Ghazaly et al. 2016. *Int. J. Vehicle Structures & Systems*, 8(1), 35-40
ISSN: 0975-3060 (Print), 0975-3540 (Online)
doi: 10.4273/ijvss.8.1.07

International Journal of Vehicle Structures & Systems
Available online at www.maftree.org/eja

H∞ Control of Active Suspension System for a Quarter Car Model

Nouby M. Ghazaly[a, c], Abd El-Nasser S. Ahmed[a], Ahmed S. Ali[b] and G.T. Abd El- Jaber[a]
[a]*Mech. Engg. Dept., Faculty of Engg., South Valley University, Qena, Egypt*
[b]*Mech. Engg. Dept., College of Engg., Assuit University, Assuit, Egypt*
[c]*Corresponding Author, Email: nouby.ghazaly@eng.svu.edu.eg*

ABSTRACT:
In recent years, the use of active control mechanisms in active suspension systems has attracted considerable attention. The main objective of this research is to develop a mathematical model of an active suspension system that is subjected to excitation from different road profiles and control it using H∞ technique for a quarter car model to improve the ride comfort and road handling. Comparison between passive and active suspension systems is performed using step, sinusoidal and random road profiles. The performance of the H∞ controller is compared with the passive suspension system. It is found that the car body acceleration, suspension deflection and tyre deflection using active suspension system with H∞ technique is better than the passive suspension system.

KEYWORDS:
Quarter car model; Active suspension system; H∞ technique; Road profile; Vertical acceleration; Suspension travel

CITATION:
N.M. Ghazaly, A.N.S. Ahmed, A.S. Ali and G.T.A. Jabera. 2016. H∞ Control of Active Suspension System for a Quarter Car Model, *Int. J. Vehicle Structures & Systems*, 8(1), 35-40. doi:10.4273/ijvss.8.1.07

1. Introduction

The purpose of suspension system is to improve the ride comfort, road handling and stability of vehicles. Suspension consists of springs, shock absorbers and linkages that connect a vehicle to its wheels. The main function of vehicle suspension system is to minimize the vertical acceleration transmitted to the passenger which directly provides road comfort. Current automobile suspension systems using passive components can only offer a compromise between these two conflicting criteria by providing spring and damping coefficients with fixed rates. A good suspension system should provide good vibration isolation and a small “rattle space” that is the maximum allowable relative displacement between the vehicle body and various suspension components [1]. A passive suspension has the ability to store energy via a spring and to dissipate it via a damper. An active suspension system has the ability to store, dissipate and to introduce energy to the system depending upon operating conditions [2]. Active suspension can give better performance of suspension by having force actuator, which is a close loop control system. The controller will decide either to add or dissipate the energy from the system with the help of sensors as an input. Sensors will give the data of road profile to the controller. Therefore, an active suspension system is needed with an active element inside the system to give both conditions so that it can improve the performance of the suspension system[3-6].

In recent years more attention has been devoted to the development of active suspensions. Various approaches have been proposed to solve the crucial problem of designing a suitable control law for these active suspension systems. In many control applications, it is expected that the behaviour of the designed system will be insensitive to external disturbance and parameter variations. It is known that feedback in conventional control system has the inherent ability of reducing the effects of external disturbances and parameter variations [7-9]. The H∞ method addresses a wide range of control problems by combining the frequency and time-domain approaches. The design is an optimal one in the sense of minimization of the H∞ norm of closed-loop transfer function. The H∞ model includes coloured measurement and process noise. It also addresses the issues of robustness due to model uncertainties. It is applicable to the SISO and MIMO systems [10]. In this study, the active suspension system is synthesized based on H∞ techniques for a quarter car model to improve the ride comfort and road handling. Comparison between the performance of passive and active suspension systems is performed using different road profile.

2. Mathematical modelling

Hydraulic actuators are added to the passive system to achieve an active suspension system as shown in Fig. 1. The advantage of such a system is that even if the active hydraulic actuator or the control system fails, the passive components come into action. The equations of motion are written as,

$$m_s \ddot{z}_s + K_s(z_s - z_{us}) + C_a(\dot{z}_s - \dot{z}_{us}) = 0 \quad (1)$$

$$m_{us}\ddot{z}_{us} + K_s(z_{us} - z_s) + C_a(\dot{z}_{us} - \dot{z}_s) + K_t(z_{us} - z_r) = 0 \quad (2)$$

If the control force from the hydraulic actuator ua = 0 then Eqns. (1) and (2) become the equations of passive suspension system [11, 12].

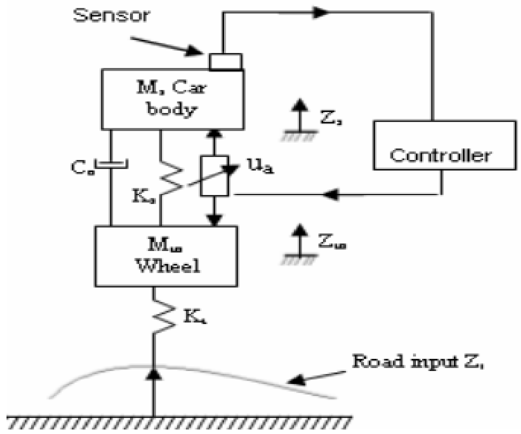

Fig. 1: Active suspension system model of a quarter car

Considering u_a as the control input, the state-space representation of Eqns. (1) and (2) becomes,

$$\dot{z}_1 = z_2 \tag{3}$$

$$\dot{z}_2 = \frac{-1}{M_s}\left(K_s(z_1 - z_3) + C_a(z_2 - z_4)\right) \tag{4}$$

$$\dot{z}_3 = z_4 \tag{5}$$

$$\dot{z}_4 = \frac{1}{M_{us}}\left(K_s(z_1 - z_3) + C_a(z_2 - z_4) + K_t(z_3 - z_r)\right) \tag{6}$$

Where $z_1 = z_s$, $z_2 = \dot{z}_s$, $z_3 = z_{us}$ and $z_4 = \dot{z}_{us}$. Block diagram [13] of control system used to develop active suspension system is shown in Fig. 2. In order to develop an active suspension system, pressurized hydraulic fluid source, pressure relief valve to control the pressure of hydraulic fluid, direction control valve and hydraulic cylinder are used. The hydraulic cylinder converts the hydraulic pressure into force to be transmitted between the sprung and the unsprung masses.

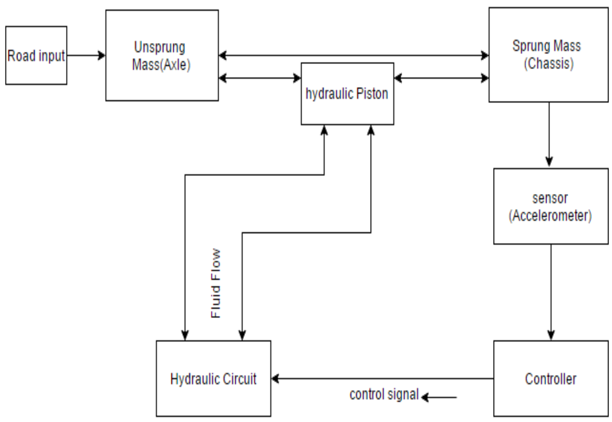

Fig. 2: Block diagram of control system

The hydraulic actuator installed in between the sprung and unsprung masses is shown in Fig. 3. It includes a valve and a cylinder. U_h is the actuator force generated by the hydraulic piston. $X_{act} = x_1 - x_3$ is actuator displacement. U_h (equal to U_a) is applied dynamically to improve the ride comfort according to the variation of the road roughness and load input [14]. The actuator force U_h is given by,

$$U_h = AP_l \tag{7}$$

Where A is the piston area and equals to upper (A_u) or lower (A_l) area of the chamber. P_l is the pressure drop across the piston. The derivative of P_l is given by,

$$\frac{v_t}{4\beta}\dot{P}_l = Q - C_{tp}P_l - A(x_2 - x_4) \tag{8}$$

Where V_t is the total actuator volume. β is the effective bulk modulus of the fluid. The hydraulic load flow is $Q = q_u + q_l$ with q_u and q_l being the flows in the upper and lower chamber respectively. C_{tp} is the total leakage coefficient of the piston. The valve load flow is given by,

$$Q = C_d \omega x_6 \sqrt{(P_s - \text{sgn}(x_6)x_5)/\rho} \tag{9}$$

Where C_d is the discharge coefficient. ω is the spool valve area gradient. x_5 is the pressure inside the chamber of hydraulic piston and $x_6 = x_{sp}$ is the valve displacement from its closed position. ρ is the hydraulic fluid density. P_s is the supply pressure. As the term $(P_s - \text{sgn}(x_6)x_5)$ may become negative, Eqn. (9) is replaced with the corrected flow equation as,

$$Q = \text{sgn}(P_s - \text{sgn}(x_6)x_5)C_d \omega x_6 \sqrt{\frac{P_s - \text{sgn}(x_6)x_5}{\rho}} \tag{10}$$

The spool valve displacement is controlled by the input to the valve U_c, which could be a current or voltage signal. Eqns. (1) – (10) are rewritten as follows,

$$\dot{x}_1 = x_2 \tag{11}$$

$$\dot{x}_2 = \frac{1}{M_s}\left(K_s x_1 + C_a x_2 + K_s x_3 + C_a x_4 + A_l x_5\right) \tag{12}$$

$$\dot{x}_3 = x_4 \tag{13}$$

$$\dot{x}_4 = \frac{K_s x_1 + C_a x_2 + (K_s + K_t)x_3 + C_a x_4}{M_{us}} + \frac{A_l x_5}{M_s} \tag{14}$$

$$\dot{x}_5 = \beta x_5 + A(x_2 - x_4) + \omega_3 x_6 \tag{15}$$

$$\dot{x}_6 = x_6 / \tau + U_c \tag{16}$$

Where $\omega_3 = \text{sgn}(P_s - \text{sgn}(x_6)x_5)/\sqrt{|P_s - \text{sgn}(x_6)x_5|}$.

Thus, Eqns. (11) – (16) become the state feedback model of active suspension system [13]. The open loop passive and closed loop active suspension systems are simulated with the parameters as given in Table 1.

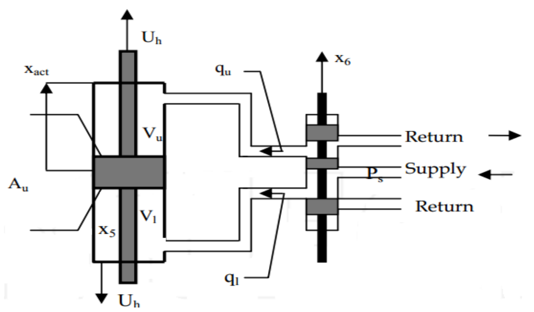

Fig. 3: Hydraulic valve and cylinder

Table 1: Suspension system model parameters

Parameter	Value	Unit	Parameter	Value	Unit
M_s	250	kg	M_{us}	50	kg
K_a	16.812	kN/m	K_t	190	kN/m
C_a	1	kNs/m	B	1	/s

An $H\infty$ design technique meets the performance objectives for the nominal actuator model. However, this model is only an approximation of the true actuator. To make sure that controller performance is maintained even with model error and uncertainty, the model with robust performance is designed. Then, μ synthesis is used to design a controller that achieves robust performance for the entire family of actuator models that takes uncertainty into account [15]. The process is shown in Fig. 4. The plant, *P*, has two inputs - the exogenous input, *w*, that includes reference signal and disturbances, and the manipulated variables, *u*. There are two outputs - the error signals, *z* and the measured variables, *v*. All these variables are generally vectors except *P* and *K* are scalar matrices [14].

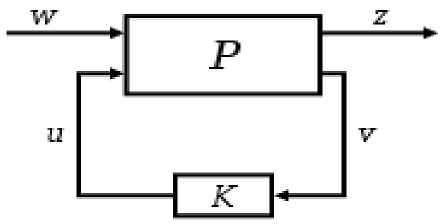

Fig. 4: Standard configuration of $H\infty$ controller

For the quarter car model, the main control objectives are formulated in terms of passenger comfort and road handling. These objectives relate to body acceleration, a_b, and suspension travel, s_d. Weighted functions [15] are used to model the external disturbances to quantify the design objectives as shown in Fig. 5. The feedback controller uses the measurements y_1 and y_2 of the suspension travel, s_d, and body acceleration, a_b, to compute the control signal, *u*. This control signal drives the hydraulic actuator. The road disturbance, *r*, which is modelled as a normalized signal, d_1, which is shaped by a weighting function W_{road}. On both measurements, sensor noise is modelled as normalized signals, d_2 and d_3, which are shaped by weighting functions W_{d2} and W_{d3}. The control objectives can be re-interpreted as a disturbance rejection goal. The goal is to minimize the impact of the disturbances, d_1, d_2, and d_3, on a weighted combination of suspension travel (s_d), body acceleration (a_b), and control effort (*u*). The $H\infty$ norm (peak gain) can be considered as the measure of the effect of the disturbances. Then, the requirements can be met by designing a controller that minimizes the $H\infty$ norm from the disturbance inputs, d_1, d_2, and d_3, to the error signals, e_1, e_2, and e_3 [13]. The Simulink models of passive and active suspension systems are shown in Fig. 6 and 7 respectively.

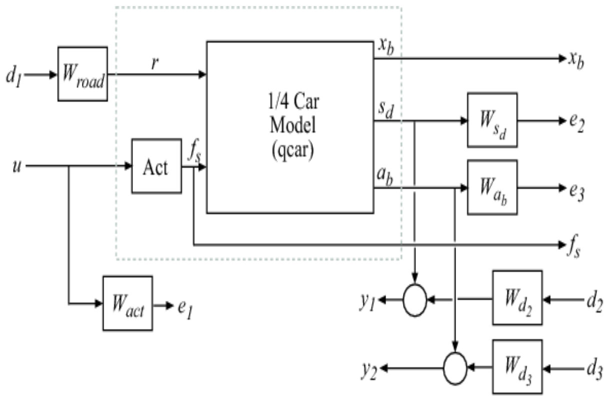

Fig. 5: Overview of active suspension system with H∞ controller

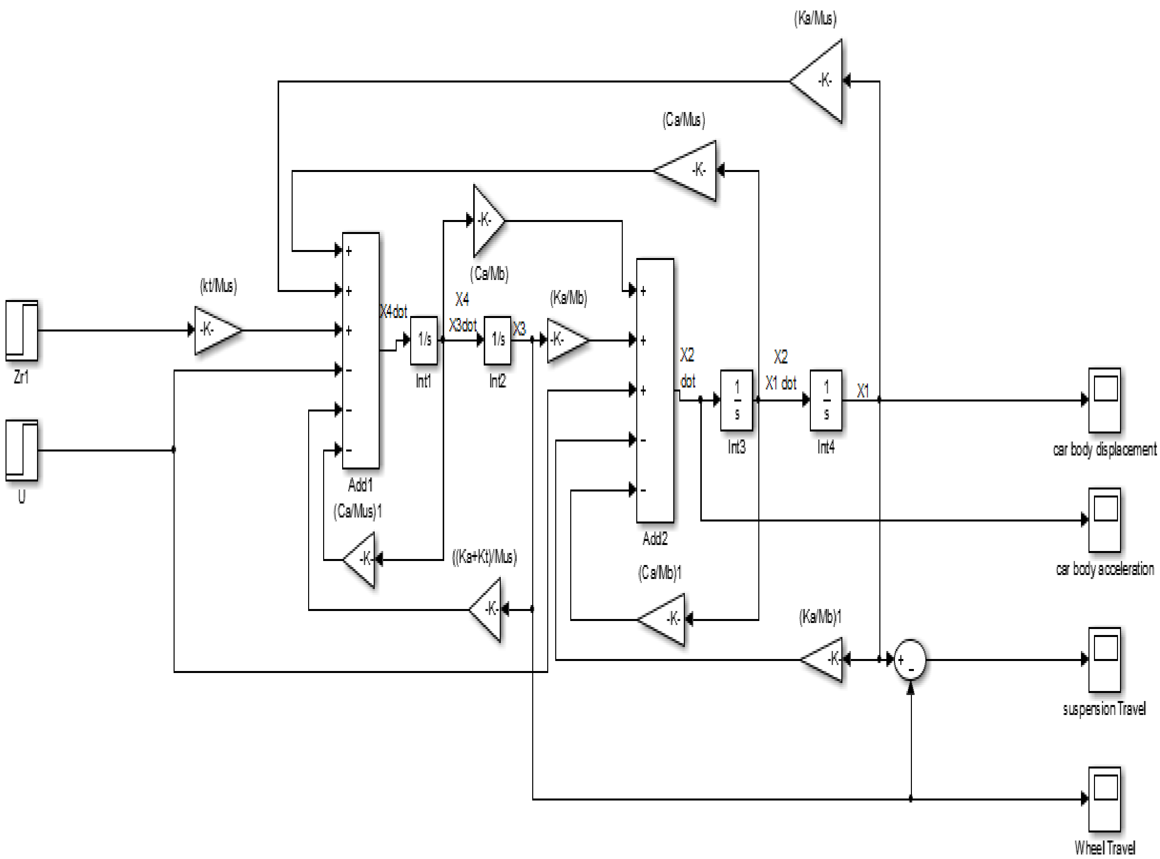

Fig. 6: Simulink model of passive suspension system

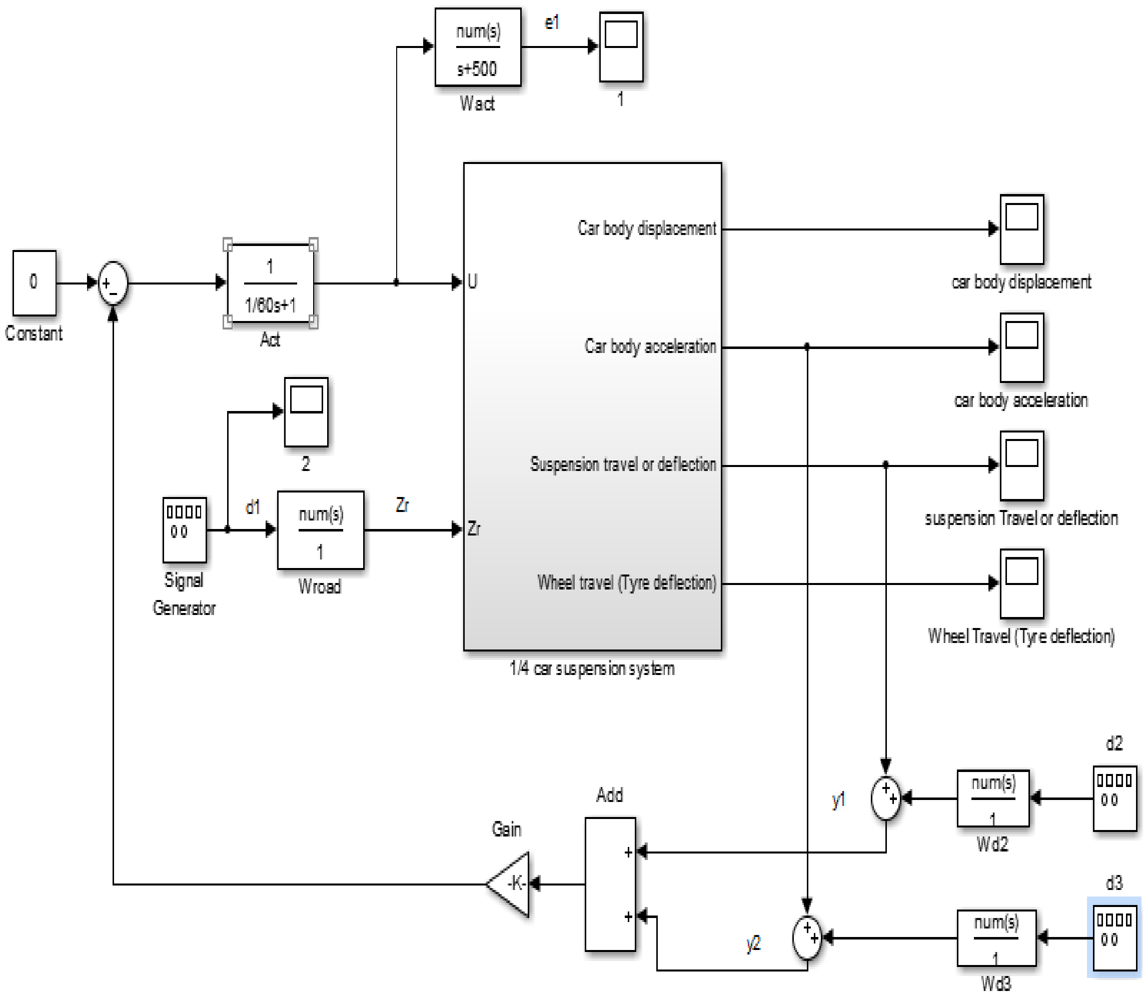

Fig. 7: Simulink model of active suspension system with *H∞* controller

3. Results and discussion

Simulations using Simulink models of quarter car with and without *H∞* controller were undertaken. Performances of the suspension systems in term of ride quality and car handling were assessed for the road disturbances input. The parameters are considered to achieve small amplitude value for suspension travel, wheel deflection and car body acceleration. The steady state for each part should be fast. Suspension travel, wheel travel, car body acceleration and displacement for quarter car are obtained as shown in Figs. 8 – 10 for step input z_r = 0.1 (P1), sinusoidal input (bumpy road, P2) and random road input (P3). For these road profiles, Table 2 gives the peak values of the various parameters. Both the peak values of system responses and settling time of excitation from the active suspension system have been reduced when compared to the passive suspension system for all the parameters of sprung mass acceleration, suspension travel (road holding) and tyre deflection for the considered step, sinusoidal and random road profiles.

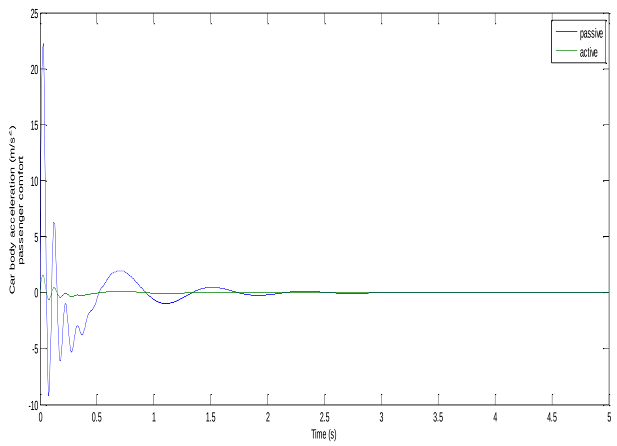

Fig. 8(a): Car body acceleration for step input z_r = 0.1

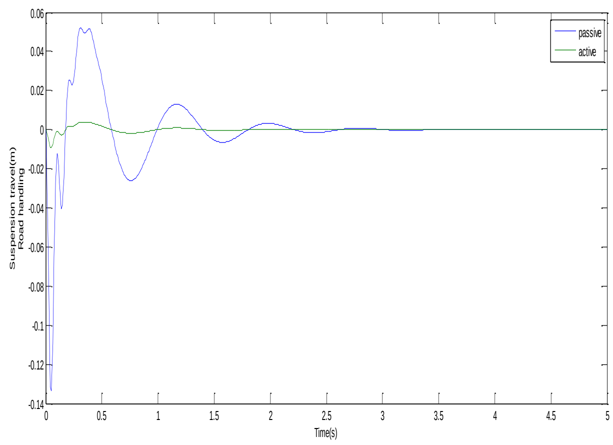

Fig. 8(b): Suspension travel for step input z_r = 0.1

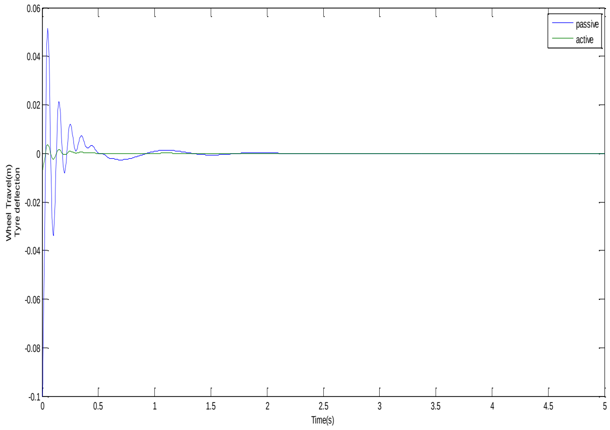

Fig. 8(c): Wheel travel for step input z_r = 0.1

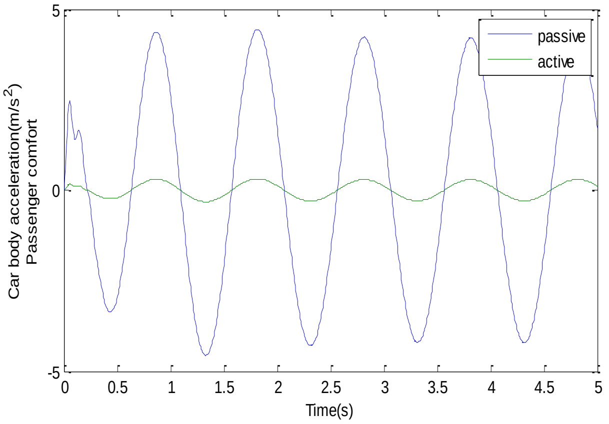

Fig. 9(a): Car body acceleration for sinusoidal input

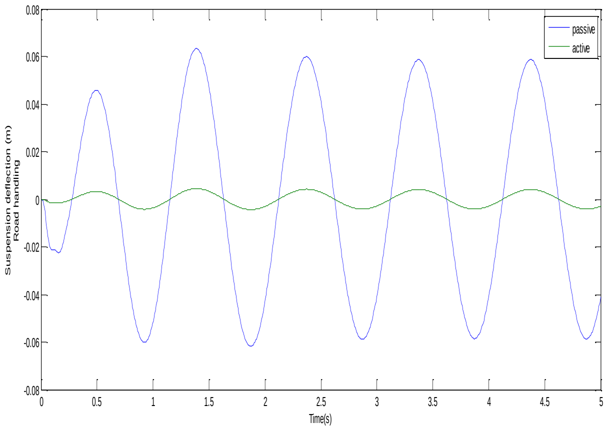

Fig. 9(b): Suspension travel for sinusoidal input

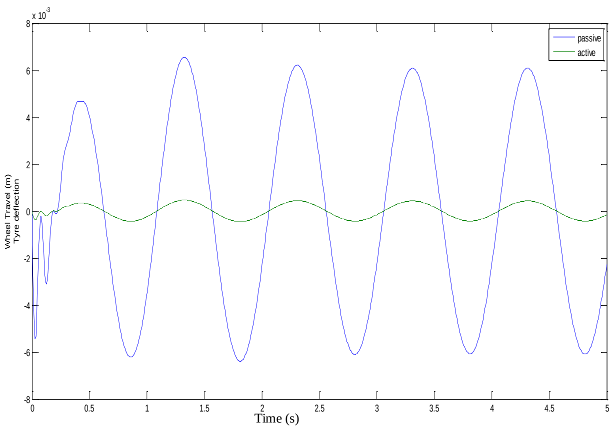

Fig. 9(c): Wheel travel for sinusoidal input

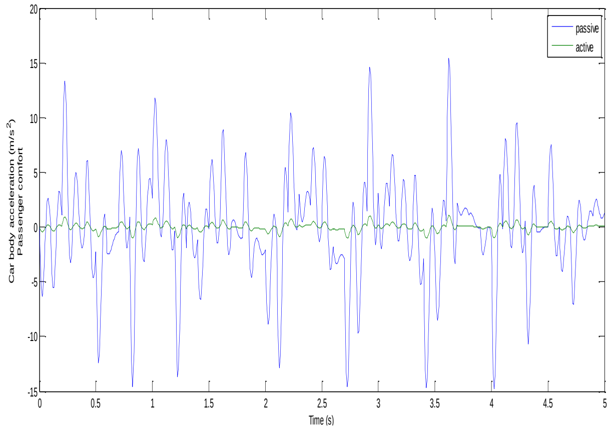

Fig. 10(a): Car body acceleration for random road input

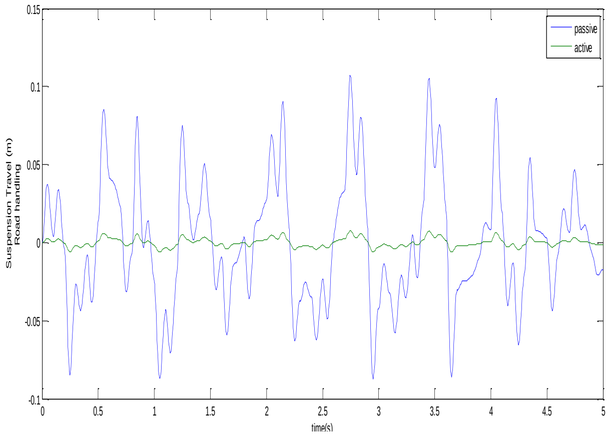

Fig. 10(b): Suspension travel for random road input

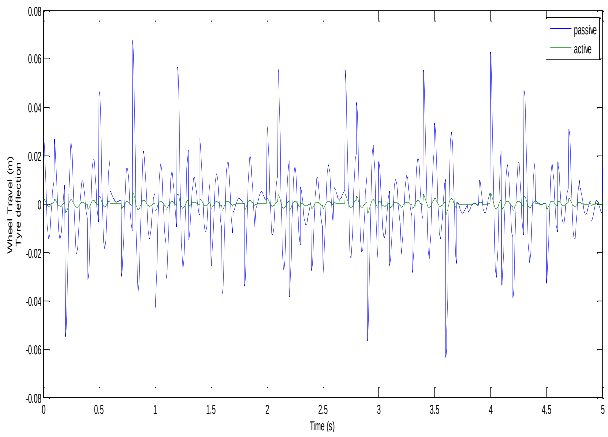

Fig. 10(c): Wheel travel for random road input

Table 2: System performance vs. Road profiles

Response vs. Road profiles	Passive suspension system			Active suspension system		
	P1	P2	P3	P1	P2	P3
Car body acceleration (m/s^2)	22.28	4.57	15.45	1.56	0.32	1.03
Suspension travel (cm)	13.35	6.32	10.75	0.93	0.44	0.75
Wheel travel (cm)	10.0	0.65	6.76	0.7	0.05	0.47

The time domain responses of active suspension system with $H\infty$ controller and passive suspension system are shown in Fig. 11. The body acceleration is the smallest to emphasize the passenger comfort. The body acceleration is the largest to emphasize the suspension deflection. The balanced design achieves a good trade-off between body acceleration and suspension deflection. In order to validate the response of $H\infty$ controller, μ controller is used and their performances are compared as shown in Fig. 12. The $H\infty$ controlled active suspension system responses correlate well with those from the suspension system with μ controller.

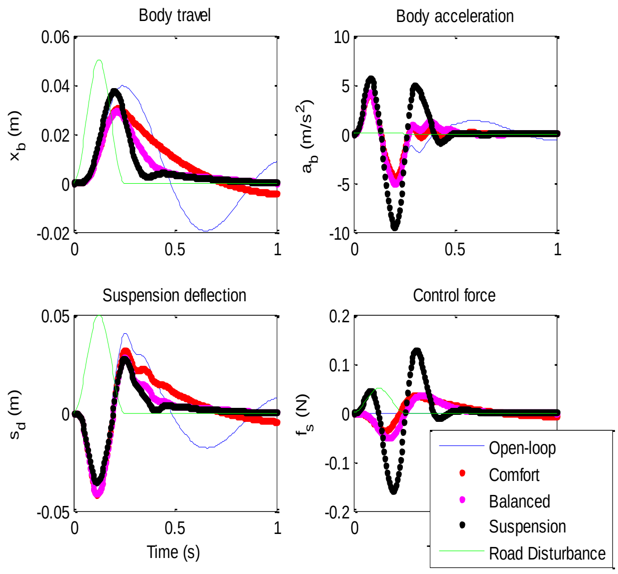

Fig. 11: Time-domain response of the closed-loop models to the road disturbance signal

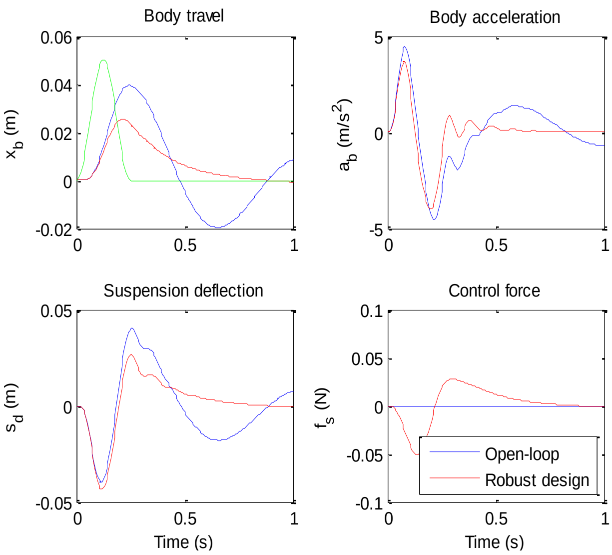

Fig. 12: Time-domain response of the closed-loop models to the road disturbance signal using μ-controller

4. Conclusions

The methodology was developed to design an active suspension for a passenger car by designing an $H\infty$ controller, which improves performance of the system with respect to design goals compared to passive suspension system. Mathematical modelling has been performed using a two degree of freedom quarter car model for passive and active suspension systems. The bounce motion was considered to evaluate the performance of suspension with respect to body acceleration, suspension travel and wheel travel. $H\infty$ controller design approach has been examined for the active system. The performance of active suspension system with $H\infty$ controller has been found to be much better than that of passive suspension system. By including an active element in the suspension, it is possible to reach a better compromise than the passive system. The μ controller has been implemented to validate the developed $H\infty$ controller. The time domain responses of the body travel, body acceleration and suspension travel with $H\infty$ controller were found to correlate well with the μ controller model.

REFERENCES:

[1] D. Hrovat. 1997. Survey of advanced suspension development and related optimal control application, *Automatica*, 30(10), 1781-1817. http://dx.doi.org/10.1016/S0005-1098(97)00101-5.

[2] H.Y. Chen and S.J. Huang. 2005. Adaptive control for active suspension system, *Proc. Int. Conf. on Control and Automatic*, Budapest, Hungary.

[3] A. Aldair and W.J. Wang. 2011. Design an intelligent controller for full vehicle nonlinear active suspension systems, *Int. J. Smart Sensing and Intelligent Systems*, 4(2), 224-243.

[4] M.A.A. Soliman, N.M. Ghazaly and F.S. Kadry. 2014. Parameters affecting truck ride comfort, *SAE Technical Paper*, 2014-01-0147.

[5] N.M. Ghazaly and A.O. Moaaz. 2014. The future development and analysis of vehicle active suspension system, *IOSR J. Mechanical and Civil Engineering*, 11(5), 19-25. http://dx.doi.org/10.9790/1684-11551925.

[6] F. Jamshidi and A. Shaabany. 2011. Robust control of an active suspension system using H2 and H∞ control methods, *J. American Science*, 7(5), 1-5.

[7] K. Zhou and J.C. Doyle. 1998. *Essentials of Robust Control*, Prentice Hall.

[8] Z. Xie, P.K. Wong, J. Zhao and T. Xu. 2015. Design of a de-noising hybrid fuzzy-PID controller for active suspension systems of heavy vehicles based on model adaptive wheelbase preview strategy, *J. Vibroengineering*, 17(2), 883-904.

[9] Z. Xie, P.K. Wong, J. Zhao, T. Xu and K.I. Wong. 2013. A noise-insensitive semi-active air suspension for heavy-duty vehicles with an integrated fuzzy-wheelbase preview control, *Mathematical Problems in Engineering*, Article ID 121953.

[10] E. Guglielmino, T. Sireteanu, C.W. Stammers, G. Ghita and M. Giuclea. 2004. *Semi-Active Suspension Control: Improved Vehicle Ride and Road Friendliness*, Springer Science & Business Media, London.

[11] L.W. Robert and L.L. Kent. 1997. *Modeling and Simulation of Dynamic System*, Second Edition, Prentice-Hall.

[12] V.V. Hung and S.R. Esfandiari. 1997. *Dynamic Systems: Modeling and Analysis*, Second Edition, McGraw-Hill.

[13] S. Mouleeswaran. 2008. Development of active suspension system for automobiles using PID controller, *Proc. World Congress on Engineering*, London, UK.

[14] D. Karnopp. 1986. Theoretical limitations in active vehicle suspensions, *Vehicle System Dynamics*, 15, 41-54. http://dx.doi.org/10.1080/00423118608968839.