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Response Surface Method based Railway Wheel Optimisation for Natural Frequency

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ABSTRACT:

In this paper, response surface method is used to find out the fatigue life equation by taking load and dimensions of the wheel as variables. The natural frequency is taken as a factor for fatigue life. An ANOVA analysis is carried out to formulate the natural frequency function. An objective function has been formulated to minimise the mass of railway wheel considering the strength and geometrical constraints of inner hub-hole, hub-web interface and hub-rim interface.

KEYWORDS:

Wheel-Rail; Natural frequency; Response surface method; Fatigue; Finite element analysis; Mass optimization

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NOMENCLATURE:

x_1	*Inner hub hole radius in mm.*
x_2	*Outer radius at hub/web interface in mm.*
x_3	*Outer radius at hub/rim interface in mm.*
E	*Young's modulus.*
ν	*Poisson's ratio.*
σ_y	*Yield strength.*
ρ	*Density of the wheel.*

1. Introduction

In recent years, higher train speeds and increased axle loads have led to larger wheel/rail contact forces. Efforts have been made to optimize the wheel and rail design to improve the performance and reduce the cost. These trends have changed the major wheel rim damage from wear to fatigue. Unlike the slow deterioration process of wear, fatigue causes abrupt fracture in wheels or the tread surface material loss. For fatigue analysis, there are three phases (a) Fatigue initiation, (b) Crack propagation, (c) Fatigue fracture, which have their occurrence time. If it is able to predict the initiation of fatigue, the wheel life can be increased by optimising various parameters [8]. The design variable is taken as parameter to optimize the required objective which is shown by Nielsen [9]. Railway vehicles are the most energy efficient, safe, and economic overland movers of passengers and heavy freight. Railway wheel is the basic component of railway vehicles, which not only provides the support to the entire vehicle but also guides and provides steerability for the train and carries the heavy load. Generally, wheel fails due to fatigue for long run. The conventional wheels have been designed for large number of load cycle but are overweight.

In this paper we have taken a natural frequency based approach [2] to optimise the wheel mass using response surface method (RSM). The mode shapes and their natural frequency are obtained using finite element (FE) method. The mode shape of interest the frequency at which the rotation is for degree of freedom of the wheel vibrates. We obtained various rotational frequencies to fit within standard central composition design (CCD) model available in MINITAB Design of Experiments (DoE) Software. A complete regression analysis is carried out by Analysis of Variance (ANOVA) and generated response surface using RSM so as to optimize the equation for natural frequency. For optimizing the mass of railway wheel an objective function considering mass with geometry and frequency constraints has been generated.

2. Finite element modelling and analysis

For obtaining natural frequency we must understand well the formulation of wheel-rail interaction and the contact model [8]. When the wheel is moving on the rail with some angular velocity, the vehicle load is transmitted from body to the rail by axle, wheel and wheel-rail interaction. The wheel is assumed to be fixed with axle by hub and there is no slipping between them. A 3D FE model of wheel and rail are simulated using ABAQUS version 6.10-1 for practical conditions such as speed, load and interaction to obtain the contact pressure and its significance on the wheel. The wheel is meshed using hexahedral element type C3D8R and wedge element type C3D6. The rail is meshed hexahedral element type C3D8R. Steel material properties as E= 206 GPa, ν = 0.3, σ_y = 250MPa and ρ = 7850kg/m^3 are used.

The wheel and rail are idealised to have surface to surface contact with finite sliding. In this contact formulation, the rail head surface is taken master surface and the wheel tread surface is taken as slave. The normal contract properties are idealised as hard with default parameters. The tangential contact properties are idealised as penalty based friction contact with friction coefficient of 0.3. A standard wheel profile with x_1= 40mm, x_2 = 76mm, and x_3 = 265mm as shown in Fig. 1 is meshed. The mesh and model summary for a standard wheel-rail contact model is presented in Fig. 2 and Table 1 respectively. When the wheel dimensions are varied, the mesh of wheel is also changed by keeping hexahedral element type. However, linear wedge elements are used by Abaqus software to fit with the change in the geometry in some places.

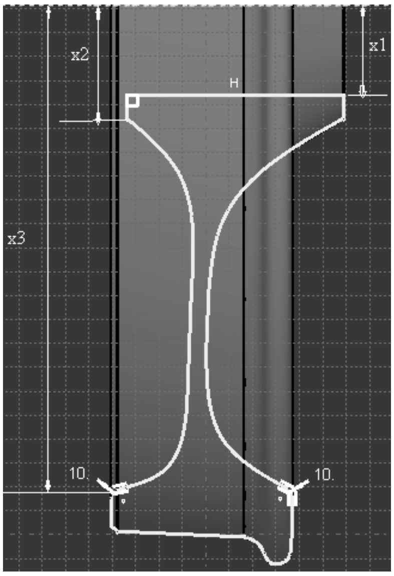

Fig. 1: Cross-section view of standard wheel profile

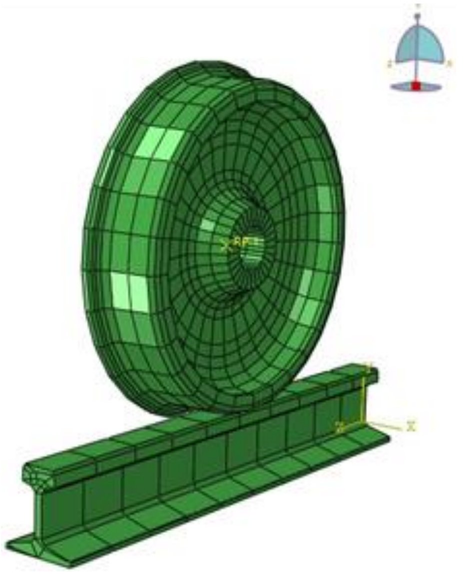

Fig. 2: 3D FE model of standard wheel and rail

Table 1: Model summary of standard wheel and rail

Component	Wheel	Rail
Nodes	2292	374
C3D8R Elements	1624	210
C3D6 Elements	87	0

The pivot point of wheel is constrained in translations TX and TZ and rotations RX and RY. The foot of rail is constrained in all degrees of freedom. The FE model discretizes the whole body in to elements to derive the stiffness and mass matrix for each element. Assembling all the element stiffness and mass matrices yields the stiffness matrix K_m, and mass matrix M_m, of the Wheel for the particular circumferential vibration mode *m*. The eigenvalue problem can be solved using,

$$\left(K_m - \omega^2 M_m\right)q = 0 \qquad (1)$$

Where ω_{mn} is the n^{th} natural frequency with *m* nodal diameters and q_{mn} is the corresponding eigenvectors. The natural frequencies for the first 10 modes of the standard wheel-rail model as obtained through ABAQUS simulation are given in Table 2. First 4 mode shapes are shown in Fig. 3 to Fig. 6.

Table 2: First 10 natural frequencies for standard wheel-rail

Mode number	Frequency(Hz)
1	0.315039
2	0.315039
3	0.497019
4	0.619044
5	0.679175
6	0.679901
7	1.744317
8	1.744317
9	1.772315
10	1.772315

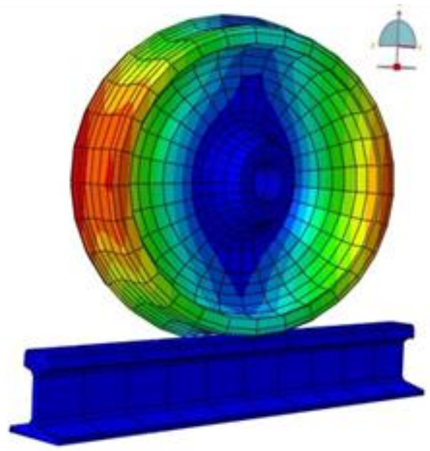

Fig. 3: First mode shape at 0.315039 Hz

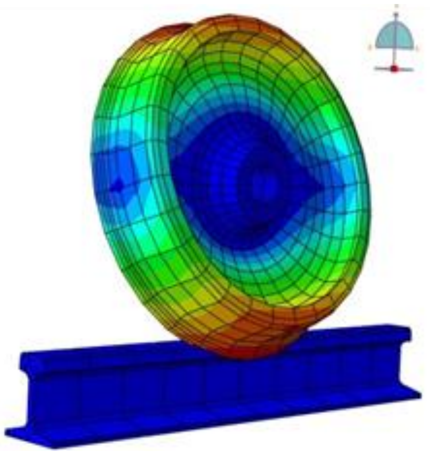

Fig. 4: Second mode shape at 0.315039 Hz

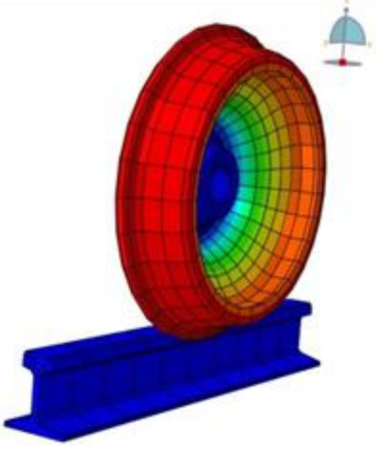

Fig. 5: Third mode shape at 0.497019 Hz

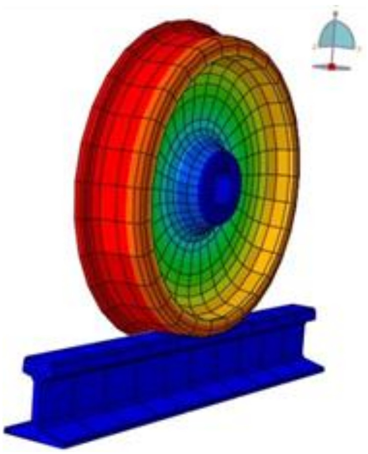

Fig. 6: Fourth mode shape at 0.619044 Hz

3. Response surface method & optimisation

In this study, RSM was used to determine the optimum design for the minimization of the wheel mass within specific fatigue life. The significant process variables were identified by using the CCD based on DoE principles. The natural frequency response y is determined as a function of multiple design variables (x_i). The behaviour in RSM is expressed by the approximation as a polynomial y=f(x) on the basis of the observed data. A quadratic response function with two variables for a regression model is expressed by,

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1^2 + \beta_4 x_2^2 + \beta_5 x_1 x_2 + \varepsilon \quad (2)$$

Where $\beta_0, \beta_1, \beta_2, \beta_3, \beta_4$ and β_5 are the regression coefficients ε is the approximation error. The method of least squares is typically used to estimate the regression coefficients in a multiple linear regression model. Suppose that n > k observations on the response variable are available, say y_1, y_2,…., y_{12}, for each observed response y_i, we will have an observation on each regression variable. Let x_{ij} denotes the i[th] observation of variable x_j. The model in terms of the observations may be written in matrix notation as

$$y = X\beta + \varepsilon \quad (3)$$

Where y is an nxl vector of the observations, X is an nxp matrix of the levels of the independent variables. β is a px1 vector of the regression coefficients. ε is an nx1 vector of random errors.

Our aim is to find the vector of least squares estimators, *b*, that minimizes the following function,

$$L = \sum_{i=1}^{n} \varepsilon_i^2 = \varepsilon' \varepsilon = (y - X\beta)'(y - X\beta) \quad (4)$$

After some simplifications, the least squares estimator of β is derived as,

$$b = (X'X)^{-1} X'y \quad (5)$$

It is easy to note that $X'X$ is a pxp symmetric matrix and $X'y$ is a px1 column vector. The diagonal elements of $X'X$ are the sums of squares of the elements in the columns of X. The off-diagonal elements of $X'X$ are the sums of cross-products of the elements in the columns of X. The elements of $X'y$ are the sums of cross-products of the columns of X and the observations $\{y_i\}$.The fitted regression model is given by,

$$\hat{y} = Xb \quad (6)$$

It is always necessary to examine the fitted model to ensure that it provides an adequate approximation to the true system. We consider several techniques for checking model adequacy and verify that none of the least squares regression assumptions are violated.

The method of least squares produces an unbiased estimator of the parameter β in the multiple linear regression models. The residual sum of squares is,

$$SS_E = \sum_{i=1}^{n} (y_i - \hat{y}_i)^2 = \sum_{i=1}^{n} e_i^2 \quad (7)$$

$$SSE = y'y - b'X'y \quad (8)$$

The total sum of squares is given by,

$$SS_T = y'y - \left(\sum_{i=1}^{n} y_i\right)^2 \Big/ n = \sum_{i=1}^{n} y_i^2 - \left(\sum_{i=1}^{n} y_i\right)^2 \Big/ n \quad (9)$$

The coefficient of multiple determination R^2 is given by,

$$R^2 = 1 - \frac{SS_E}{SS_T} \quad (10)$$

Where R^2 is a measure of the amount of reduction in the variability of y obtained by using the regression variables x_1, x_2, ..., x_k in the model. Where x_1, x_2 and x_3 are the radius of inner hub hole, hub web outer interface and hub/rim outer interface respectively.

The objective function for minimization of mass of the railway wheel is as follows,

$$f(x_1, x_2, x_3) = \rho[4.8235 x_1^3 + 1.011 x_1 x_2^2 + 3.03 x_1 x_3^2] \quad (11)$$

The constraints for the minimisation are as follows, $\sigma_{applied} \le \sigma_y$, $x_2 - x_1 \le 0$, $4x_3 - 3x_1 \le 0$, $x_1, x_2, x_3 \ge 0$, and $y(x_1, x_2, x_3) \le 0.62$.Where y is natural frequency. The applied stress applied is derived from FEA. The variation of the natural frequency for the considered design variable ranges by running the simulation in ABAQUS software many times. A ±20% variation (-1 for -20% and 0 for standard value and 1 for +20%) in design variables is used for the FE simulations based on CCD [7]. Table 3 the frequency results of the CCD array and highlighted in bold are the rotational mode frequencies of interest to the fatigue problem.

Table 3: Result of frequency analysis with design model (values in bold refer rotational mode)

STD Order	Coded values of design variables			Mode No. with frequency (in Hz)					
	x1	x2	x3	1	2	3	4	5	6
1	-1	-1	-1	0.2931	0.2931	0.4711	**0.56697**	0.6958	0.6963
2	1	-1	-1	0.2947	0.2947	0.4728	**0.61633**	0.6958	0.6964
3	-1	1	-1	0.3253	0.3253	0.494	**0.61579**	0.6432	0.6442
4	1	1	-1	0.3277	0.3277	0.4963	0.64248	0.6435	**0.6759**
5	-1	-1	1	0.3252	0.3252	0.4941	**0.61582**	0.6391	0.6401
6	1	-1	1	0.3271	0.3271	0.4959	0.63923	0.6402	**0.6744**
7	-1	1	1	0.341	0.341	0.5166	**0.62849**	0.64	0.6407
8	1	1	1	0.3425	0.3425	0.5181	0.64005	0.6408	**0.69**
9	-1.68179	0	0	0.3326	0.3326	0.51	**0.59531**	0.6531	0.6539
10	1.68179	0	0	0.3357	0.3357	0.513	0.65329	0.654	**0.6937**
11	0	-1.68179	0	0.3221	0.3221	0.5021	**0.62456**	0.6719	0.6727
12	0	1.68179	0	0.3402	0.3402	0.5158	0.64486	0.6456	**0.657**
13	0	0	-1.68179	0.2997	0.2997	0.4792	**0.58815**	0.7221	0.7229
14	0	0	1.68179	0.3213	0.3213	0.4987	**0.64426**	0.645	0.6456
15	0	0	0	0.315	0.315	0.497	**0.61904**	0.6792	0.6799
16	0	0	0	0.315	0.315	0.497	**0.61904**	0.6792	0.6799
17	0	0	0	0.315	0.315	0.497	**0.61904**	0.6792	0.6799
18	0	0	0	0.315	0.315	0.497	**0.61904**	0.6792	0.6799
19	0	0	0	0.315	0.315	0.497	**0.61904**	0.6792	0.6799
20	0	0	0	0.315	0.315	0.497	**0.61904**	0.6792	0.6799

In order to evaluate influential factors in the quadratic RSM, an ANOVA table was established as given in Table 4. The variance analysis was statistically significant at a 99.1% confidence level. The design variables x_1, x_2 and x_3 have significant effects on the natural frequency of the wheel. Finally, the assumptions for regression and variances of the residuals are verified. Therefore it can be concluded that the quadratic model is adequate enough to describe the natural frequency response surface. The equation for the fitted quadratic regression model is given by,

$$\begin{aligned} y(x_1,x_2,x_3) &= 0.619+0.0289\,x_1+0.0139\,x_2 \\ &+0.0167\,x_3+0.009158\,x_1^2+0.0078\,x_2^2 \\ &-0.000847\,x_3^2+0.001703\,x_1x_2 \\ &+0.001330\,x_2x_3-0.01002\,x_1x_3 \end{aligned} \quad (12)$$

Table 4: ANOVA for fatigue initiation life

Factor	Regression	Error
Sum of square	0.020744	0.000193
DOF	9	10
Adj sum of square	0.020744	0.000193
Adj Mean square	0.002305	0.000019
F	119.15	-
P	0	-

The effects of design variables on the natural frequency can be understood from the response function using FE simulation in Abaqus. To determine the optimal natural frequency, 3D response surface plots were plotted using the responses of the natural frequency. The 3D surface plots were generated as a function of a pair of significant design variables while holding the third significant variable constant for each response. Figs. 6 to 8 show the 3D response surface at the three levels of x_1, x_2 and x_3 respectively. The relationship between the responses and design variables and the nature of the stationary point can be easily predicted. From the predicted responses it is clear that the stationary point is representing a point of minimum response for natural frequency.

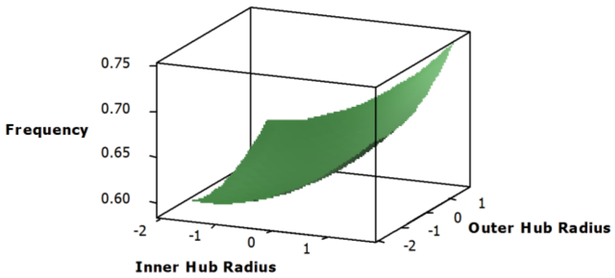

Fig. 6: Response surface of natural frequency for constant x_1

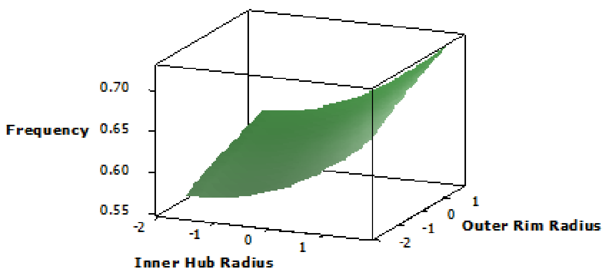

Fig. 7: Response surface of natural frequency for constant x_2

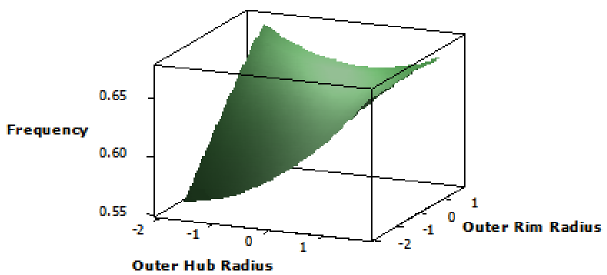

Fig. 8: Response surface of natural frequency for constant x_3

Eqn. (12) is further used as a constraint for the objective function for mass optimization of the wheel. The objective function and constraint equations were solved using Matlab code. Standard wheel mass was 210.38kg. After running the MATLAB code, the optimum mass obtained is 200.8724 kg (4%reduction).

4. Conclusions

The main objective of this study was to carry out parametric analysis of a railway wheel to minimize the weight of the railway wheel. The wheel was modelled by some initial dimensions subjected to impose constraints like natural frequency that was obtained through FE analysis. The design variables were the inner hub hole radius, outer radius of fillet at hub/web interface and outer radius of fillet at web/rim interface. The natural frequency values for varying the design variable has been inspected using RSM and ANOVA and an equation is obtained to understand the behaviour of the responses. The responses have been plotted and found that the equation is fit for prediction of natural frequency at any point of variation of design variable.

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