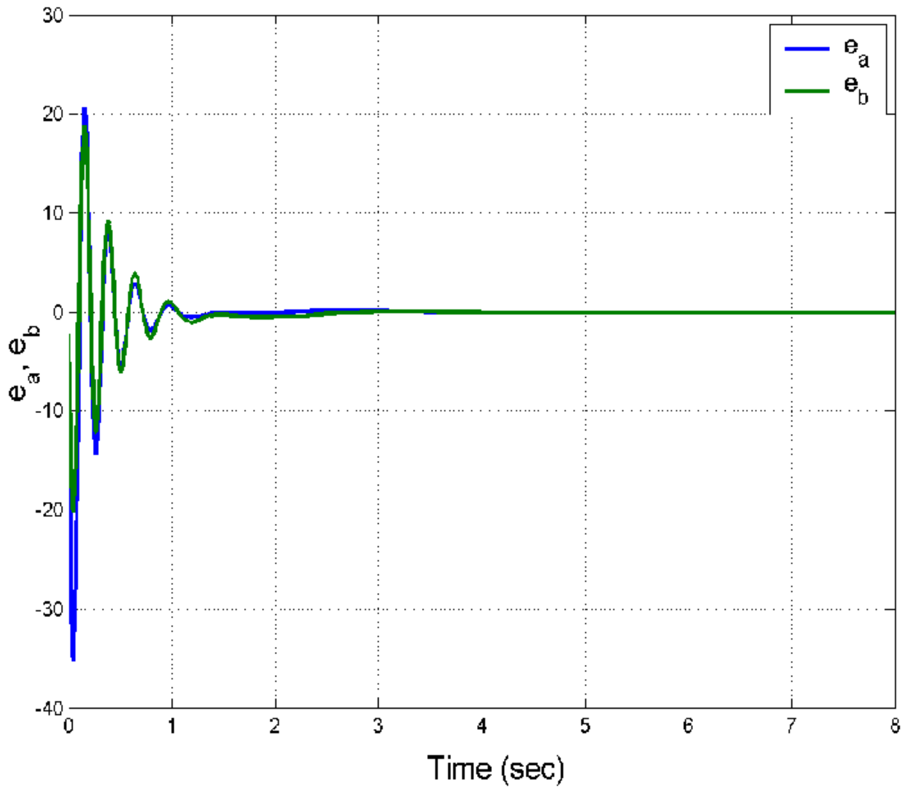

Figure 4. Time-History of the Parameter Estimation Error e_a, e_b

4. ADAPTIVE BACKSTEPPING SYNCHRONIZATION OF IDENTICAL ARNEODO CHAOTIC SYSTEMS

4.1 Main Results

In this section, we derive new results for the adaptive backstepping synchronization of identical Arneodo systems (1980) with unknown parameters.

As the master system, we take the Arneodo dynamics described by

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ \dot{x}_3 &= ax_1 - bx_2 - x_3 - x_1^2 \end{aligned} \tag{16}$$

where $x_i, (i = 1, 2, 3)$ are the state variables and a, b are unknown system parameters.

The system (16) is chaotic when the parameter values are taken as

$$a = 7.5, \quad b = 3.8$$

As the slave system, we consider the controlled Arneodo dynamics described by

$$\begin{aligned}\dot{y}_1 &= y_2\\ \dot{y}_2 &= y_3\\ \dot{y}_3 &= ay_1 - by_2 - y_3 - y_1^2 + u\end{aligned} \tag{17}$$

where $y_i, \ (i = 1,2,3,4)$ are the state variables and $u_i, \ (i = 1,2,3,4)$ are the nonlinear controllers to be designed.

The synchronization error is defined by

$$e_i = y_i - x_i, \quad (i = 1,2,3,4) \tag{18}$$

Then the error dynamics is obtained as

$$\begin{aligned}\dot{e}_1 &= e_2\\ \dot{e}_2 &= e_3\\ \dot{e}_3 &= ae_1 - be_2 - e_3 - (y_1 + x_1)e_1 + u\end{aligned} \tag{19}$$

We use backstepping control method to find an adaptive synchronizer $u(t)$ which uses the states of the master and slave systems and also the estimates $\hat{a}(t), \hat{b}(t)$ of the unknown parameters $a, b.$

We define parameter estimation errors as

$$\begin{aligned}e_a(t) &= a - \hat{a}(t)\\ e_b(t) &= b - \hat{b}(t)\end{aligned} \tag{20}$$

Note that

$$\begin{aligned}\dot{e}_a(t) &= -\dot{\hat{a}}(t)\\ \dot{e}_b(t) &= -\dot{\hat{b}}(t)\end{aligned} \tag{21}$$

The main result for the adaptive backstepping synchronizer design for the Arneodo systems (16) and (17) is described by the following theorem.

Theorem 2. *The identical Arneodo chaotic systems (16) and (17) with unknown parameters and is globally and exponentially stabilized for all values of* $x(0), y(0) \in R^3$ *by the backstepping controller*

$$u(t) = -(\hat{a} + 3 - y_1 - x_1)e_1 - (5 - \hat{b})e_2 - 2e_3 \tag{22}$$

where $\hat{a}(t), \hat{b}(t)$ *are estimates of the unknown parameters* a, b *and the parameter update law is given by*

$$\begin{aligned} \dot{\hat{a}} &= (2e_1 + 2e_2 + e_3)e_1 + k_a e_a \\ \dot{\hat{b}} &= -(2e_1 + 2e_2 + e_3)e_2 + k_b e_b \end{aligned} \tag{23}$$

with gains $k_a, k_b > 0.$

Proof. We establish the main result using Lyapunov stability theory [48].

First, we define a Lyapunov function candidate

$$V_1 = \frac{1}{2} z_1^2, \tag{24}$$

where $z_1 = e_1.$

Differentiating V_1 along the solutions of the error system (19), we get

$$\dot{V}_1 = z_1 \dot{z}_1 = e_1 e_2 = -e_1^2 + e_1(e_1 + e_2) \tag{25}$$

Secondly, we choose the second Lyapunov function candidate as

$$V_2 = V_1 + \frac{1}{2} z_2^2 = \frac{1}{2}\left(z_1^2 + z_2^2\right) \tag{26}$$

where $z_2 = e_1 + e_2.$

Differentiating V_2 along the solutions of the error system (19), we get

$$\dot{V}_2 = -z_1^2 - z_2^2 + z_2(2e_1 + 2e_2 + e_3) \tag{27}$$

Finally, for the systems (19) and (21), we consider the Lyapunov function candidate

$$V = V_2 + \frac{1}{2} z_3^2 + \frac{1}{2} e_a^2 + \frac{1}{2} e_b^2 = \frac{1}{2}\left(z_1^2 + z_2^2 + z_3^2 + e_a^2 + e_b^2\right) \tag{28}$$

where $z_3 = 2e_1 + 2e_2 + e_3$.

Differentiating V along the solutions of the systems (19) and (21), we obtain

$$\dot{V} = -z_1^2 - z_2^2 - z_3^2 + z_3\left[(a + 3 - y_1 - x_1)e_1 + (5 - b)e_2 + 2e_3 + u\right] - e_a \dot{\hat{a}} - e_b \dot{\hat{b}} \tag{29}$$

Substituting the backstepping controller (22) into (29), we obtain

$$\dot{V} = -z_1^2 - z_2^2 - z_3^2 + z_3\left(e_a e_1 - e_b e_2\right) - e_a \dot{\hat{a}} - e_b \dot{\hat{b}}$$

That is,

$$\dot{V} = -z_1^2 - z_2^2 - z_3^2 + e_a \left(z_3 e_1 - \dot{\hat{a}} \right) + e_b \left(-z_3 e_2 - \dot{\hat{b}} \right) \tag{30}$$

Substituting the parameter update law (23) into (30), we get

$$\dot{V} = -z_1^2 - z_2^2 - z_3^2 - k_a e_a^2 - k_b e_b^2 \tag{31}$$

which is a negative definite function.

Hence, by Lyapunov stability theory [48], the proof is complete.

4.2 Numerical Results

For the numerical simulations, the fourth order Runge-Kutta method is used to solve the two systems of differential equations (16) and (17) with the adaptive control law (22) and the parameter update law (23).

We take the parameter values as in the chaotic case, viz.

$$a = 7.5, \quad b = 3.8$$

We take the positive gains as $k_a = 6$ and $k_b = 6$.

Suppose that the initial values of the estimated parameters are

$$\hat{a}(0) = 2, \quad \hat{b}(0) = 5$$

We take the initial values of the master system (16) as

$$x_1(0) = 3, \quad x_2(0) = 8, \quad x_3(0) = -1$$

We take the initial values of the slave system (17) as

$$y_1(0) = 5, \quad y_2(0) = -10, \quad y_3(0) = 9$$

Figure 5 shows the chaos synchronization of the identical Arneodo systems.

Figure 6 shows the time-history of the synchronization error e_1, e_2, e_3.

Figure 7 shows the time-history of the parameter estimates $\hat{a}(t), \hat{b}(t)$.

From this figure, it is clear that the parameter estimates converge to the original values $a = 7.5$ and $b = 3.8$, respectively.

Figure 8 shows the time-history of the parameter estimation errors e_a, e_b.

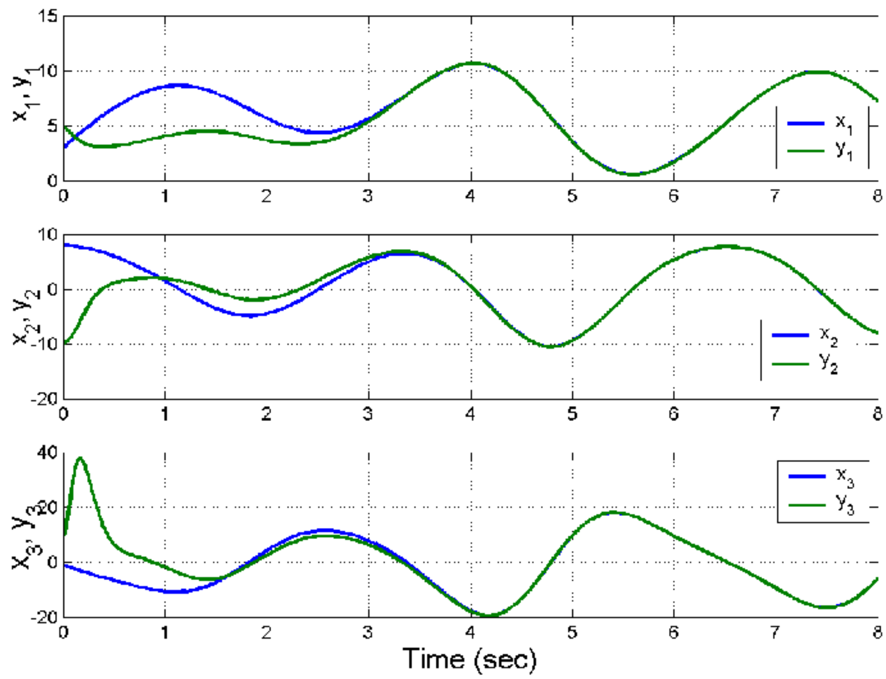

Figure 5. Adaptive Synchronization of the Arneodo Systems

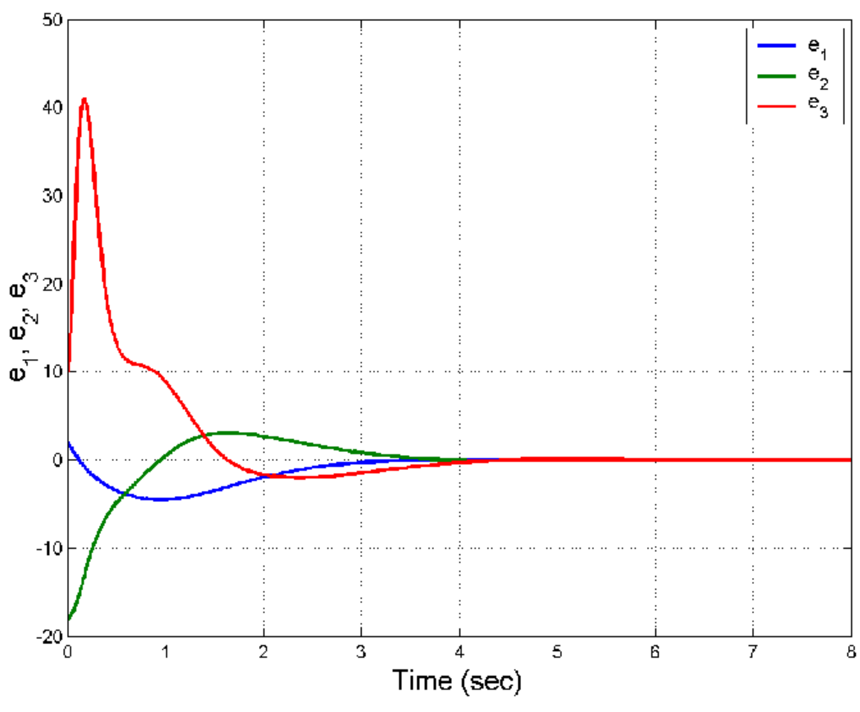

Figure 6. Time-History of the Synchronization Error e_1, e_2, e_3

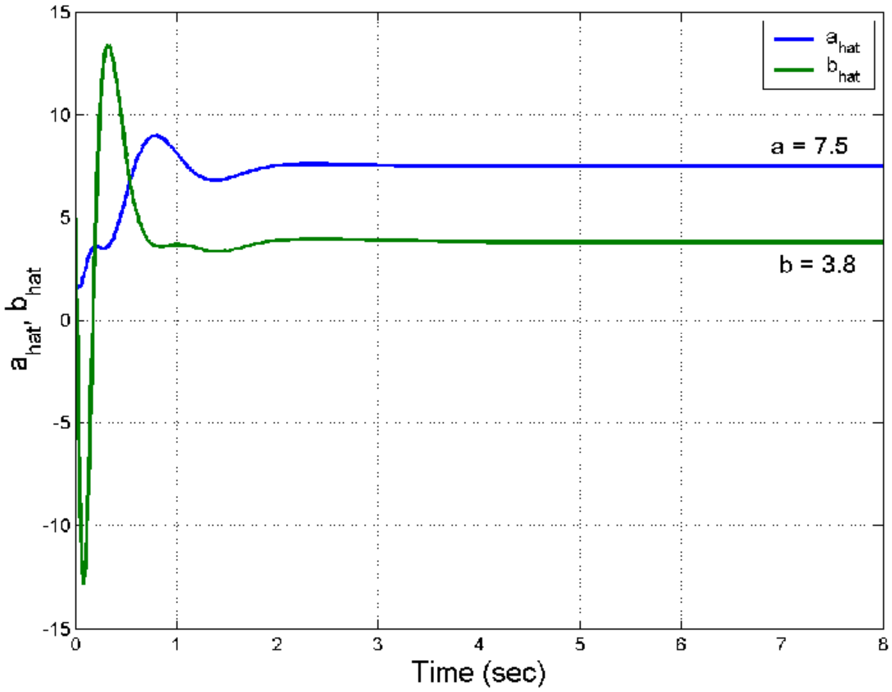

Figure 7. Time-History of the Parameter Estimates $\hat{a}(t), \hat{b}(t)$

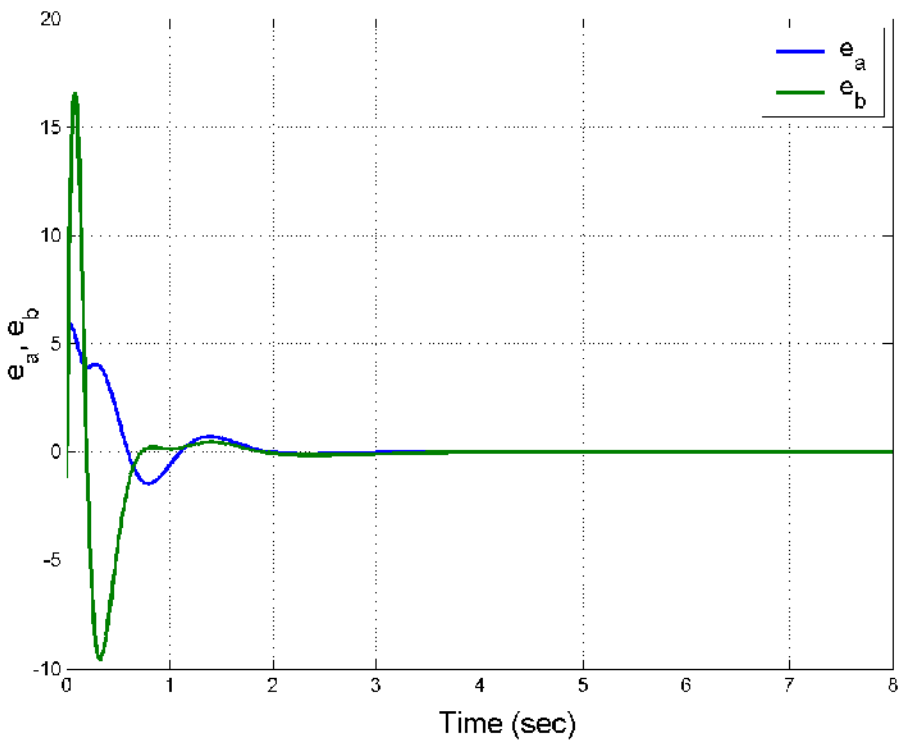

Figure 8. Time-History of the Parameter Estimation Error e_a, e_b

5. CONCLUSIONS

In this paper, we have applied backstepping control method to derive new results for the adaptive stabilization and synchronization of the Arneodo system (1980) with unknown system parameters. First, an adaptive controller law was designed via backstepping control method for stabilizing the Arneodo system (1980) to its unstable equilibrium at the origin. Next, an adaptive synchronizer law was designed via backstepping control method for synchronizing identical Arneodo systems. The main results derived in this paper were proved using Lyapunov stability theory. Numerical simulations using MATLAB have been provided to validate and demonstrate the effectiveness of the proposed adaptive backstepping control and backstepping synchronization schemes for the Arneodo chaotic system.

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