Multibody Modelling of Engine and Minimization of Engine Mount Vibration using Ant Colony Algorithm Optimization

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ABSTRACT:

In this research work, multibody modelling of engine is carried out and the elastomeric mounts are used to isolate the vibration caused in the diesel engine during its running. Vibration in the engine is measured as displacement at the engine mounts and the reduction of vibration displacement provides both the passive isolation and balancing of unbalanced forces in the engine. The elastomeric mounts not only provide the passive isolation but also the supplement the force balancing system. The engine dynamic vibration force components like inertial forces by moving components and piston slap is modelled and resolved in the Cartesian coordinates. The set of equations arrived are solved to find the displacement at the mounts using Runge-Kutta method for a specific period. The mount location and orientation are used as the optimizing parameters that minimize the mount displacement. The optimization is carried out using one of the most promising heuristic optimization are obtained for different control parameters of the ACA.

KEYWORDS:

Elastomeric mount; Ant colony algorithm; Unbalanced forces; Slap forces; Equation of motion

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ACRONYMS AND NOMENCLATURE:

P_{gas}	Cylinder pressure
m_p	Mass of the piston
δ	Angle difference between the cranks
ϕ	Angle of connecting rod with the cylinder axis
ω	Angular velocity of the crankshaft
L	Connecting rod length
$m_{1,}m_{2}$	Mass of crank, connecting rod, piston respectively
θ, r	Crank angle and radius respectively
<i>x</i> , <i>y</i> , <i>z</i>	Displacement along x,y and z axes
a, b, c	Distance of connecting rod mass centre from crank pin, piston pin, crankshaft axis respectively
d	Engine mount displacement
$m_{A,} m_B$	Equivalent mass about mass centre of connecting rod and crank respectively
A_{2}, A_{4}, A_{6}	Fourier constants
F_i	Inertia force
$m_{rec,} m_{rot}$	Mass of reciprocating & rotating parts respectively
М	Mass of the engine
$\theta_{x,y,z}$	Mount orientation angles
x_m, y_m, z_m	Mount position with respect to the mass centre
$k_{x_{x}} k_{y_{y}} k_{z}$	Mount stiffness in x, y and z axis respectively
n	Number of cylinders
F_m	Force of the mount
F_s	Slap force
Ν	Speed of the engine
β	V-angle of the engine

1. Introduction

The problem of vibrations exists in all engineering applications and is usually associated with unbalanced forces of the moving components. Internal combustion engines receive the vibration from the unbalanced forces of the moving components, which are interconnected. Mechanical systems are described by such type of equations are called linear parametrically excited systems [3]. The development of a multibody model for the four-cylinder engine is presented in this work. The requirement for evaluation of the loads acting on the engine components at rotational speed makes it necessary to take order to identify the unbalanced effects of forces, which might have a major impact on the dynamics of the system. In the mechanical systems of Internal Combustion (IC) engines, there are two additional time-varying factors due to the piston reciprocating motion, which is the periodic change of the direction of motion and the sudden impacts of the piston on the cylinder due to the variation in the cylinder pressure [5]. The engine of an automobile develops and transfers the unbalanced forces to the supporting structures. The unbalanced forces from the engine are due to the rotating and reciprocating parts of the engine that reduce the durability and reliability of automobiles.

The engine unbalanced forces are measured in terms of mount displacement at the engine mounts. The

unbalanced forces are measured as engine mount displacement at the mounts [7]. The reduction of mount displacement can be done many ways and two of the methods are chosen in this research work. One of the methods is balancing the unbalanced forces at the engine itself and the other is using the good vibration damper [6]. In the prediction of vibration, transmission from the mounting system to the chassis is one of important problem and true in the design of a resilient mounting system since the selection of appropriate mounting elements is highly constrained by unbalanced forces. The exciting sources of an engine are due to rotational imbalance and reciprocating masses. A mathematical model is developed from the engine multibody physics [1] such that the engine piston cylinder arrangement configurations and mount parameters are used as inputs and the output will be the displacement. Finally, ant colony algorithm (ACA) and artificial bee colony algorithm is used to predict the optimized engine parameter such that the engine mount displacement at the mounts is minimized.

2. Multibody modelling of engine and mount system

The use of vibration isolator for the IC engines is of most important to damp out the vibratory forces impinged by the unbalanced forces of the engine system. A mathematical model need to be developed for enginemount systems and have described in terms mass M, stiffness K and viscous damping matrix C. The minimization of forces transmitted from the engine to the mounts is carried out in this work for that the engine is modelled as a rigid body subjected to periodic loadings and the engine mounts assumed to be viscously damped. The formulated equation of motion contained three constant matrices M, K and C of coupled system of differential equations with the consideration of the inertia effects of rotating and reciprocating parts. The engine is modelled as a rigid body [17] that is supported on three engine mounts of vibration isolators as shown in Fig. 1. A rigid-body model of a structure with geometry points remains fixed relative to one another and global coordinate of engine system G_{xyz} is located at the mass centre of the engine when the engine system is statically equilibrium. The three orthogonal coordinate axes, which are shown in Fig. 1, are X axis parallel to the cylinder axis, Z-axis parallel to the crank axis and Y axis is orthogonal to both X and Z axes.



Fig. 1: Rigid body model of the engine-mount system

The rigid-body model consists of six DOFs that include three translation and three rotation modes respectively. In this paper, a modelling of a dynamic equation of an engine-mount system is presented. In this dynamic model, the contribution of inertial effects of the rotating and reciprocating parts are considered into account and leads to the mass matrix and the velocity vector are developed. The prediction of the responses of the unbalanced forces is necessary to provide required input information for the optimization problems [9]. The vibration responses of an engine-mount model are used to evaluate the displacement of the engine-mount system, and for further consideration of optimization problems, where the response of the system is utilized as input information. The equation of motion of the enginemount system is given by,

$$[M]\ddot{x} + [K]x = \{F\}e^{i\omega t} \tag{1}$$

Where [M] and [K] are mass stiffness matrices and x is the displacement. In the case of crankshafts with asymmetric mass distribution due the reciprocating parts, a mass balancing system has been incorporated. It is important to include time-varying components generated by crankshaft rotation and by reciprocating pistons [10]. The matrices are incorporated with not only the inertial effects of the moving components but also with the slap of the piston and the effect of constant damping of the resilient mounts.

The inertial effect of the moving components are formulated by considering the inertial effects of the rotating and reciprocating components i.e. crank shaft, piston and connecting rod. The inertial forces are determined for every component with respect to their individual mass centres. The equivalent masses are also calculated and then the reciprocating and rotating masses are determined. The inertial forces of the moving components as shown in Fig. 2 are given by,

$$\begin{pmatrix} m_{rot} r \dot{\theta}_{2}^{2} \sin \theta_{2} - m_{rot} r \ddot{\theta}_{2} \cos \theta_{2} \\ \left(m_{rot} + m_{rec} r \dot{\theta}_{2}^{2} \cos \theta_{2} + m_{rec} r \dot{\theta}_{2}^{2} (A_{2} \\ \cos 2\theta_{2} - A_{4} \cos 4\theta_{2} + A_{6} \cos 6\theta_{2}) \\ + (m_{rot} + m_{rec}) r \dot{\theta}_{2}^{2} \sin \theta_{2} + m_{rec} r \dot{\theta}_{2}^{2} \\ \left(\frac{A_{2}}{2} \sin 2\theta_{2} - \frac{A_{4}}{4} \sin 4\theta_{2} + \frac{A_{6}}{6} \sin 6\theta_{2} \right) \end{pmatrix}$$
(2)

Where $m_A = m_2 b/L$, $m_{rec} = m_3 + m_B = m_3 + m_2 a/L$, $m_B = m_2 a/L$ and $m_{rot} = m_1 c/r + m_A = m_1 c/r + m_2 b/L$.

The crank angle (θ) of subsequent cylinders with reference to the prior one for the given 4-cylinder engine is determined from the firing order of the engine as per Table 1 is given by,

 $\theta_1 = \theta, \ \theta_2 = \theta_1 + \delta, \ \theta_3 = \theta_2 + \delta \text{ and } \theta_4 = \theta_3 + \delta.$

Table 1: Firing order of the diesel engine ($P-Power,\,E-Exhaust,\,I-Intake$ and C-Compression)

Crankshaft	Cylinder Number			
angle (°)	1	3	4	2
0 - 180	Р	С	Ι	Е
180 - 360	Е	Р	С	Ι
360 - 540	Ι	Е	Р	С
540 - 720	С	Ι	Е	Р



Fig. 2: Inertial forces due to the moving components

Piston slap phenomenon is the one of the important factor that induces the vibratory forces in the engine block. Cylinder pressure controlled piston-slap force depends on the pressure-crank angle relation of each cylinder. It is generally difficult to predict the slap induced vibration in the block during the pendulum motion of the piston. Based on the analytical equation of the kinetic motion of the piston connecting rod assembly [11] with respect to the cylinder pressure and the inertial forces the piston slap is calculated. The slap forces induced in the cylinder block are given by,

$$\frac{r}{L}\sin\theta \left[\frac{\pi}{4}D^2P_{gas} - m_p r\omega^2 \left(\cos\theta + \frac{r}{L}\cos 2\theta\right)\right] = F_s \quad (3)$$

The mounts are designed to meet the specific demands of the vibratory system such that the forces exerted from the system should be damped out and the reaction forces exhibited by mount should be minimum [4]. Reaction forces released from the mount will lead to the structure borne vibration. These vibration effects vehicle structure combined with road excitations increase the noise and reduce performance and reliability of the engine [14]. The mount should be placed in right position and orientation so that the dynamic excitations released from the engine is balanced and minimized at the mount. A typical mount used in this work is shown in Fig. 3.



Fig 3: Engine mount system of the engine

The force system of the mount is resolved in the Cartesian coordinates with respect to the orientation angles at the chassis as shown in Fig. 4 is given by,

$$-k_i(x_{mi} + z_{mi}\theta_y - y_{mi}\theta_z) = F_m$$
(4)

The equation of forces from the inertial, slap and mount are summarized to form equation of motion and are deduced to the acceleration equation. These equations are solved for the determination of the reaction forces excited from the engine mount and are given below,

$$\begin{cases} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{cases} = \frac{1}{M} \begin{cases} f_{xm} + f_{xc} + f_{xs} + f_{xb} \\ f_{ym} + f_{yc} + f_{ys} + f_{yb} \\ f_{zm} + f_{zc} + f_{zs} + f_{zb} \end{cases}$$
(5)

The above equation is solved to find the displacement caused at the mount. These summarized equations are second order differential equations with that the mount orientations are the input functions and the reaction force exerted by the mount is objective functions. The model developed to determine the displacement at the mount is simulated with the parameters of the case study engine considered (see Table 2). The displacement is calculated from the reaction forces and these forces are minimized to reduce the displacement at the mount. The equations are solved using the MATLAB to determine the displacements at the mount. The resultant displacement at the mount is determined using,

$$D = \sqrt{x^2 + y^2 + z^2}$$
(6)



Fig. 4: Mount orientation with respect to engine mass centre

Table 2: Parameters of the case study engine

Parameter	Value	Parameter	Value
m_l	2.35 kg	L	120 mm
m_2	1.1 kg	Μ	250 kg
m_3	1.5 kg	N	2500 rpm
m_A	0.76 kg	ω	159 rad/s
m_B	0.34 kg	п	4
m _{rot}	1.935 kg	β	0
m_{rec}	2.94 kg	δ	180°
a	36 mm	P_{gas}	500 kN/m^2
b	81 mm	\tilde{k}_x	35 kN/m
с	10 mm	k_{v}	79 kN/m
r	30 mm	k_{z}	35 kN/m

3. Optimization of the mount orientations with the engine using ACA

The case study has been considered for the optimisation of the engine parameters (variables) using ACA with an objective of minimising the engine mount displacement. To apply ACA optimisation methodology for a system of parameters, the domain has to be divided first into a specific number of randomly distributed regions, say for example R. These regions are indeed the trail solutions and act as local stations for the ants to move and explore. The fitness of these regions are first evaluated and stored on the descending fitness. Totally, "N" number of ants explores these regions and updating the regions is done locally and globally with the local search and global search respectively. Thus, these ants are divided into "G" global ants and "L" local ants. The ACA generates the initial population of design variables in their ranges defined for the case study. In ACA, a set of design variables (mount orientations) is considered to be an ant (Fig. 5) and there are 20 ants considered in this optimisation. The design parameters of the model are given in the Table 2.



Fig. 5: Representation of ant in ACA

3.1. Initialization

To initialize the population of ants a set of twenty masses and lead angles are randomly generated, and their displacements are found using the mathematical model. Then, they are sorted according to ascending order of solutions. The solutions 1-12 and 13-20 are named superior and inferior solutions respectively.

3.2. Global search

The global search is carried out to improve the inferior solutions. This search includes a crossover or random walk, mutation, and trail diffusion. In the crossover or random walk, the inferior solutions from 13 to18 are replaced by the superior solutions. The Replacement of each inferior solution by a superior solution is decided based on the crossover probability. To replace the 13th solution, a random number between 1 and 12 is generated. Then, the corresponding solution in the superior region replaces the 13th inferior solution. The selected solutions in the superior region should be excluded, so that it is not selected again for replacement. The above procedure is repeated up to the 18th solution. The mutation process further improves the replaced solutions. The masses, lead angles and mount orientation angles of each element in the replaced 13th set is modified by adding or subtracting with mutation step size (Δ). The mutation step size (Δ) is obtained by,

$$\Delta = R\left(1 - r^{(1-T)b}\right) \tag{7}$$

Where $R = V_{max} - V$ and V_{max} is maximum range the variable defined, V_i is the variable of the corresponding

to the ith iteration, r is a random number, T is the ratio of current iteration to the total number of iterations and b is a constant.

Then, a random number is generated between 0 and 1. If the random number generated is less than Pm, the mutation step size (Δ) is subtracted from the corresponding variable of the respective set, or else it is added to the corresponding variable of the respective set. The same procedure is repeated up to 18th solution. The trail diffusion improves the 19th and 20th solutions. In this process, two sets are randomly selected from the superior solutions. The new set obtained from parent 1 and parent 2 is termed the Child. The variables of each set of the Child are termed VC. For each set of variables a random number is generated if the number is between 0 and 0.5, the new variables of each set is obtained by,

$$VC = \alpha VP_1 + (1 - \alpha)VP_2 \tag{8}$$

If α is between 0.5 and 0.75, the new value of each new set is obtained by $VC = VP_1$. If α is between 0.75 and 1, the value of the variable the new set is obtained by $VC = VP_2$. The above procedure is repeated for the 20th solution also. After the crossover or random walk, mutation, and trail diffusion processes, the solutions for the modified sets from 13th to 20th are found from the mathematical model.

3.3. Local search

The local search is done to improve the superior solutions from 1 to 12. The average pheromone value is given by,

$$P_{avg} = \sum P / N_s \tag{9}$$

Where *P* is pheromone value of each solution and N_s is the number of superior solutions. A random number is generated between 0 and 1. If the number generated is less than the average pheromone value (P_{avg}), the search is further pursued or else the ant quits and then leaves the solution without any alteration. A limiting step value L_s , which is added to the value of the respective variable of the set when the random number generated is greater than 0.5 and subtracted to the value of the respective set when the random number generated is less than 0.5, is calculated as follows,

$$L_{\rm s} = K_1 - AK_2 \tag{10}$$

Where K_1 and K_2 are the values chosen such that $K_1 > K_2$. 'A' is the age of the ant. All the set corresponding to the superior solutions are modified by a local search, and solutions for the modified variable sets from 1 to 12 are found using the mathematical model. If the current solution is less than the previous solution, the age for the new solution is $A_i = A_{i-1} + 1$. If the new solution is greater than the old solution, the age for the new solution is $A_i = A_{i-1} - 1$. The new pheromone value of the ant for the next iteration is,

$$P_i = \frac{S_i - S_{i-1}}{S_{i-1}} + P_{i-1} \tag{11}$$

Where P_i is the pheromone value for the new solution and S_i is the response of current solution. The objective function of the ACA simulation is the engine mount displacement with input variables as given in Table 3.

Table 3: Input parameters of the developed model

Input Variables				
$\varDelta_1, \varDelta_2, \varDelta_3, \varDelta_4$	Lead angle of the Balancing masses			
θ_{x} , θ_{y} , θ_{z}	Mount orientation angles			

In the simulation of ACA, the pheromone value of each ant is initialised as 1 in the population. The each ant in the population is determined for the displacement then segregation of the superior and inferior ants and global and the ants for the global and local searches are decided. The various crossover (P_c) and mutation (P_m) operations are performed on the population of the ants with different probabilities, as they are the control parameters for the ACA. After performing the optimization using different runs of ACA with cross over probability (P_c) ranging from 0.6-0.8 and the mutation probability (P_m) ranging from 0.02-0.05, the results of the ACA are analysed for the least displacement in concurrence with the pheromone value and the age of the ants. The promising ants that are produced the least displacement are identified by the least pheromone evaporation and increased age. The results of the convergence for the different crossover (P_c) and mutation (P_m) are shown in Figs. 6-9. It has been found that the crossover $P_c = 0.8$ produced the better convergence for different mutation probabilities.



Fig. 6: Convergence of the $P_c = 0.6$ with different mutation probabilities of ACA



Fig. 7: Convergence of the $P_{\rm c}=0.7$ with different mutation probabilities of ACA



Fig. 8: Convergence of the $P_{\rm c}$ = 0.8 with different mutation probabilities of ACA



Fig. 9: Convergence of the $P_{\rm c}$ = 0.85 with different mutation probabilities of ACA

The ACA simulation with $P_c = 0.8$ and $P_m = 0.05$ produced the least displacement for each run as shown in Fig. 10. The results of ACA for the least displacement are verified through the five different runs of the ACA probabilities correspond to the least displacement. The least maximum mount displacement obtained is 81.45 µm. The respective optimized parameters corresponding to the least displacement is given in the Table 4. The results of the multimodal runs proved to be prominent in both the convergence and the least displacement.



Fig. 10: Convergence of the P_{c} = 0.8 and $P_{\rm m}$ = 0.05 for five different runs of ACA

Table 4.	Design	narameters	of the	develor	ned model
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Run /	1	2	2	4	5
Parameter	1	Ζ	3	4	3
$X_l(cm)$	42	23	40	35	44
$Y_1(cm)$	15	13	13	15	14
$Z_l(cm)$	14	8	18	18	17
$X_2(cm)$	42	48	47	30	41
$Y_2(cm)$	13	11	11	15	10
$Z_2(cm)$	14	8	18	28	17
$X_3(cm)$	26	20	43	35	34
$Y_3(cm)$	5	4	7	6	6
$Z_3(cm)$	-5	-6	-7	-9	-9
θ_{xI}	3	1	6	3	2
θ_{yI}	2	1	4	2	0
θ_{zI}	1	0	1	1	0
θ_{x2}	3	2	5	3	5
θ_{v2}	2	5	2	2	1
θ_{z2}	1	2	2	1	2
θ_{x3}	1	7	6	1	0
θ_{y3}	6	3	5	6	1
θ_{z3}	1	0	1	0	0
D(µm)	81.45	81.64	81.65	81.60	81.67

4. Conclusions

In this research work, a 4-cylinder 4-stroke diesel engine mounted on the elastomeric mounts is considered. The engine parameters are optimised with an objective of minimising the displacement caused at each mount. A mathematical model is developed considering the various unbalanced forces produced by the inertia of the rotating and reciprocating components, slap of the piston and reaction forces from the mount. The output of the model, mount displacement, is based on the reaction forces at the mounts. The output displacement of the model is determined by solving the equation using Runge-Kutta method in which the model input parameters are the mount positions and orientations. The model developed is coded and numerically simulated in the MATLAB for a specified period to find the displacement. The ant colony algorithm is identified and employed for optimising the mount parameters. The results are obtained for different control parameters and the control parameter Pc 0.8 and Pm 0.05 produced the least mount displacement and the convergence of the control parameters are verified through five different runs. It has been concluded that the model will be able to determine the mount displacement for any engine category with that the mode constants are to be reassigned. The optimization considered in the article also works well for the given range of the design variables. For the future scope, the model can be extended with crank web unbalance, cylinder frictional force, and hydraulic mounts.

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