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Reduction of Body Acceleration in the Quarter Car Model using Fractional Order Fuzzy Sliding Mode Controller

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ABSTRACT:

Vehicle vibration can be controlled by Active Suspension System (ASS). The performances of ASS are better than the conventional Passive Suspension System (PSS). The effectiveness of ASS is based on the type of controllers used. In this paper, a quarter car model with ASS is considered for analysis. To reduce the vibration and improve the ride quality, Fractional order Fuzzy Sliding Mode Controller (FrFSMC) is proposed and its performances are compared with Fuzzy Sliding Mode Controller (FSMC) and passive system. While testing the performance of the controllers three types of road disturbances are given to the quarter car model to stimulate the vibration. The results of the proposed controllers are also compared against the existing Gray Fuzzy Sliding Mode Controller (GFSMC). From the time responses and root mean square indices, FrFSMC performs better than the FSMC, GFSMC and PSS.

KEYWORDS:

Active suspension system; Fractional order fuzzy sliding mode control; Fuzzy sliding mode control; Quarter car model; Vibration control

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NOMENCLATURE AND ABBREVIATIONS:

ASS	*Active suspension system*
BA	*Body acceleration*
FLC	*Fuzzy logic control*
FrFSMC	*Fractional order fuzzy sliding mode controller*
FrSMC	*Fractional order sliding mode controller*
FSMC	*Fuzzy Sliding mode controller*
GFSMC	*Gray fuzzy sliding mode controller*
PSD	*Power spectrum density*
QCM	*Quarter car model*
RMS	*Root mean square*
SMC	*Sliding mode controller*
c_s	*Damping coefficient*
c_t	*Tyre damping*
D	*Differentiation*
f_s	*Force*
k	*Switching gain*
k_s	*Spring constant*
k_t	*Tyre stiffness*
m_s	*Sprung mass*
m_u	*Unsprng mass*
s	*Sliding surface*
Z_s	*Sprung mass displacement*
Z_u	*Unsprung mass displacement*
λ	*Sliding surface gain*

1. Introduction

An automotive suspension system is one of the important components in a vehicle to reduce the unwanted vibration and improve the ride quality. Vibrations are mainly generated by irregularities in the road which cannot be avoided. Therefore, the suspension system is designed to keep a firm contact between the road and the tire, to have good handling performance and safety to the driver. In a car, four independent suspensions are mounted in each wheel assembly. The suspension system supports the vehicle body and reduces the vibration. In a passive suspension system, springs and dampers are placed between the vehicle body and axles. In the case of an active suspension system, a control force is generated and applied to oppose the vibration. Considering one-fourth of the vehicle model is the common practice [1] to analyze the vertical vibration in the vehicle and the same is called as the Quarter Car Model (QCM) [2-3]. Though pitch, roll and yaw are causes of discomfort in a travel but the vertical displacement (yaw) is the main factor for the ride discomfort.

Therefore importance is given to analyze the vertical displacement and its derivatives such as vibration and acceleration. The control strategies for the QCM are reviewed in [4]. A semi-active suspension controller using Sliding Mode Control (SMC) for quarter car with driver model [5], Grey Fuzzy SMC [6], a type-2 Fuzzy Logic Controller (FLC) based SMC [7], a robust FLC-based SMC [8] and a model-free adaptive SMC [9] are designed for active suspension system of the QCM. The car body mass was estimated by SMC [10] for the QCM. Different kinds of FLC are designed and analysed for a QCM [11-13]. Fuzzy based SMC (FSMC) is designed for the quarter car with driver model works better than the FLC in [14]. The PID and FLC are simulated for the quarter car with driver model in [15]. In general, PID, FLC are most common controllers. In [14], the effectiveness of the FLC over PID is compared and [15] indicates the effectiveness of the FSMC over SMC. Therefore, in this paper FSMC is compared against the fractional order SMC and fractional order Fuzzy SMC.

In the case of a driver model, second actuator can be included in between the sprung mass and mass of the cushion. The concept of the dual actuator is not considered in [14-15]. The quarter car model with and without driver model is almost similar. The merits of the proposed work over ref [14-15] are the effectiveness of the FrFSMC against FrSMC and FSMC. The Fractional order Fuzzy Sliding Model Control (FrFSMC) is proposed to reduce the vibration for the QCM for different road profiles and its performances are compared with FSMC and passive system. This paper is organized as follows. In section 2, Quarter car model is discussed. In section 3, controllers design approaches for the proposed model is presented. In section 4, the numerical simulations are discussed. Finally, results and conclusions are summarized in section 5.

2. Quarter car model

The QCM has 2 degrees of freedom. It has made of two solid masses namely sprung and unsprung masses which are denoted as m_s and m_u respectively. The m_s represents one fourth of the body of the vehicle, the m_u represents one wheel of the vehicle. A spring of stiffness k_s, and a shock absorber with damping coefficient c_s, support the sprung mass and are called the main suspension. The m_u is in direct contact with the ground through the spring. k_u, representing the tire stiffness. Fig. 1 shows the QCM with the active suspension system. f_s is the force produced by the actuator to oppose the vibration. The equations are motion are given by,

$$\begin{aligned} m_u \ddot{z}_u &= -k_t(z_u - z_r) - c_t(\dot{z}_u - \dot{z}_r) \\ &\quad + k_s(z_s - z_u) + c_s(\dot{z}_s - \dot{z}_u) - f_s \end{aligned} \tag{1}$$

$$m_s \ddot{z}_s = -k_s(z_s - z_u) - c_s(\dot{z}_s - \dot{z}_u) + f_s \tag{2}$$

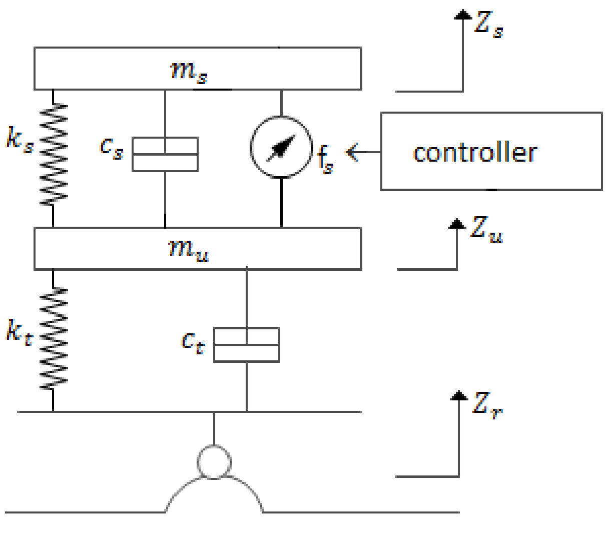

Fig. 1: Quarter car model

3. Design of controllers

3.1. Fuzzy sliding mode controller

SMC is a type of Variable Structure Control Systems (VSCS) which are characterised by a set of feedback control laws and decision rules. These rules are termed as the switching functions. Its input has a measure of the system behaviour at the present instant and produces as an output to the feedback controller which should be used at that instant in time. Therefore VSCS is valid for specified region of system behaviour. In SMC, VSCS are designed to drive and then constrain the system state to lie within a neighbourhood of the switching function. There are two main advantages of this approach. Firstly, the dynamic behaviour of the system may be tailored by the particular choice of switching function. Secondly, the closed loop response becomes totally insensitive to a particular class of uncertainty. The latter invariance property clearly makes the methodology an appropriate choice for robust control. The vehicle suspension is subjected to uncertainty in terms of variation of the driver mass (part of sprung mass). The sliding mode design approach consists of two components. The first involves the design of a switching function so that the sliding motion satisfies design specifications. The second is the selection of a control law which will make the switching function attractive to the system state. This control law is not necessarily discontinuous [11], [23].

Fuzzy control has been proposed to tackle the problem of car suspension for the unknown environmental parameters. However, the large amount of the fuzzy rules makes the analysis complex. Therefore FLC-based SMC scheme is proposed. The main strategy of tuning the controller is to tune the slope of the sliding surface. Because of the sliding surface is brought to the neighbourhood of the system states, the need for a high control gain in the conventional SMC is eliminated. The advantage of the FSMC is that it requires fewer fuzzy rules than FLC [8]. It forces the sliding surface to go to the state errors and make them zero rapidly. Thus while improving the ride quality, smaller control forces are obtained by using the FLC. FSMC is implemented by considering the property of driving FLC into the sliding mode in which the controlled system is invariant to parameter fluctuations and disturbances. The Lyapunov's function $V = 0.5s^2$ is considered to design the SMC. Where s is the sliding surface. The existence condition for sliding mode is possible.

$$\dot{V} = s\dot{s} < 0 \tag{3}$$

The state variables - suspension deflection x_1 and car body velocity - x_2 for the QCM is chosen as follows,

$$x_1 = z_s - z_u \tag{4}$$

$$x_2 = \dot{z}_s \tag{5}$$

The necessary condition to drive the state trajectory toward the sliding surface [17] is given by,

$$\dot{s}(x,t) = 0 \tag{6}$$

The sliding surface is chosen as

$$s = x_2 + \lambda x_1 \tag{7}$$

Where λ is the sliding surface gain.

Taking derivative on both sides of the Eqn. (7) becomes,

$$\dot{s} = \dot{x}_2 + \lambda \dot{x}_1 \quad (8)$$

When the system moves on the sliding mode at,

$$\dot{s} = 0 \quad (9)$$

$$0 = \dot{x}_2 + \lambda \dot{x}_1 \quad (10)$$

By substituting the state variables,

$$0 = -\frac{k_s}{m_s}(z_s - z_u) - \frac{c_s}{m_s}(\dot{z}_s - \dot{z}_u) + \frac{f_s}{m_s} + \lambda(\dot{z}_s - \dot{z}_u) \quad (11)$$

The equivalent control force derived from the above Eqn. (11) is,

$$f_{s\_equ} = k_s(z_s - z_u) + c_s(\dot{z}_s - \dot{z}_u) - \lambda m_s(\dot{z}_s - \dot{z}_u) \quad (12)$$

Hence the desired control force is,

$$f_s = f_{s\_equ} - k * sign(s) \quad (13)$$

$$f_s = k_s(z_s - z_u) + c_s(\dot{z}_s - \dot{z}_u) - \lambda m_s(\dot{z}_s - \dot{z}_u) - k * sign(s) \quad (14)$$

Where *k*sign(s)* is the switching function derived from the FLC which brings the system into the sliding surface and converges to zero. The input and output membership functions of the FLC have three membership functions such as Positive (P), Negative (N) and Zero (Z) which are shown in the Fig. 2. These inputs and output membership functions forms a set of 9 fuzzy rules [18] as shown in Table 1. This fuzzy rule base is, in the form of linguistic variables using fuzzy conditional statements. The centre of gravity method is used to de-fuzzify the inferred output. The value of 'k' in FLC is the range of the output which is -20 to 20.

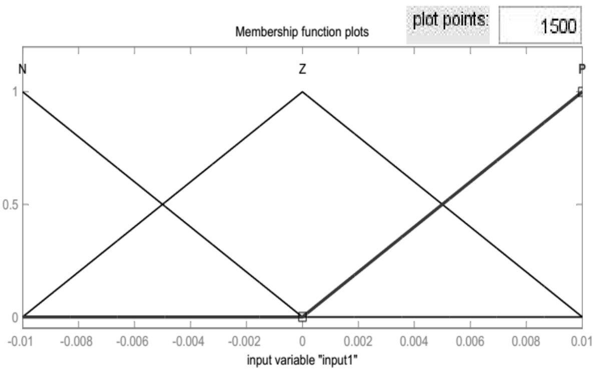

Fig. 2: Membership function for the inputs and output

Table 1: Rule base for FSMC

Outputs		s N	s Z	s P
s	N	P	P	Z
	Z	P	Z	N
	P	Z	N	N

3.2. Fractional fuzzy sliding mode controller

Fractional calculus is one of the commonly used techniques to extend the integer order calculus into non integer order calculus. Which means the order can be in the non-integer form [19-21]. The commonly used definition is Caputo fractional calculus; it is clear that fractional calculus has higher degrees of freedom than integer order calculus. Therefore an appropriate fractional order is given to get the better control performance. The essential discontinuous switching characteristic of the conventional SMC causes high frequency chatter leading to the steady state error. Therefore fractional calculus is implemented which extends the conventional integer order calculus to any order. Fractional calculus and SMC have been combined and applied to system control. Thus weakening the chatter and maintaining the robustness and response characteristics of the traditional SMC. A fractional order sliding surface is designed with regard to the slow energy transfer of the fractional order sliding surface.

The real time self-tuning sliding mode switch gain of the FLC is designed with regard to the uncertainty of parameters and disturbance changes. The objective of such a design is to weaken the chatter caused by the SMC while improving the accuracy of the control and the robustness against load disturbance, thereby realising better control performance than the traditional integer order SMC [22]. The state variables x_1 and x_2 for the quarter car system are chosen per the Eqns. (4) and (5). The fractional order sliding surface is

$$s = x_2 + \lambda x_1,\ \dot{x}_1 = x_2,\ s = \dot{x}_1 + \lambda x_1$$

$$s = D^{\alpha} x_1 + \lambda x_1 \quad (15)$$

Where D represents derivative. Taking derivative on both sides and simplification becomes,

$$s = D^{\alpha} D^{-1} D^{1} x_1 + \lambda x_1,\ s = D^{\alpha-1} D x_1 + \lambda x_1$$

$$\dot{s} = D^{\alpha-1} \dot{x}_2 + \lambda \dot{x}_1 \quad (16)$$

As per the Eqn. (9),

$$0 = D^{\alpha-1} \dot{x}_2 + \lambda \dot{x}_1 \quad (17)$$

By substituting the state variables and simplification, the equivalent control force is,

$$f_{s\_equ} = k_s(z_s - z_u) + c_s(\dot{z}_s - \dot{z}_u) - \lambda m_s D^{1-\alpha}(\dot{z}_s - \dot{z}_u) \quad (18)$$

Hence the desired active control force f_s is,

$$f_s = f_{s\_equ} - k * sign(s) \quad (19)$$

The switching function *k*sign(s)* is derived by the FLC which is used as same as the FLC used in FSMC.

4. Numerical simulations results

The QCM is simulated in Simulink blocks of MATLAB R2012b and the required functions for the road profiles are written as m files and used in interpreted functional block of the Simulink. Figs. 3 to 6 show the Matlab Simulink model of the proposed controllers and the QCM model. The vehicle parameters used in this work are from [6] and summarised in Table 2. While testing the performances of the QCM, the system is subjected to single bump, sinusoidal road and a random road profile. All the 3 road profiles are from [11] and [23] and shown in Fig. 7. Initially, the SMC is designed by adjusting the λ in trial and error method and then fixed at 1.5. Next, the FrFSMC parameter α is adjusted by fixing the λ value and finally α is fixed at 0.9. Since the ride quality mainly depends on vertical displacement and its derivatives of the QCM, the reduction of Body Acceleration (BA) is analysed for the designed controllers.

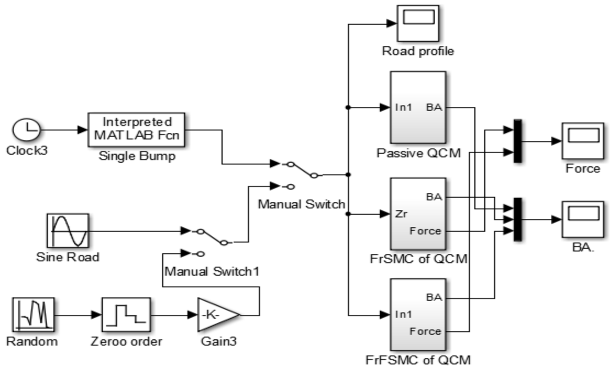

Fig. 3: Simulink model of the proposed system

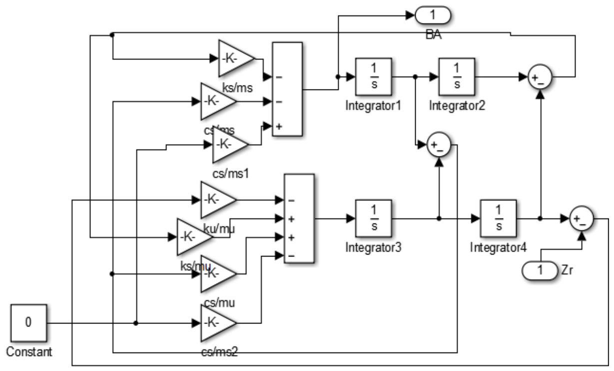

Fig. 4: Simulink model of the QCM

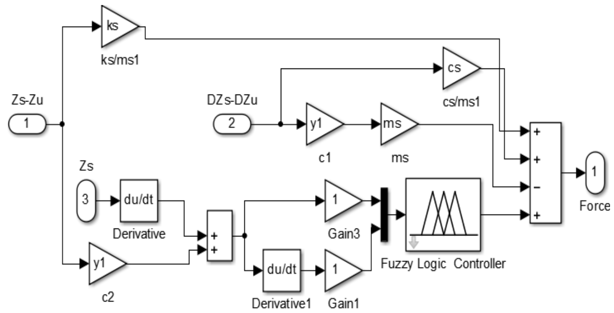

Fig. 5: Simulink model of the FSMC

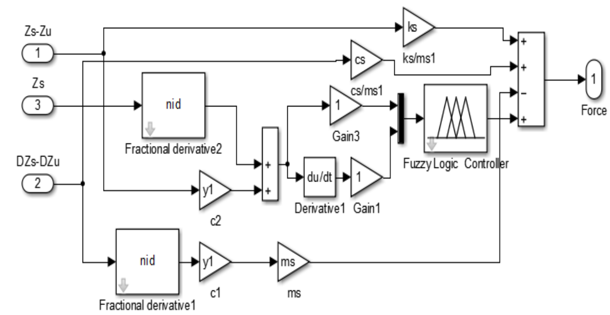

Fig. 6: Simulink model of the FrFSMC

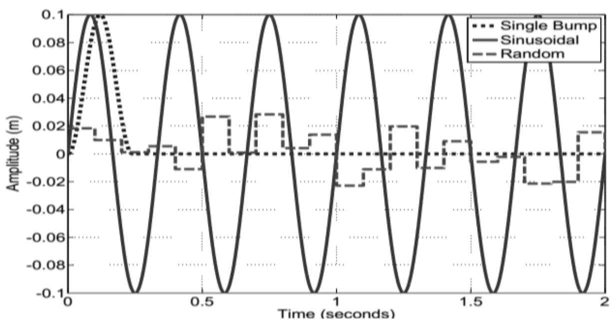

Fig. 7: Road profiles

Table 2: Quarter car model parameters

Mass (kg)		Damping coefficient (Ns/m)		Spring stiffness (kN/m)	
m_u	59	c_t	0	k_t	190
m_s	290	c_s	1000	k_s	16.812

The BA is the final control element considered for analysis. The time response of the QCM with the controllers is shown in Fig. 8 for the single bump input and its corresponding forces are shown in Fig. 9. The RMS values are shown in Table 3. From the RMS values, it is derived as FrFSMC reduces the BA by 79.29% and FSMC reduces the BA by 70.81%. FrFSMC controls the BA 8.48 % better than the FSMC. The control force produced by the FrFSMC is slightly more than the FSMC to reduce the BA.

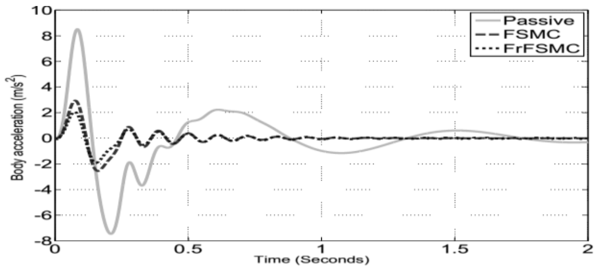

Fig. 8: Time response of the BA for single bump input

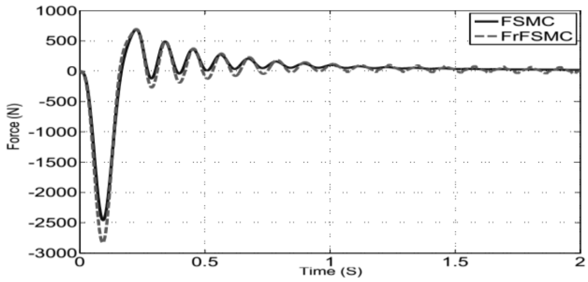

Fig. 9: Force produced by controllers for single bump input

Table 3: RMS values of BA (m/s^2)

Controller	Road Profile		
	Single bump	Sinusoidal	Random
Passive	2.233	7.411	2.234
FSMC	0.6518	2.316	1.026
FrFSMC	0.4624	1.585	0.8145

The second road profile considered for analysis is the sinusoidal road which gives the periodic disturbance with a magnitude of 0.1m and 3 Hz. Fig. 10 shows the time response of controllers for sinusoidal road input. FrFSMC reduces the BA by 78.61% and FSMC reduces the BA by 68.75%. FrFSMC controls the BA 9.86 % better than the FSMC. The control force shown in Fig. 11 produce by the FrFSMC is slightly more than the FSMC to reduce the BA. Compared with the single bump the magnitude of the force is much higher due to continuous road disturbance. While driving the vehicle on the road any kind of disturbances are expected therefore it is necessary to consider the random road profile. Fig. 12 shows the time response of the controllers with random road input. In this case, FrFSMC reduces the BA by 63.54% and FSMC reduces the BA by 54.07%. FrFSMC controls the BA 9.47 %

better than the FSMC. The control force shown in Fig. 13 produce by the FrFSMC is slightly more than the FSMC to reduce the BA. The magnitude of the force depends on the magnitude of the road input which is varying rapidly in 0.1 seconds.

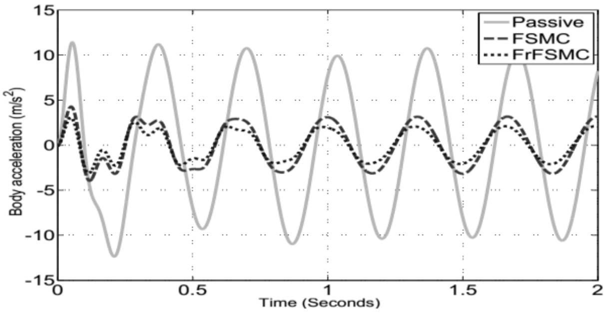

Fig. 10: Time response of the BA for sinusoidal road input

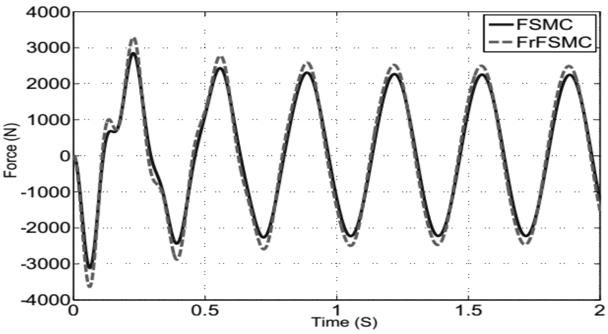

Fig. 11: Force produced by controllers for single bump input

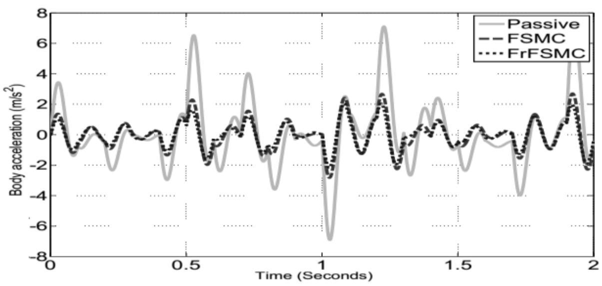

Fig. 12: Time response of the BA for random road input

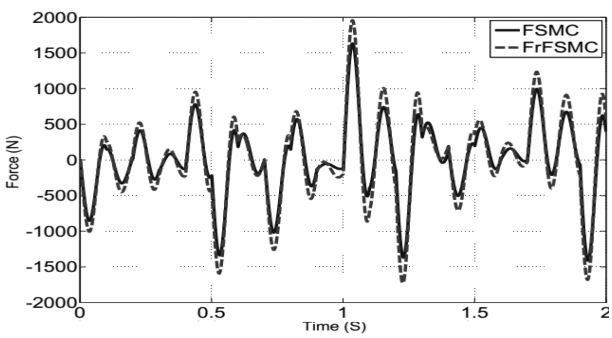

Fig. 13: Force produced by controllers for random road input

The performance of the controllers is tested under the perturbed condition as shown in Fig. 14. The driver body mass which is also part of the sprung mass is varied from 260kg to 320kg. Therefore the driver mass is varied from 40kg to 100kg. FrFSMC performs well for all the driver masses. Fig. 15 shows that the percentage of the reduction in BA with different speed of the vehicle. The bump in designed with respect to speed. Therefore the variation in the speed affects the response of the system. FrFSMC performs better than all the controllers in all the considered speed. The performance of the FrFSMC and FSMC are also compared with the existing FSMC and Gray Fuzzy SMC (GFSMC) from [6]. The FSMC proposed in this paper reduces the BA 64.45% whereas the existing FSMC in [6] reduces only 47.7%. Similarly, the FrFSMC reduces the BA by 73.82 % whereas the GFSMC reduces only 56.50%. Therefore, the FrFSMC reduces the BA by 17.32 % better than the existing GFSMC.

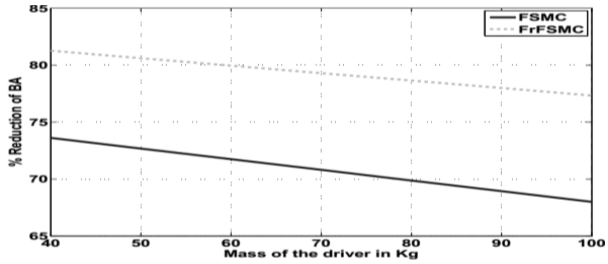

Fig. 14: Performance of the controllers with driver's mass variation for single bump

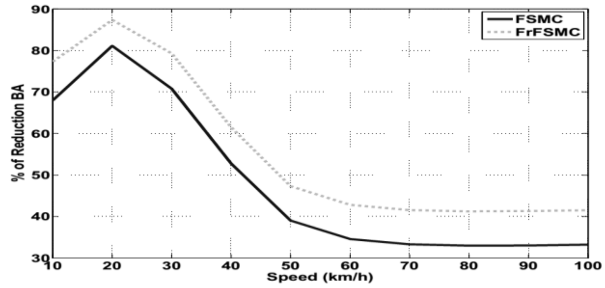

Fig. 15: Performance of the controllers with different speed of the vehicle for single bump

The Power Spectrum Density (PSD) is a plot of the BA as a function of frequency. The PSD is plotted for all three types of road profiles which are shown from Figs. 16 to 18. In all the three graphs the FrFSMC has reduced the BA effectively in the human sensitive frequency range [2] of 4 to 8Hz.

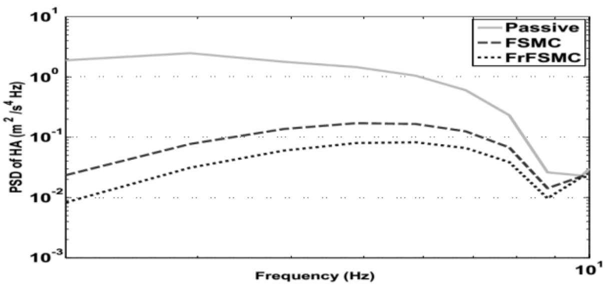

Fig. 16: PSD for single bump input

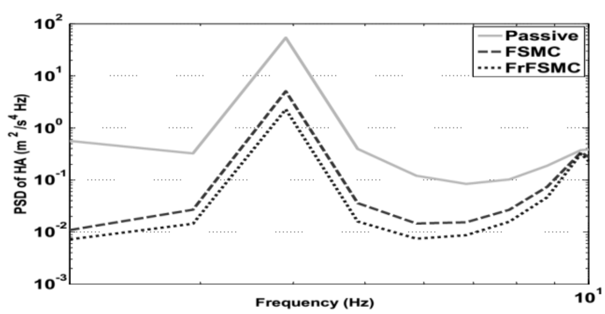

Fig. 17: PSD for sinusoidal road input

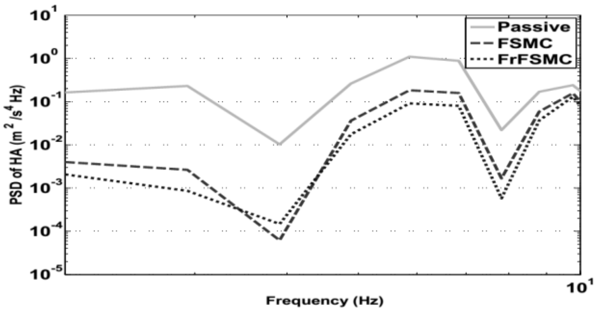

Fig. 18: PSD for Random road input

5. Conclusion

The control strategies for the QCM with FrFSMC are designed and simulated. The performances of the FrFSMC are compared against the FSMC and passive system. FrFSMC reduces BA better than FSMC and passive suspension system. FrFSMC reduces BA by 17.32 % better than the existing GFSMC. The results are analysed in terms of RMS and PSD. Hence the reduction of BA is improved by FrFSMC. The QCM can be considered with driver model and integrated seat suspension system. The other types of advanced SMCs such as Terminal SMC, Fractional Terminal SMC can be designed to improve the performance further.

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