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Golden Section Search Based Optimization of Road Vehicle Suspension System

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ABSTRACT:

Suspension system is an important part of a vehicle which connects the road wheels and vehicle body. The major function of suspension is to isolate vehicle body from road disturbances. The design of suspension system is generally a compromise between many design requirements that aim to provide a comfortable ride and good vehicle handling. An optimization technique is used to choose the suspension parameters that meet these design requirements. In this present work a two degree of freedom quarter car vehicle vibration model is considered for optimization. Sprung mass acceleration and relative displacement of quarter car are considered as the measure of ride comfort and vehicle handling respectively. Golden section search optimization technique is used for single objective optimization of quarter car considering sprung mass acceleration as objective function and relative displacement as constraint. It is noticed that the accuracy level in getting the optimized value using this approach is comparatively high and reliable.

KEYWORDS:

Quarter car; Optimization; Golden section search; Ride comfort; Vehicle handling

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ACRONYMS AND NOMENCLATURE:

m_s	*Sprung mass (kg)*
m_u	*Unsprung mass (kg)*
k_s	*Suspension stiffness (N/m^2)*
c_s	*Suspension damping coefficient (Ns/m^2)*
k_u	*Tire stiffness (N/m^2)*
f	*Frequency of excitation (Hz)*
$\omega = 2\pi f$	*Frequency of excitation (rad)*
$x_s(t)$	*Sprung mass displacement (m)*
$x_u(t)$	*Unsprung mass displacement (m)*
X_u	*Unsprung mass displacement amplitude (m)*
X_s	*Sprung mass displacement amplitude (m)*
$y(t)$	*Road displacement (m)*
Y	*Road displacement amplitude (m)*
$\xi = c_s/2m_s\omega_s$	*Damping ratio*
$r = \omega/\omega_s$	*External frequency ratio*
$\alpha = \omega_s/\omega_u$	*Natural frequency ratio*
$\varepsilon = m_s/m_u$	*Mass ratio*
$\omega_s = \sqrt{k_s/m_s}$	*Sprung mass natural frequency (rad)*
$\omega_u = \sqrt{k_u/m_u}$	*Unsprung mass natural frequency (rad)*
$Z = X_s - X_u$	*Relative displacement (m)*
$\ddot{X}_s$	*Sprung mass acceleration amplitude (m/s^2)*

1. Introduction

Suspension system is an assembly of spring and damper which connects the road wheels and vehicle body. When a vehicle moves over a bump or pothole the vertical excitation from road transferred to the suspension. The suspension system takes up this energy and isolates vehicle body from these road disturbances. For any road vehicle it very much essential to keep these road disturbances transferred to passenger or vehicle body minimum for good ride comfort [11]. There are many constraints and conflicting criteria's which oppose to achieve good ride comfort [6]. One of the important conflicting criteria is vehicle handling [4]. The suspension parameters namely spring and dampers are designed in such a way that it should meet these requirements. Hence a design optimization is adopted to select these parameters meeting the desired requirements. Literature survey is carried out focusing on vehicle dynamics, vehicle vibration, quarter car model and optimization. During last few years a lot of work is carried out in the area of suspension optimization.

Alkhatib et al [8] presented optimization problem of a linear one degree-of-freedom vibration isolator mount and the method is extended to the optimization of a linear quarter car suspension model. Gobbi et al [4] presented a multi-objective stochastic optimization method and proposed a two degree of freedom linear quarter car model running on a randomly profiled road. Jazar et al [1] proposed a new optimization technique called Root Mean Square (RMS) optimization a linear quarter car model moving over a harmonically bumped road. Indicating the shortcomings of classical optimization technique he demonstrated the RMS optimization technique which is based on minimizing the absolute acceleration RMS with respect to relative displacement RMS. Then he used the proposed optimization technique to create design charts for suspension parameters which is used to find absolute acceleration RMS in the presence of physical constraint such as limit on relative displacement is given.

Lu et al [2] presented a passive suspension optimization using genetic algorithms. Optimization problem is formulated considering linear quarter car truck model with three design variables the suspension spring constant, the suspension damping and the tire stiffness. Instead of using ride quality as the performance measure of suspension systems, in this paper the dynamic load generated by vehicle-pavement interaction is used as the objective function to be minimized. Zhongzhe Chi et al [14] presented a comparative study of three optimization algorithms, namely Genetic Algorithms (GA), Pattern Search Algorithm (PSA), and Sequential Quadratic Programming (SQP), for a design optimization of a vehicle suspension based on a quarter car vehicle model. In this paper three objective functions (vertical vehicle body acceleration, suspension working space, and the dynamic tire load) and five design variables (sprung mass, unsprung mass, suspension spring stiffness, suspension damping coefficient and the tire stiffness) are considered for design optimization.

Mahmoodabadi et al [3] formulated an optimal vehicle suspension design problem with quarter car vehicle dynamic model. Multi-objective genetic algorithm (MOGA) is used for pareto optimization of a two-degree of freedom vehicle vibration model considering the two conflicting functions namely sprung mass acceleration and relative displacement between sprung mass and tire. Multi-objective genetic algorithm is used for Pareto optimization of a two-degree of freedom vehicle vibration model considering the two conflicting functions namely sprung mass acceleration and relative displacement between sprung mass and tire. Papalambros et al [6] presented a nonlinear half car suspension model subjected to random road excitations. Mathematical model formulation is done in the form of nonlinear programming problem which is used for optimization study. Constrained optimization problem is formulated considering sprung mass acceleration as objective function, suspension stiffness and damping as design variables subjected to road holding, rolling angle, suspension working space and tire life. Prasad et al [7] proposed an 8 degree of freedom full car model for design optimization. Number of objectives such as maximum bouncing acceleration of seat and sprung mass, RMS weighted acceleration of seat and sprung mass as per ISO 2631 standards, jerk, suspension travel, road holding and tyre deflection are minimized subjected to a number of constraints.

Goga and Marian [11] presented a half car model with passive vehicle suspension and system optimization is done using genetic algorithms. Passive suspension parameters (damping and stiffness coefficients) were optimized. Objective functions considered were minimization of vertical acceleration of body, wheel and angular acceleration of vehicle body. Optimized results were compared with the original values and indicated that results for model with optimized parameters show significant decreasing of amplitudes and faster stabilization of measured quantities against results of model with original parameters. Farid et al [12] proposed an optimization of four degrees-of-freedom vehicle's human with seat suspension system using genetic algorithms (GA) to determine vehicle suspension parameters to achieve the best comfort of the human body. Optimal design parameters of suspension are found that minimizes a seat suspension deflection, seat body acceleration and human body head acceleration for step and sinusoidal excitation inputs. Gundogodu [5] proposed an optimization of a four degrees-of-freedom quarter car seat and suspension system using genetic algorithms to determine a set of parameters to achieve the best performance of the driver. In this work, a quarter car model is selected for optimization considering sprung mass acceleration as objective function, relative displacement as constraint and optimum values of suspension parameters i.e. suspension stiffness and suspension damping coefficients are evaluated.

2. Mathematical model and equation of motion of quarter car

A vehicle represents a complex vibration system with many degrees of freedom. Commonly used vehicle models include quarter car, half car, and full vehicle models. To obtain the qualitative insight into the functions of suspension, particularly the effect of sprung mass and unsprung mass, spring stiffness, and damping on vehicle vibrations a linear model with two degree-of-freedom, a quarter car is considered [13]. Fig. 1 shows a simplified two degree-of-freedom quarter car model. 1/4th of the vehicle body is represented by m_s which is called sprung mass. Mass of one tire is represented by m_u. k_s and c_s represents suspension stiffness and suspension damping coefficient respectively. Tire stiffness is represented by k_u. Tire damping coefficient is neglected since it is very small compared to suspension damping coefficient [9]. By applying Newton's second law of motion, the governing differential equations of quarter car for sprung and unsprung mass respectively are given by,

$$m_s\ddot{x}_s + c_s(\dot{x}_s - \dot{x}_u) + k_s(x_s - x_u) = 0 \tag{1}$$

$$m_u\ddot{x}_u + c_s(\dot{x}_u - \dot{x}_s) + k_s(x_u - x_s) + k_u x_u - k_u y = 0 \tag{2}$$

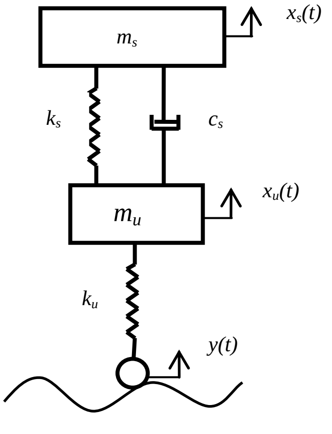

Fig. 1: Quarter car model

Comfort of driver/passenger is often represented by acceleration of the sprung mass [14]. The expression for sprung mass acceleration is given by,

$$\ddot{X}_s = \sqrt{\frac{4\xi^2 r^2 + 1}{A^2 + B^2}}\, Y\omega_u^2 r^2 \alpha^2 \tag{3}$$

Where $A = r^2(r^2\alpha^2 - 1) + 1 - (1+\varepsilon)r^2\alpha^2$.

$B = 2\xi r(1-(1+\varepsilon)r^2\alpha^2)$. Handling of vehicle is represented by relative displacement between sprung and unsprung masses [13] which is given by,

$$Z = \frac{r^2 Y}{\sqrt{A^2 + B^2}} \tag{4}$$

3. Golden section search optimization

Optimization problem with objective function and constraint which are fairly simple terms of design variables can be easily solved by analytical optimization techniques. If the optimization problem involves the objective function or constraints that are not stated as explicit functions of the design variables or which are too complicated to manipulate, it cannot be solved by using the classical analytical methods. In such cases numerical method of optimization is applied [10]. Numerical methods are classified as elimination method and interpolation method. Elimination method is used when the initial interval of uncertainty within which optimum value lies is known and the function to be optimized is uni-modal i.e. it has only one peak (maximum) or one valley (minimum) in a given interval. In elimination method interval of uncertainty within which optimum lies is bracketed such that interval of uncertainty which is not feasible is rejected in each iteration. Golden section search can be applied to solve when the optimum solution is known to lie within restricted ranges of the design variables [10].

Problem considered in the present work consists of nonlinear functions of design variables, one of the numerical methods called Golden section search is adopted to solve optimization problem. Optimal values of quarter car suspension parameters are found using single objective optimization considering sprung mass acceleration ($\ddot{X}_s$) as objective function to be minimized subjected to constraint relative displacement (Z). Golden section search Algorithm is used for optimization of suspension parameters, viz., suspension stiffness (k_s) and suspension damping coefficient (c_s). Sprung mass (m_s), unsprung mass (m_u) and tire stiffness (k_u) considered as fixed parameters during optimization. Road profile is considered as sinusoidal and amplitude (Y) and frequency (f) of road excitation are given in Table 1. Suspension parameters stiffness (k_s) and damping (c_s) are the design variables known to exist in a range [3]. The optimization problem is to minimize $f(x)$ = $\{\ddot{X}_s\}$ when subjected to constraints as below,

$$Z \le 0.0528,\ 10000 \le k_s \le 16000,$$
$$500 \le c_s \le 2000,\ k_s > 0 \text{ and } c_s > 0.$$

Table 1: Quarter car and road parameters

Parameters	m_s	m_u	k_t	Y	f
Value	240	36	160000	0.05	1

4. Results and discussions

Optimization algorithm is used to minimize the sprung mass acceleration ($\ddot{X}_s$) subjected to constraint relative displacement (Z) for the given range of design variables i.e. suspensions stiffness (k_s) and suspension damping coefficient (c_s).The path followed and number of iteration taken to evaluate the optimum values design variables ($k_s$and c_s), constraint (Z) and objective function ($\ddot{X}_s$) by the algorithm is shown in the form of graph as shown in Figs. 2 to 5. From Fig. 2, it is noticed that the optimum value of k_s obtained as 12.313 kN/m at 27th iteration. The convergence to this value has started at 12th iteration. Similarly from Fig. 3, the optimum value of c_s obtained as 2000 Ns/m at 27th iteration. The convergence to this value has started at 14th iteration.

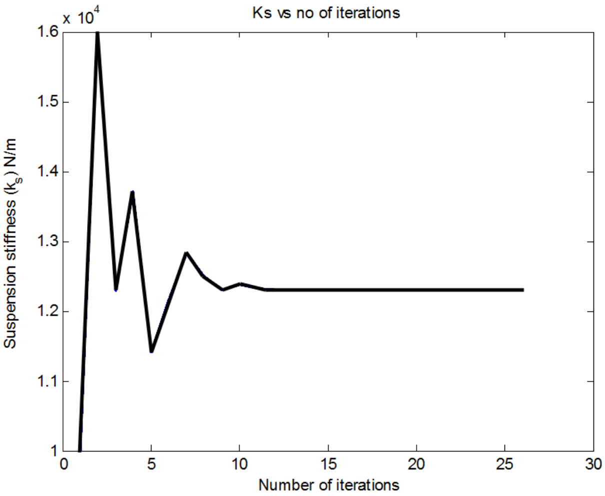

Fig. 2: Suspension stiffness (k_s) vs. No of iterations

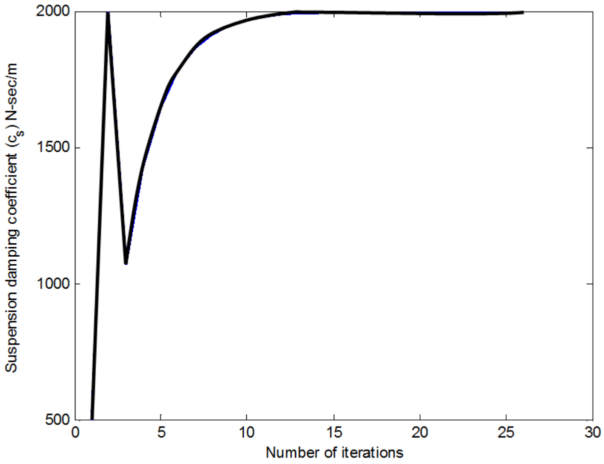

Fig. 3: Suspension damping coefficient (c_s) vs. No of iterations

Fig. 4 indicates the relative displacement vs. number of iterations. The graph shows the changes in constraint values during optimization. Optimization terminates after 27 iterations with constraint tolerance value of 0.12×10^{-9}. From Fig. 5, it is noticed that the optimum value of $\ddot{X}_s$ obtained 3.9826 m/s^2 at 27th iterations. The convergence to this value has started at 17th iterations. The optimal values of objective function ($\ddot{X}_s$) and design variables (k_s and c_s) closely match with the optimized values reported in [3]. The comparison of optimization results are given in Table 2.

Table 2: Comparison of optimization results

Parameters	Golden section search method	Ref. [3]	% error
k_s (N/m)	12313	12305	0.0005
c_s (Ns/m)	1998	2000	0.001
$\ddot{X}_s$ (m/s^2)	3.9826	3.9799	0.0007

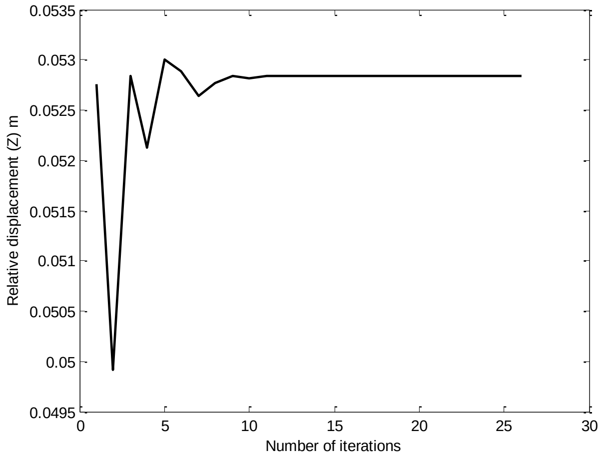

Fig. 4: Relative displacement (*Z*) vs. No of iterations

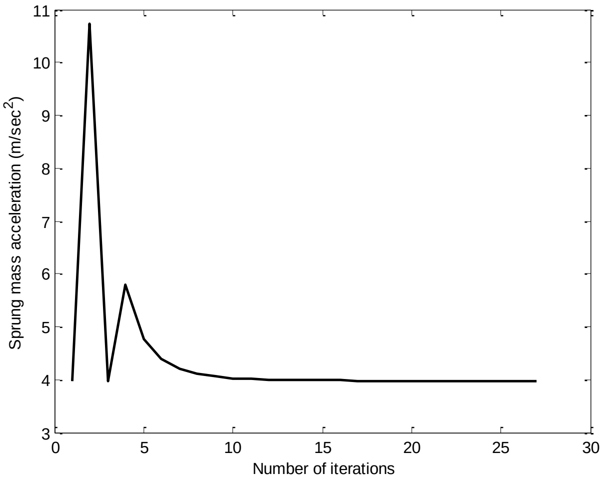

Fig. 5: Sprung mass acceleration ($\ddot{X}_s$) vs. No of iterations

5. Conclusions

Optimization problem involving the objective function or constraints that are not stated as explicit functions of the design variables or which are too complicated to manipulate, have been addressed using golden section search optimization technique. Computing time taken for obtaining the result is very less and it is about 27 seconds. It is noticed that the accuracy level in getting the optimized value using golden section approach is comparatively high and reliable.

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EDITORIAL NOTES:

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