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System Optimization Algorithm for 3DOF Quarter Car Active Suspension

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ABSTRACT:

This paper handles the synergy between the design and control optimization problem for an active car suspension system consisting both active and passive components. The dynamics of the suspension system are modeled utilizing a three degree of freedom (3DOF), linear with time invariant quarter car model with capability to capture the impact of the passive stiffness on suspension deflection depending up on the spectral density of road disturbances. Direct transcription, a strategy which guarantees system optimality, is presented and utilized to find the optimal design of the suspension system. The active system dynamics were analyzed with modified level of control force to examine how dynamic system should be designed accordingly when the active control force is introduced.

KEYWORDS:

Active suspension system; System optimization; Quarter car model; Direct transcription; Optimal design

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1. Introduction

Ride comfort is provided by car suspension system by isolating the passengers from ground disturbances and enhances the vehicle handling by adjusting the contact forces between the vehicle body, tires and the road. These desired requirements are alternately conflicting with the strength and stiffness. Softer suspension offers more comfort at the expense of degraded handling [1]. These requirements are traded off by arranging them in a weighted performance function, Bloza form, for optimization [2-8]. The optimal performance of a suspension system depends on whether it is passive or active. External energy sources are utilized by active suspension (e.g. hydraulic actuators), while passive suspension consists solely of power storage and dissipation components (e.g. springs and dampers) [1]. Suspension passive elements can only transmit forces which depend on relative vehicle chassis/tire motion, while active elements can produce forces that rely on absolute chassis motion.

Consequently, active system can perform better than their passive counterparts significantly, at the cost of consuming external energy [1-2, 8-9]. When a suspension system consists both passive and active elements, these elements compete, instead of help each other [10], so failing to achieve the suspension's full performance potential [11]. This competition exemplifies the coupling between the controller and plant optimization problems [10, 12-16]. Sequential optimization of the suspension's controller (e.g. active elements) and plant (passive elements) does not account for this synergy and becomes unsuccessful to guarantee the system optimality. To overcome this sub-optimality and to design synergy systems, one must optimize both passive and active components simultaneously. Such a simultaneous optimization strategy is lacking in the previous works. This paper contributes an integrated coupling between the passive and active components for 3DOF suspension optimization problem with its solution utilizing a direct transcription method.

2. System-level optimization problem

To optimize a passive and active sub-systems of a suspension system simultaneously, the designer must utilize a system-level model to capture the impact of the two sub-systems on performance index. The proposed model should show design aims in terms of design variables (e.g., spring wire diameter, spring helix diameter, pitch, number of active coils, valve diameter, working piston diameter, damper stroke) to determine the passive stiffness and damping coefficient rather than the obtained ones (natural frequencies and damping ratios). It must be simple when capturing the suspension system design tradeoff between the control cost, handling and the ride comfort. The dynamics of the suspension system are characterized by a linear time invariant quarter car 3DOF model as shown in Fig. 1. In the proposed model $m_p = 90kg, m_s = 240kg$ and

$m_u = 36$ are the passenger with seat, sprung and unsprung masses. $k_p = 8000 N/m$ and $c_p = 3000 Ns/m$ are the passenger seat stiffness and seat damping coefficient, k_s and c_s are the passive stiffness and damping coefficient, $k_{us} = 160000$N/m is the tire stiffness where the tire damping coefficient is neglected (due to its small value), $u(t)$ is the active force $z_p(t), z_s(t), z_u(t)$ and $z_o(t)$ are the vertical displacements of the passenger and seat mass, the vehicle body mass, the wheel assembly mass and the road vertical disturbances respectively.

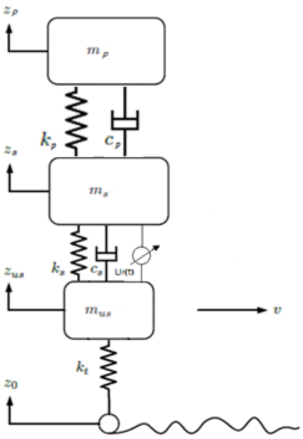

Fig. 1: Combined active/passive suspension dynamics model

The equations of motion of the active suspension system state space model with the control force and road disturbances is given by,

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\\ \dot{x}_5\\ \dot{x}_6\end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 \\ \frac{-k_t}{m_u} & \frac{k_s}{m_u} & 0 & \frac{-c_s}{m_u} & \frac{c_s}{m_u} & 0 \\ 0 & \frac{-k_s}{m_s} & \frac{k_p}{m_s} & \frac{c_s}{m_s} & \frac{-c_s}{m_s} & \frac{-c_p}{m_s} \\ 0 & 0 & \frac{-k_p}{m_p} & 0 & \frac{c_p}{m_p} & \frac{-c_p}{m_p} \end{bmatrix} \begin{Bmatrix} x_1\\ x_2\\ x_3\\ x_4\\ x_5\\ x_6\end{Bmatrix} + \begin{Bmatrix} 0\\ 0\\ 0\\ \frac{-1}{m_u}\\ \frac{1}{m_u}\\ 0\end{Bmatrix} u + \begin{Bmatrix} -1\\ 0\\ 0\\ 0\\ 0\\ 0\end{Bmatrix} \dot{x} \qquad (1)$$

Where $x(t) = \lfloor x_u - x_o, x_s - x_u, x_p - x_s, \dot{x}_u, \dot{x}_s, \dot{x}_p \rfloor$ is the suspension system states vector. The suspension system is excited by variances in road elevation z_o when the vehicle moves at speed $v = 20 m/s$. A rough road input [17] and a ramp input are utilized to check the suspension. Simultaneous approach is explored for designing this active suspension system utilizing direct transcription method. Most previous studies dealt with sprung mass stiffness and damping coefficient as independent variable. Allison's 2DOF model [21] treated the sprung mass stiffness and damping coefficient as dependent variable since they are dependent on the geometric variables (spring and damper design variables). The variation of co-design problem structure is a significant step towards co-design strategies that can handle the plant design with extra realistic details. The system level combined passive and active suspension components optimization is posed by using the equation of motion in state space Eqn. (1). The control force *u(t)* to the system is an active component between the vehicle body and the wheel assembly.

A control force could be produced via an actuator (for e.g. an electromagnetic linear motor) [18, 19]. However; the actuator details won't be discussed in this work which will be assumed to be an arbitrary control force trajectory *u(t)* that could be attained by imposing maximum force limits. The control variables vector x_c is a time discretisation of the actuation force trajectory,

$$x_c = [u_1, u_2, ...u_{nt}]^T$$

The objective function of the system incorporates the ride comfort, handling and control utilizing the Lagrange term as follows,

$$J = \int_0^{t_F} (\rho_1(\ddot{z}_p)^2 + \rho_2(z_{us} - z_r)^2 + \rho_3(u)^2) dt \qquad (2)$$

Subject to: $\dot{x}(t) = Ax(t) + Bu(t) + D\dot{x}_o$ (3)

$$u_{\min}(t) \le u(t) \ge u_{\max}(t) \qquad (4)$$

$$\underline{x}_p \le x_p \le \overline{x}_p \qquad (5)$$

$$g_p \le 0 \qquad (6)$$

Where $\rho_1 = 0.5, \rho_2 = 10^5$ and $\rho_3 = 10^{-5}$ are the optimization parameters [21], x_p is the plant design variables vector, and g_p are the plant design constraints. The performance index, optimization objective, is a weighted summation of the RMS of passenger and seat acceleration, wheel hop and control force.

Variables of optimization for the plant are the spring and damping geometric variables used to determine the passive stiffness k_s and damping coefficient c_s and for the control design variables, the time discretisation of control force. This work is assumed a linear with time invariant full state feedback where the control force bounds are imposed. The function of active control force in the system can be changed by making explicit bounds on the maximum control force. System results with bounds equal zero will be near to the results of passive system, thus the bounds increased gradually to monitor how the structure design with dynamic characteristics change as the active force plays an important role. An optimal solution to the introduced model will be indicated as the optimal active suspension design.

3. Direct transcription method

Direct Transcription (DT) method is guaranteed to find single system optimization design which is a category of "discretize-then-optimize" optimal control methods. DT method that is implemented to the active suspension optimization problem is given by,

$$\left.\begin{array}{c} \min\limits_{\Xi, x_c, x_p, t_F} J = f(\Xi, x_c, x_p, t_F) \\ S.T: g_p(\Xi, x_p) \le 0 \\ \zeta_i(\Xi, x_c, x_p, t_F) = 0 \\ Where: i = 1, 2, ...n_{t-1}, n_t \end{array}\right\} \qquad (7)$$

Where the dimension of Ξ is $(n_t.n_s)$. n_t are the time discretisation points, n_s is the number of states variables,

x_c are the control design variables, x_p is the plant design, t_F is the simulation time, g_p are plant design constraints, ζ_i are the defect constraints to preserve the continuity for the states and control variables discretisation. The structure of the problem can be analyzed utilizing graph theoretic techniques [20]. Fig. 2 depicts a flowchart of DT method that handles the active suspension as a co-design problem. The physical system analysis depends on the plant design and state design variables to determine the physical design constraints $g_p(.)$ with the intermediate variables (p) utilized in calculating the objective function with the dynamic constraints.

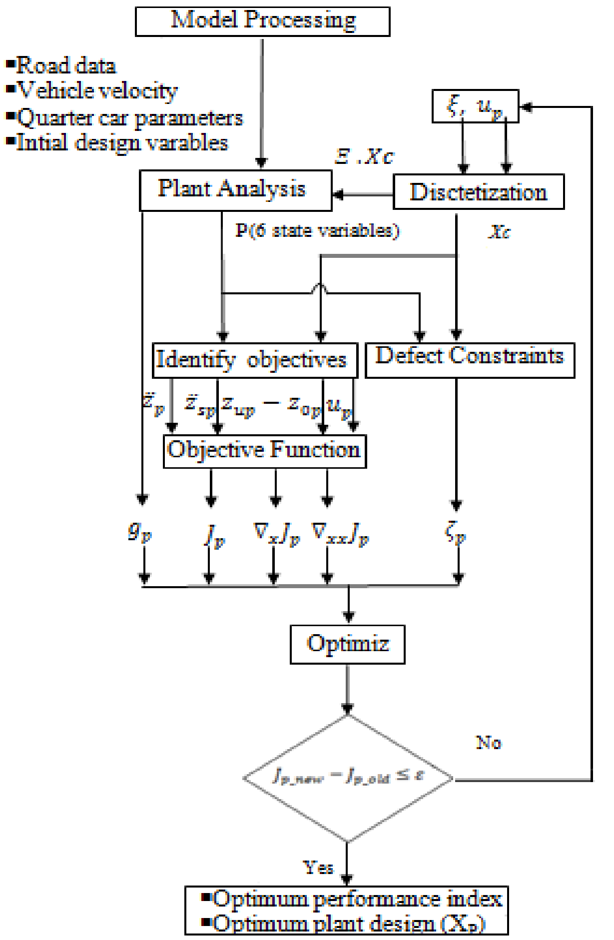

Fig. 2: Analysis structure of DT for 3DOF active suspension system

This structure of problem accounts for the coupling between the physical and control design variables which is a more realistic problem representation since it takes into account of the synergy between the plant and control design variables. While DT's dimension is large ($n_t.n_s$ elements), its problem structure shows important utilities where every iteration of the optimization is a complete constraint Jacobean as given in Table 1. The DT co-design algorithm is summarized as follows,

1. Set $k=1$, initial x^k and ε
2. $$\left.\begin{array}{c} x_p^{k+1} \arg \min\limits_{\Xi, x_p, x_c, t_F} J \\ S.T.: g_p(\zeta(t), x_c^k, x_p) \le 0 \end{array}\right\}$$
3. If $\left\| J^{k+1} - J^k \right\| \le \varepsilon$, terminate.
4. $k = k+1$, go to step 2.

Where k is an optimization iteration counter, ε is termination tolerance and simulation satisfies the system dynamics.

Table 1: Constraint Jacobean Structures

	$d[t_0]$	…	$d[t_{nt}]$	$x_1[t_0]$	…	$x_1[t_{nt}]$	$x_c[t_0]$	…	$x_c[t_{nt}]$
$g_1[t_0]$	x	⋯ ⋯	x	x	…	0	x	…	0
$g_1[t_1]$	x	…	x	0	…	0	0	…	0
⋮	⋮	…	⋮	⋮	…	⋮	⋮	…	⋮
$g_1[t_{nt}]$	x	…	x	0	…	x	0	…	x

4. Plant design constraints

The vehicle active suspension system in the introduced model uses a helical compression spring component with squared and ground end as shown in Fig. 3. The suspension coil spring hedges the absorber as it has the same axis. The spring design variables are D and d (the helix and wire diameters respectively), p (spring pitch), and N_a (the number of active coils) while the damper design variables are D_o (the spool valve outer circumference), D_p (working piston diameter), and D_s (damper stroke). The complete physical design vector can be defined $x_p = \lfloor d, D, p, N_a, D_o, D_p, D_s \rfloor$ as the suspension spring stiffness and damping coefficient depend on the elements of physical design vector Eqn. (8 and 9) respectively:

$$k_s = \frac{d^4 G}{8D^3 N_a (1 + \frac{1}{2C^2})} \quad (8)$$

$$c_s = \frac{D_p^4}{8C_d C_2 D_o^2} \sqrt{\frac{\pi k_v \rho_1}{2}} \quad (9)$$

Where G = 77.2 GPa is the shear modulus, $C_d = 0.7$ is the discharge coefficient, C_2 is the absorber valve coefficient, k_v is the spool valve spring stiffness, and ρ_1 the absorber fluid density. A fully reproducible detail of active suspension system important constraints is found in [21]. The first tow spring constraints depend on the spring index $(C = D/d)$ and are given as follows,

$$g_1(x_p) = 4 - C \le 4 \quad (10)$$

$$g_2(x_p) = 4 - C \le 12 \quad (11)$$

The constraint to prevent buckling is given by,

$$g_3(x_p) = L_o - 5.26D \le 0 \quad (12)$$

The spring free length must be fit with the specified pocket length ($L_{o\max}$) to the vehicle as follows,

$$g_4(x_p) = L_o - L_{o\max} \le 0 \quad (13)$$

The outer diameter of the spring must not exceed $D_{o\max}$ to avoid interference with vehicle components.

$$g_5(x_p) = d + D - D_{o\max} \le 0 \quad (14)$$

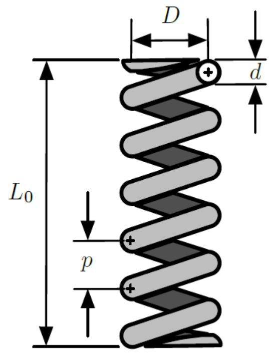

Fig. 3: Helical compression spring used in the suspension model

The spring internal diameter ought to be large to fit around the absorber within δ_{dc} clearance as follows,

$$g_6(x_p)=d\_D+D_p+2(\delta_{dc}+t_d)\le 0 \tag{15}$$

Where t_d is the absorber wall thickness. The following constraint is the suspension rattle space (permissible peak-to-peak displacement),

$$g_7(x_p,\Xi)=\delta_{\max}-L_o+L_s+L_B+\delta_g\le 0 \tag{16}$$

Where L_B is the bump stop thickness, and δ_g is the static suspension deflection. While the stress constraint is given by,

$$g_8(x_p)=\frac{(\tau n_d-S_{sy})}{S_{sy}}\le 0 \tag{17}$$

Where τ is the shear stress, n_d is the design factor, and S_{sy} is shear yield stress. To ensure spring validity and linearity of equations of motions, Eqn. (17), the next constraint must be satisfied as,

$$g_9(x_p,\Xi)=0.15+1-\frac{L_o-L_s}{\delta_g+1.1\delta_{\max f}}\le 0 \tag{18}$$

Where $\delta_{\max f}$ is the maximum spring deflection when the quarter car travels at *20 m/s* over a rough road [17].

The shock absorber and its orifice are shown in Fig. 4 and Fig. 5 respectively. To ensure adequate absorber range of motion and fitment with the specific length the following two constraints must be satisfied,

$$g_{10}(x_p)=L_o-L_s-D_s\le 0 \tag{19}$$

$$g_{11}(x_p)=2D_s+l_{d1}+l_{d2}-L_{o\max}\le 0 \tag{20}$$

Where l_{d1},l_{d2} quantify the distances required for absorber parts above and below the range of working piston. To ensure the maximum absorber pressure does not exceed the seal maximum pressure P_{allow}, the following constraint must be used,

$$g_{12}(x_p,\Xi)=p_{\max}-p_{allow}\le 0 \tag{21}$$

While the following constraint is to prevent the absorber from excessive velocity $\dot{\xi}_{3allow}$ as,

$$g_{13}(x_p,\Xi)=\dot{\xi}_{3\max}-\dot{\xi}_{3allow}\le 0 \tag{22}$$

The last absorber constraint is related to the clearance requirement to the maximum displacement of the spool valve lift x_{vallow} as,

$$g_{14}(x_p,\Xi)=x_{v\max}-x_{vallow}\le 0 \tag{23}$$

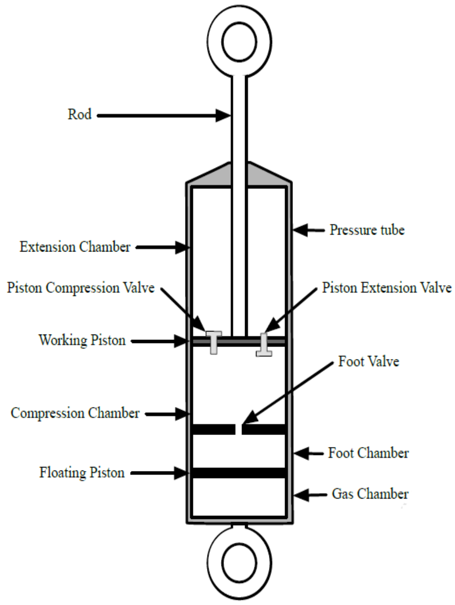

Fig. 4: Single tube telescope absorber section

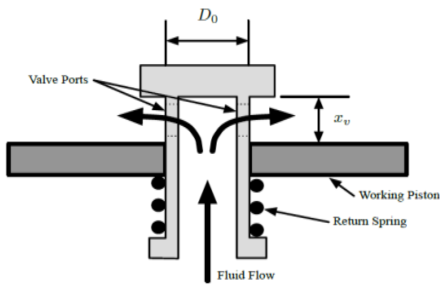

Fig. 5: Piston compression valve section

5. Results and discussion

An extension of DT to co-design problem was explained using 3DOF active suspension system. The problem of active suspension design was demonstrated utilizing DT method with full plant constraints. The DT solution for the plant design variables is given by,

$$x_{DT*}=\begin{bmatrix}0.0181,0.1373,0.0304,11.838,0.0092,\\0.0303,0.1697\end{bmatrix}$$

The optimum plant design achieved has the best performance index, as the plant passive components values are $k_s=3.37e4N/m$ and $c_s=303.5Ns/m$ when the actuator control force is *1200N.*

6. Conclusion

In this paper, an integrated active suspension system model with 3 masses has been developed and used for active suspension system. Direct transcription method is utilized where both state variables and control variable are discretized simultaneously by considering the plant design constraints. The numerical simulations showed that the optimal design based on the performance index of the suspension system is achieved when the actuator force is equal to 1200 N. DT method adoption as a strategy to calculate the optimal active suspension can assist the engineers to design passive system dynamics which combine in a perfect manner with the active element to obtain the best system performance.

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