Ghazaly et al. 2014. *Int. J. Vehicle Structures & Systems*, 6(3), 58-63
ISSN: 0975-3060 (Print), 0975-3540 (Online)
doi: 10.4273/ijvss.6.3.03

International Journal of Vehicle Structures & Systems
Available online at www.maftree.org/eja

Experimental Evaluation of an Empirical Model for Wedge Disc Brake using Box-Behnken Design

N.M. Ghazaly[a,c], M.M. Makrahy[b], K.A.A.E. Gwwad[b], K.R. Mahmoud[b] and A.M.A.E. Tawwab[b]

[a]*Mechanical Engineering Dept., Faculty of Engineering, South Valley University, Egypt.*

[b]*Automotive and Tractor Engg. Dept., College of Engineering, Minia University, Egypt.*
[c]*Corresponding Author, Email: noubyluxor@gmail.com*

ABSTRACT:

This article is a case study of Box-Behnken design application for development and evaluation a new empirical model to predict the performance of wedge disc brake. Box-Behnken design is used to study the applied pressure, rotational speed, wedge inclination angle and quantity of water spraying. A total of 27 experimental tests are conducted using brake dynamometer to estimate the main and interaction effect of these parameters. The results of analysis of variance for the braking forces from the input parameters are obtained to evaluate the goodness of the developed empirical model through F- and P-values. It is found that a strong correlation between the empirical regression model and the actual results. Moreover, in order to confirm the validity of the empirical model, additional 16 experimental tests are carried out using Taguchi method. The results showed a good agreement between the predicted empirical model and experimental results. This indicates that the developed empirical model based on operating parameters can be used successfully to predict the wedge disc brake performance.

KEYWORDS:

Box-Behnken design; Wedge disc brake; Empirical model; Brake dynamometer; Design of experiments

CITATION:

N.M. Ghazaly, M.M. Makrahy, K.A.A.E. Gwwad, K.R. Mahmoud and A.M.A.E. Tawwab. 2014. Experimental Evaluation of an Empirical Model for Wedge Disc Brake using Box-Behnken Design, *Int. J. Vehicle Structures & Systems*, 6(3), 58-63. doi:10.4273/ijvss.6.3.03.

1. Introduction

Brake performance remains an important economic and technical issue in the automotive industry despite much progress being made especially in the last decade. Many of the brake mechanisms are far from being fully improved their performance. Experimental approaches using brake dynamometers have been widely used to investigate the brake performance, to study the influence of different design parameters at various operating conditions, to understand the characteristics of the brake system during braking event and to verify solutions of simulation models [1-3]. The conventional and classical methods of studying the brake performance by maintaining other factors constant does not describe the combined effect of all the factors involved. The conventional technique for the optimization of a multivariable system usually sets one-factor at a time (OFAT). This approach needs to conduct a lot of experiments and could not give the alternative effects between factors. The OFAT method is not only time and energy consuming, but also usually incapable of reaching the true optimal conditions because it neglects their interactions [4].

Recently, design of experiments (DOE) has been utilized in design process optimization. The main objective of DOE is to optimize a response which is influenced by several independent input factors by understanding their interactions and their effects on the output response which could not be accomplished through a tedious, time consuming, and ineffective traditional system of OFAT optimization [5-8]. DOE is a very useful tool for this purpose as it provides statistical models for designing and analyzing the results [9]. Response surface methodology (RSM) is one suitable method of DOE that is used in various researches and scientific fields [10]. RSM can be defined as statistical techniques useful for developing, improving, and optimizing processes [11]. The main advantage of RSM is the reduced number of experimental trials needed to evaluate multiple parameters and their interactions. It is often employed to fit a polynomial model to the given data and to obtain the optimal factors [12, 13].

RSM is more practical compared to theoretical models as it arises from experimental methodology which includes interactive effects of the variables and eventually it depicts the overall effects of the parameters on the process. There are several types of RSM designs [14-17] such as full factorial designs, central composite design and Box-Behnken design (BBD). BBD is an independent, rotatable or nearly rotatable quadratic design that contains no embedded factorial or fractional factorial design. The design is based on combinations from the midpoint of edges of the process space and at the center. Since the BBD is almost rotatable it needs three levels of each parameter. This design is preferable when the number of runs needs to be reduced. BBD is cheaper than full factorial designs or central composite designs. BBD space does not include the extreme points of the cube [18].

In the present work, experimental investigations are carried out using BBD to develop an empirical model to predict the performance of wedge disc brake performance based on operating parameters. Four factors are studied and three levels are selected. Twenty seven experiments are conducted according to BBD. Minitab program is used for regression and graphical analysis of the data obtained. The variability in dependent variables is explained by the multiple coefficient of determination, R^2 and the empirical equation is used to predict the model and subsequently the interaction between the factors within the specified range is clarified. The effect of operating parameters is analyzed using the RSM plots.

2. Experimental setup

A schematic representation of the test rig is shown in Fig. 1. The brake test rig is designed to provide the necessary disc rotation speed, applied pressure, wedge angle and quantity of water spray to evaluate the performance of the wedge disc brake system. Full details of the design are outlined in our previous work [19]. The test rig is divided into driving unit, braking unit and measurement facilities. Fig. 2 shows the test rig with its different units. The driving unit consists of an A.C. motor of 18.56 kW and 1500 rpm, that rotates the driving shaft at different rotating speeds. The braking unit comprises the new wedge disc brake assembly, as shown in Fig. 2. The instruments for measurement include - rotational speed (tachometer), applied pressure (a pressure gauge), temperature (thermocouple) and tangential force (load cell).

To examine the performance of the wedge brake at different operating parameters, various vehicle speeds at 6, 11.6, 22.6 and 36.3 km/hr. are achieved through gearbox reduction ratios and measured by speed tachometer. Different brake pressure in the range 2.5 to 10 bar is controlled and wedge angle is adjusted manually between 15˚ to 45˚.Quantity of water spraying is controlled up to 240 mm^3 through all performance tests. Four-channel data acquisition system is used to monitor braking forces. The acquired signals are transferred to a computer in digital form for storage and further analysis, see Ref. [19] for more details.

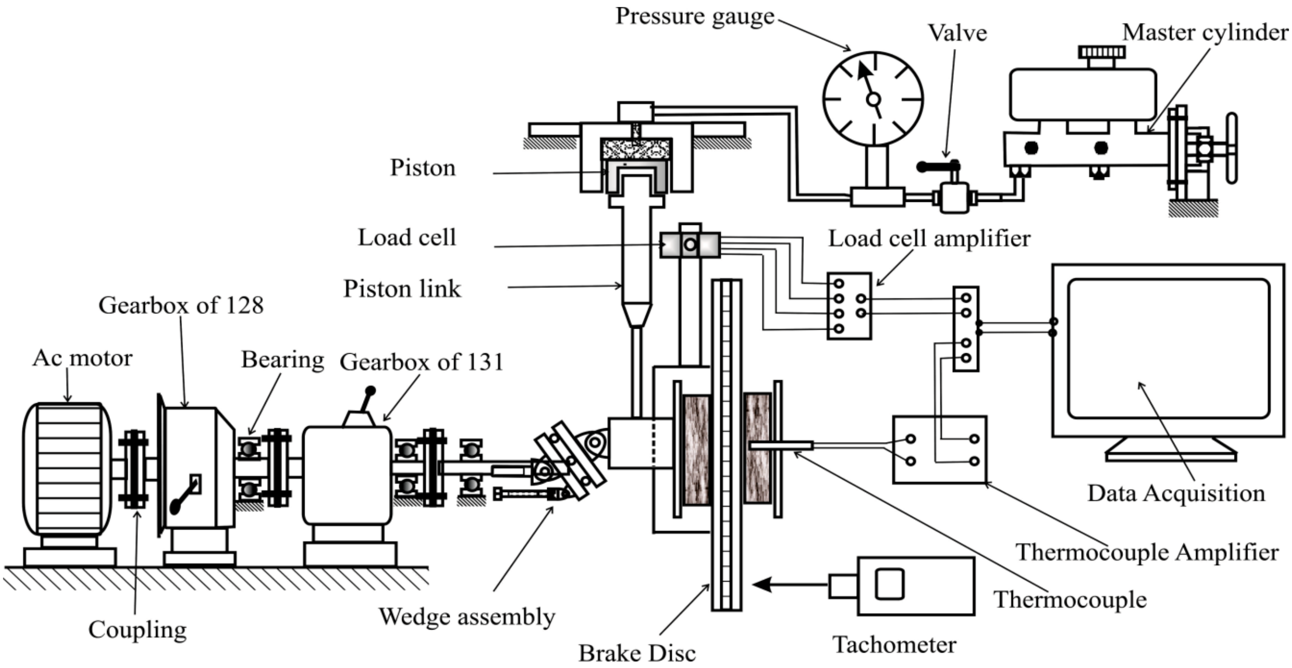

Fig. 1: Schematic of brake dynamometer test setup and measurement instrumentation

Fig. 2: Main components of the brake dynamometer

3. BBD based optimization

BBD based on RSM is used to determine the optimum operational conditions for the system. The behaviour of the system is explained by a 2nd order polynomial using:

$$y = \beta_0 + \sum_{i=1}^{k} \beta_i \chi_i + \sum_{i=1}^{k} \beta_{ii} \chi_i^2 + \sum_{i<j=2}^{k} \beta_{i_j} \chi_i \chi_j \pm \varepsilon \qquad (1)$$

Where y is the output response of the brake system, x_i, x_j,... x_k are the coded independent variables, k is the number of the patterns, i and j are the index numbers for pattern, β_0 is called intercept term, β_i is the first-order main effect, β_{ii} is the quadratic effect, β_{ij} is the interaction effect, and εis the random error between the predicted and measured value. It is assumed that the independent variables are continuous and controllable by experiments with negligible errors. It is required to find a suitable approximation for the true relationship between independent variables and the response surfaces.

In this study, the BBD is chosen for finding out the relationship between the brake force response and input operation parameters. A 3-level 4-parameter BBD is used to identify the best combinations of the operation parameters which involved in the experimental tests to obtain high brake performance. The input variables are applied pressure, rotational speed, wedge inclination angle and quantity of water spraying. Each parameter is coded as -1, 0, and 1 level for the BBD as given in Table 1. These experiments are run according to a random sequence or run order provided by the DOE fractional technique. Twenty seven tests are performed and used to find out the relationship between braking force of the wedge disc brake and the operating parameters using BBD. The actual levels of each variable for the experiments in the design matrix are carried out and experimental results using brake test rig are obtained and given in Table 2.

Table 1: Coded and actual values of independent variables

Parameters		Coded levels of variables		
		-1	0	1
Applied pressure (bar)	X_1 (A)	2.5	6.25	10
Rotational speed (rpm)	X_2 (B)	54	205	329
Wedge angle (°)	X_3 (C)	15	30	45
Water spraying (mm^3)	X_4 (D)	0	120	240

Table 2: BBD matrix, experimental and predicted braking force

Runs	Pressure (bar)	Speed (rpm)	Angle (°)	Water (mm^3)	$Y_{exp.}$ (N)	$Y_{pred.}$ (N)	Diff. %
1	6.25	54	30	240	2030	1923	5.56
2	2.5	329	30	120	806	811	-0.62
3	6.25	205	45	0	1543	1559	-1.03
4	2.5	205	45	120	842	768	9.64
5	10	329	30	120	3087	2956	4.43
6	6.25	329	30	240	1968	1806	8.97
7	6.25	205	15	0	3140	3028	3.70
8	6.25	54	30	0	2325	2250	3.33
9	6.25	205	30	120	2137	2065	3.49
10	2.5	205	15	120	1562	1608	-2.86
11	10	205	45	120	2723	2420	12.52
12	6.25	205	15	240	2883	2807	2.71
13	10	205	30	240	3109	2986	4.12
14	2.5	54	30	120	936	1017	-7.96
15	2.5	205	30	240	847	821	3.17
16	6.25	329	45	120	1408	1387	1.51
17	6.25	205	30	120	2137	2065	3.49
18	6.25	54	15	120	3199	3056	4.68
19	6.25	329	15	120	2993	2859	4.69
20	10	205	15	120	4111	4377	-6.08
21	10	54	30	120	3336	3306	0.91
22	6.25	329	30	0	1933	1811	6.74
23	6.25	205	30	120	2137	2065	3.49
24	2.5	205	30	0	951	926	2.70
25	6.25	205	45	240	1426	1478	-3.52
26	6.25	54	45	120	1776	1746	1.72
27	10	205	30	0	3305	3182	3.87

4. Results and discussions

The main objective of analysis of variance (ANOVA) is to analyze the accessibility of the predicted model. The significance test of regression model and single model coefficients and distortion test are carried out using Minitab 16 statistical software. Based on BBD, 27 combination tests are developed and processed to obtain brake force of the wedge disc brake. The results of ANOVA for braking forces from input parameters are given in Table 3. To evaluate the goodness of the model the statistical significance of a quadratic model is tested through F- and P-values. A large F-value indicates that most of the variation can be explained by a regression equation. P-value less than 0.05 indicates that the model is considered to be statistically significant [20].The F value is the ratio of model mean square (MS) to the appropriate error mean square. From Table 3 and based on P-values the applied pressure (A), the wedge angle (C), interaction between applied pressure and wedge angle (A*C), and square terms for wedge angle (C*C) are the most significant parameters.

Table 3: ANOVA for braking force quadratic model

Source	DF	Seq. SS	Adj. SS	Adj. MS	F-value	P-value
Regression	14	20768740	20768740	1483481	40.84	0
A	**1**	**14687502**	**14563710**	**14563710**	**400.9**	**0**
B	1	153372	164817	164817	4.54	0.055
C	**1**	**4970017**	**4877621**	**4877621**	**134.27**	**0**
D	1	72601	81763	81763	2.25	0.159
A*A	1	5250	169	169	0	0.947
B*B	1	52901	13585	13585	0.37	0.552
C*C	**1**	**438306**	**315659**	**315659**	**8.69**	**0.012**
D*D	1	34986	34986	34986	0.96	0.346
A*B	1	5145	5145	5145	0.14	0.713
A*C	**1**	**311788**	**311788**	**311788**	**8.58**	**0.013**
A*D	1	2102	2102	2102	0.06	0.814
B*C	1	3581	3581	3581	0.1	0.759
B*D	1	26269	26269	26269	0.72	0.412
C*D	1	4919	4919	4919	0.14	0.719
Res. error	12	435931	435931	36328		
Lack of fit	10	435931	435931	43593		
Pure error	2	0	0	0		
Total	26	21204671				

The coefficient of determination, R^2, is defined as the ratio of the explained variation to the total variation and is a measure of the fit degree. When R^2 approaches to unity, it indicates a good correlation between the experimental and the predicted values. As shown in Table 4, the goodness of the model can be confirmed by R^2(0.9794) and the adjusted R^2 (0.9555). Both values are closer to 1, which indicates a good correlation between the observed and the predicted values. Value of adjusted R^2(0.8811) means that the total variation of 88.11% $Y_{exp.}$ is attributed to the independent variables and the remainder 10.99% of the total variation cannot be explained by the model.

Table 4: R^2 analysis

Parameter	SD	PRESS	R^2	Adj. R^2	Pred. R^2
Value	109.598	2520253	97.94%	95.55%	88.11%

Regression analysis is performed to determine the surface response as function of second order polynomial equation. The empirical relationship between the braking force response and the independent variables in terms of coded parameters is obtained using,

$$\begin{aligned} Y_{pred} &= 1655.48 + 468.941A + 0.46188B - 76.1967C \\ &- 0.54596D - 0.4003A^2 - 0.002706B^2 \\ &+ 1.03812C^2 - 0.00562D^2 - 0.06933AB \\ &- 4.9633AC - 0.0509AD - 0.0197BC \\ &+ 0.00489BD + 0.01948CD \end{aligned} \quad (1)$$

The adequacy of quadratic model is tested through the correlation between predicted and experimental values of braking forceas shown shown in scattered plot Fig. 3. From correlation between $Y_{pred.}$ and $Y_{exp.}$ values of braking force is evident that the regression model can represent the experimental data well. On the basis of the evaluation of ANOVA outputs, the statistical significance of a quadratic model for the response is confirmed and it can be concluded that the model can be used for analysing the effect of process variables.

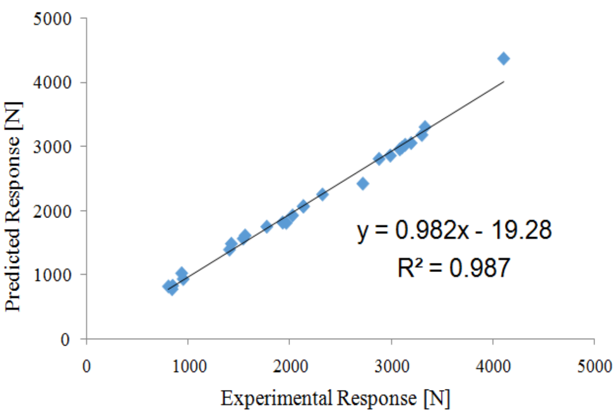

Fig. 3: Predicted vs. Experimental braking force values

3D surface plots of the braking force that is obtained from response surface plots as a function of two parameters at a time maintaining all other parameters at a fixed level are shown in Fig.4 (a) to (f). The results of Fig. 4 (a) indicate that, the braking force increases for an increase in the applied pressure. The braking force decreases with increasing the rotation speed. From Fig. 4(b), it can be seen that the braking force increases for an increase in the applied pressure and a reduction in the wedge angle. This indicates that both the applied pressure and wedge angle has the significant effect on increasing the brake performance. From Fig. 4(c) the braking force increases for an increase in the pressure force and a reduction in the quantity of water spray. From Fig. 4(d), the braking force increases for a reduction in the wedge angle and vehicle speed. From Fig. 4(e), the braking force decreases for an increase in the vehicle speed and the quantity of water spray. From Fig. 4(f), the braking force increases for a reduction in wedge angle and the quantity of water spray.

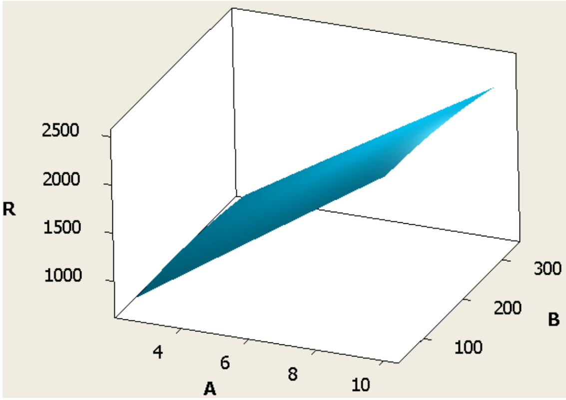

Fig. 4(a): Applied pressure and rotational speed vs. Braking force

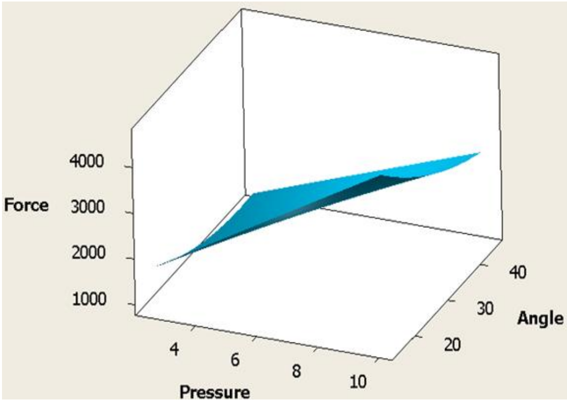

Fig. 4(b): Applied pressure and wedge angle vs. Braking force

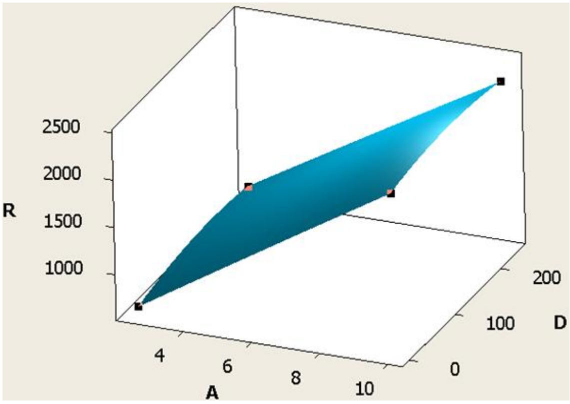

Fig. 4(c): Applied pressure and water spray vs. Braking force

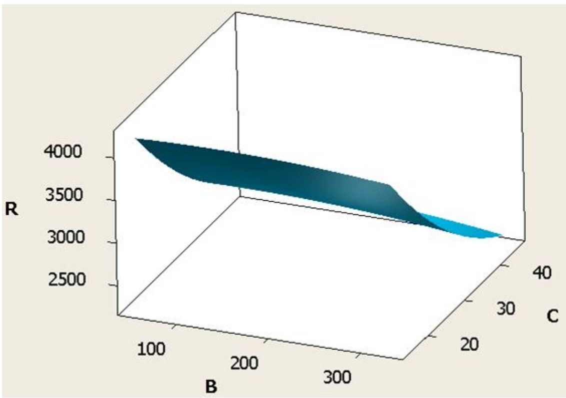

Fig. 4(d): Rotational speed and wedge angle vs. Braking force

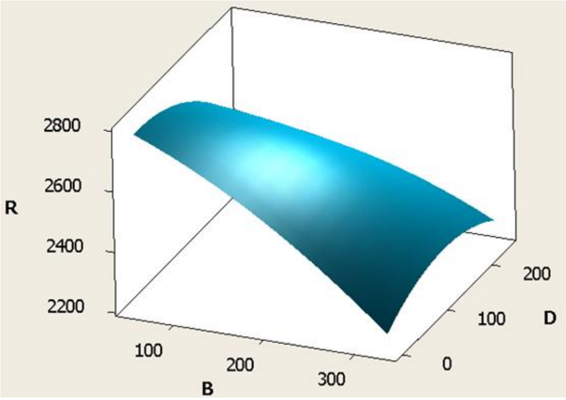

Fig. 4(e): Rotational speed and water spray vs. Braking force

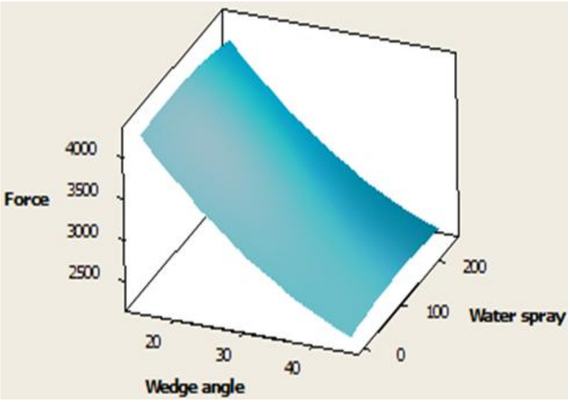

Fig. 4(f): Wedge angle and water spray vs. Braking force

In order to confirm the validity of regression equation, additional sixteen runs were carried out through Taguchi approach [21]. The difference between the predicted results using the emprical equation and experiments are shown in Table 5. The predicted responses show good agreement with actual results. The average absolute percentage deviation is found to be 5.91. This indicates that designed model space can be navigated well for prediction of the braking force.

Table 5: Difference between predicted and experimental results

Runs	Pressure (bar)	Speed (rpm)	Angle (°)	Water (mm^3)	$Y_{exp.}$ (N)	$Y_{pred.}$ (N)	Diff. %
1	2.5	54	45	0	873	900.12	-3.01
2	2.5	105	35	80	918	929.72	-1.26
3	2.5	205	25	160	1089	1090.23	-0.11
4	2.5	329	15	240	1420	1405.61	1.02
5	5	54	35	160	1754	1566.9	11.94
6	5	105	45	240	1242	1253.75	-0.94
7	5	205	15	0	2483	2557.88	-2.93
8	5	329	25	80	1767	1764.31	0.15
9	7.5	54	25	240	2782	2559.58	8.69
10	7.5	105	15	160	3255	3451.91	-5.70
11	7.5	205	45	80	2102	1881.03	11.75
12	7.5	329	35	0	2145	1913.11	12.12
13	10	54	15	80	4298	4538.78	-5.30
14	10	105	25	0	4099	3702.1	10.72
15	10	205	35	240	3016	2697.33	11.81
16	10	329	45	160	2497	2169.02	15.12

5. Conclusions

In this work, the Box-Behnken design is conducted to develop and evaluate a new empirical model based on operating parameters in order to predict the performance of wedge disc brake. According to BBD, the applied pressure, the rotational speed, the wedge angle and quantity of water spray are selected as the independent variables and the braking force is considered as the output response. With minimum number of experiments, data is collected and the models are developed. The developed BBD model show the effect of each input parameter and its interaction with other parameters, depicting the trend of braking force response. The significance test of regression model is carried out using Minitab statistical package. Verification of the fitness of the model using ANOVA technique shows that the model can be used with confidence level of 0.95, for navigating the design space. Surface plots generated show the trend of different responses by varying the two input parameters keeping the other parameter constant. Results indicated that the BBD is reliable for optimizing the braking forces of wedge disc brake precisely.

REFERENCES:

[1] E.N.S.M. Tayeb and K.W. Liew. 2009. On the dry and wet sliding performance of potentially new frictional brake pad materials for automotive industry, *Wear*, 266, 275-287. http://dx.doi.org/10.1016/j.wear.2008.07.003.

[2] M. Nouby and K. Srinivasan. 2011. Simulation of structural modifications of a disc brake system to reduce brake squeal, *J. Automobile Engineering*, 225(5), 653-672. http://dx.doi.org/10.1177/2041299110394515.

[3] D. Zimmer. 1982. ATE friction test machine and other methods of lining screening, *SAE Tech. Paper* 820163.

[4] M.N. Ghazaly. 2011. *Study on Automotive Disc Brake Squeal using Finite Element Analysis and Design of Experiments*, PhD Thesis, Dept. Mech. Engg., Anna University, India.

[5] D.C. Montgomery (Ed.). 2001. *Design and Analysis of Experiments*, 5th Edition, John Wiley & Sons, New York.

[6] G.E.P. Box, W.G Hunter and J.S. Hunter. 1978. *Statistics for Experimenters - An Introduction to Design, Data Analysis and Model Building*, 2nd Edition, John Wiley & Sons, New York.

[7] V. Czitrom. 1999. One-factor-at-a-time versus designed experiments, *The American Statistician*, 53, 126-131.

[8] D.D. Frey, F. Engelhardt and E.M. Greitzer. 2003. A role for one-factor-at-a-time experimentation in parameter design, *Research in Engineering Design*, 14, 65-74.

[9] R.L. Mason, R.F. Gunst and J.L. Hess. 2003. *Statistical Design and Analysis of Experiments with Applications to Engineering and Science*, 2nd Edition, John Wiley and Sons, New York. http://dx.doi.org/10.1002/0471458503.

[10] E.J. Contreras, D.T. Salinas, R.B. Moreno, R.R. Baños and E.D.L. Cózar. 2007. Response Surface Methodology and its application in evaluating scientific activity, *Proc. 11th Int. Conf. Scientometrics and Informetrics*, Madrid.

[11] R.H. Myers, D.C. Montgomery and M.A. Christine. 2009. *Response Surface Methodology, Process and Product Optimization using Designed Experiments*, Wiley, New York.

[12] L. Basil, R.R. Ev, C.D. Milcharek, L.C. Martins, F.A. Pavan, A.A. Dos Santos Jr., S.L.P. Dias, J. Dupont, C.P.Z. Noreña and E.C. Lima. 2006. statistical design of experiments as a tool for optimizing the batch conditions to Cr(VI) biosorption on araucaria angustifolia wastes, *J.*

Hazard. Mater., 133(1-3), 143-153. http://dx.doi.org/10.1016/j.jhazmat.2005.10.002.

[13] D.C. Montgomery. 2005. *Design and Analysis of Experiments: Response Surface Method and Designs*, John Wiley and Sons, New Jersey.

[14] G.E.P. Box and D.W. Behnken. 1960. Some new three level designs for the study of quantitative variables, *Technometrics*, 2, 455-475. http://dx.doi.org/10.1080/00401706.1960.10489912.

[15] G.E.P. Box, W.G. Hunter, J.S. Hunter. 2005. *Statistics for Experimenters: An Introduction to Design, Data Analysis, and Model Building*, 2nd Edition, John Wiley & Sons, New York.

[16] C. Daniel. 1976. *Applications of Statistics to Industrial Experiments*, Wiley, New York. http://dx.doi.org/10.1002/9780470316467.

[17] D.C. Montgomery, 1997. *Design and Analysis of Experiments*, 4th Edition, John Wiley & Sons, New York.

[18] T.B. Beielstein, M. Chiarandini, L. Paquete and M. Preuss. 2010. *Experimental Methods for the Analysis of optimization Algorithms*, Springer-Verlag, Berlin. http://dx.doi.org/10.1007/978-3-642-02538-9.

[19] M.M. Mostafa, M.G. Nouby, K.A.A.E. Gwwad, K.R. Mahmoud. and A.M.A.E. Tawwab. 2013. A preliminary experimental investigation of a new wedge disc brake, *Int. J. Engg Research and Applications*,. 3 (6), 735-744.

[20] R.H. Myers and D.C. Montgomery. 2002. *Response Surface Methodology: Process and Product Optimization using Designed Experiments*, John Wiley & Sons, New York.

[21] M.M. Mostafa, K.A.N.A.E. Gwwad, K.R. Mahmoud and A.M.A.E. Tawwab. 2013. Optimization of operation parameters on a novel wedge disc brake by Taguchi method, *American J. Vehicle Design*, 1(2), 21-24.