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Single Motor Mechanical Power-Split Transmission for Hybrid Racing Cars

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ABSTRACT:

A hybrid vehicle derives its power from two or more distinct sources such as an internal combustion (IC) engine and an electric motor. Based on the mechanical architecture, Hybrid Electric Vehicles can be divided into three categories: parallel hybrids, series hybrids, and power-split hybrids. Parallel hybrids require frequent role reversal due to restrictions on motor power, whereas series hybrids lead to lower efficiency of the whole power train. The power-split hybrids combine the advantages of these two configurations by using one IC engine and two motors. The main objective of this paper is to design a transmission system for a hybrid racing car which is powered by an IC engine and a single electric motor which are arranged so as to represent a unique power-split system. This configuration reduces the need of one motor and allows seamless transition between engine-only with regeneration mode, motor only mode and parallel mode.

KEYWORDS:

Hybrid racing car; Transmission system; Regeneration; Power-split system; Gear box; Finite element analysis

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NOMENCLATURE:

R,S,P	No. of teeth of ring, sun and planet gear
N_r/T_r	Turns ratio
W^t	Tangential transmitted load (N)
K_o	Overload factor
K_v	Dynamic factor
K_s	Size factor
b	Face width, mm
m_t	Transverse metric module
K_H	Load-distribution factor
K_B	Rim-thickness factor
Y_J	Geometric factor for bending strength
Z_E	Elastic coefficient, $\sqrt{(N/mm^2)}$
Z_R	Surface condition factor
Z_I	Geometric factor for pitting resistance
d_{w1}	Pitch diameter, mm
Y_N	Stress cycle factor for bending stress
Y_θ	Temperature factor
Y_Z	Reliability factor
Z_N	Stress cycle life factor
Z_W	Hardness ratio factors for pitting resistance
$\propto$	Cone angle (°)
β	Chamfer angle (°)
μ	Friction coeff. of cone
μ_D	Friction coeff. of chamfers
d_m	Mean cone diameter (mm)
d_D	Pitch diameter (mm)
F_a	Shift force at sleeve (N)
n_c	No. of cones
τ_{max}	Shear stress
C_m, C_t	Service factors
M, T	Moment and torque (Nmm)
K	Outer to inner diameter ratio (d_o)
S	Speed
F'+D	Friction roll + drag force
GR	Overall gear ratio
E/M	Engine/Motor rpm
F	Driving force at wheel
Inst a	Instantaneous acceleration
D	Distance
R	Rolling friction coefficient
W	Wheel rpm
ω	Angular velocity of wheel
P	Power from engine/motor performance graph
F-	Difference between forces (F-(F'+D))
Avg a	Average acceleration
T	Time from instantaneous acceleration

1. Introduction

The power split hybrid systems combine a motor and an engine to drive the vehicle in addition to a separate alternator that is used to generate power and store in battery packs when the vehicle is running on engine. So this architecture can behave as both series and parallel hybrid, but results in great electrical and mechanical complexities. In this paper, a unique hybrid transmission system has been designed. This transmission system uses only one motor and one engine to run the car and to generate power by recharging the batteries at the same time as shown in Fig. 1(a). To achieve this, a motor can work as an alternator when given a rotation that is opposed to the direction in which it provides the power. Planetary gear set has been used to couple the engine and the motor as shown in Fig. 1(b). Sun gear is connected to the motor through solid shaft. The planetary carrier is connected to the engine through hollow shaft. Continuously Variable Transmission (CVT) of ratio ranging from 3.9 to 1 takes care of multiple speed options for engine. Motor controller takes care of speed requirements of motor. Spooling of shafts has been done to allow multiple inputs and outputs. Compound gears are connected to the shafts to achieve the required reductions for each mode. Two sets of Ratchet and Pawl are used, each on solid shaft and hollow shaft to ensure unidirectional rotation.

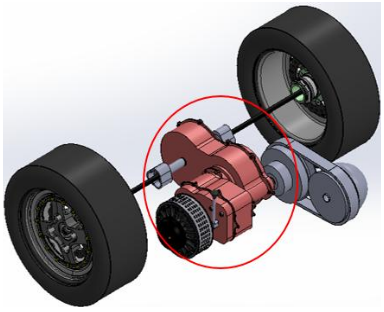

Fig. 1(a): Hybrid kit

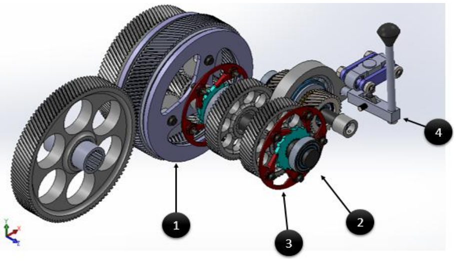

Fig. 1(b): Components of hybrid kit 1. Planetary gear set, 2. Shaft, 3. Ratchet and pawl, 4. Switching mechanism

A smooth shifting mechanism using synchronizers has been designed to shift between the modes. The shifting mechanism has the following connections:

- Motor shaft to the hollow shaft for regeneration mode.
- Motor shaft to the sun shaft for motor only and parallel modes.

In this paper, the transmission system has been designed to the requirements of Formula Student Hybrid competition organised by SAE. For a car to be qualified as a hybrid, it has to traverse 0-100m distance in 10s. A target of 8s has been set in the design presented here. A Bajaj Pulsar 220 DTSi engine having 19.12 Nm torque and 14kW of power at 7000 rpm and an AGNI motor 95r of 10 Nm torque and 4 kW of power at 4000rpm are used. Required torque is found by considering the rolling resistance, accelerating force of vehicle and aerodynamic force which sums up to 271.63 Nm. The reduction gears' values are aligned to achieve this torque. The following specifications are chosen:

- Mass of the vehicle, m = 300 kg
- Tire diameter, d = 0.510 m
- Tire pressure, p = 1.568 Pa
- Co-efficient of drag, cd = 0.9
- Frontal area, A = 0.235 m^2
- Density of air, ρ =1.22 kg/m^3

Fig. 2 shows the architecture of the system. Fig. 3 shows the engine only mode at which the power from the engine enters the gear that is mounted on the shaft connected to planetary carrier. The power from the engine is given to wheels whereas a small amount of power reaches the motor through the shifting mechanism when it shifts to connect the gear on hollow shaft with motor shaft thus making it run as an alternator achieving regeneration. Sun shaft is kept stationary by the ratchet and pawl mechanism. As shown in Fig. 4, motor powers the gear through the switching mechanism which is shifted to engage with the sun shaft. This gives rotation to the sun gear through the solid shaft, and hence the ring gear. Hollow shaft is kept stationary by the ratchet and pawl mechanism. Fig. 5 shows the mode in which both the planetary carrier and the sun gear receive rotation from engine and motor respectively but in opposite directions. This sums up the power from both the machines. The shifter is in same position as in the motor only mode. This mode gives higher torque output and top speed. The inherent nature of the planetary gear set allows this coupling.

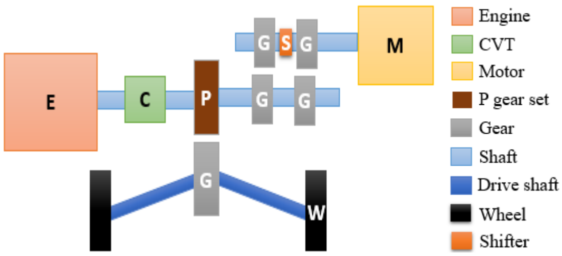

Fig. 2: Final architecture

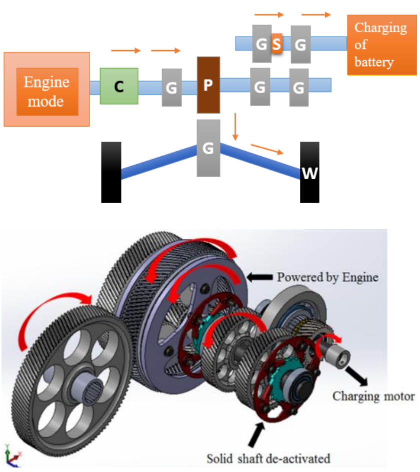

Fig. 3: Engine only with regeneration mode

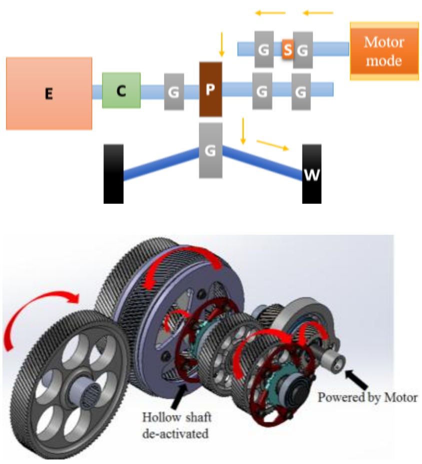

Fig. 4: Motor only mode

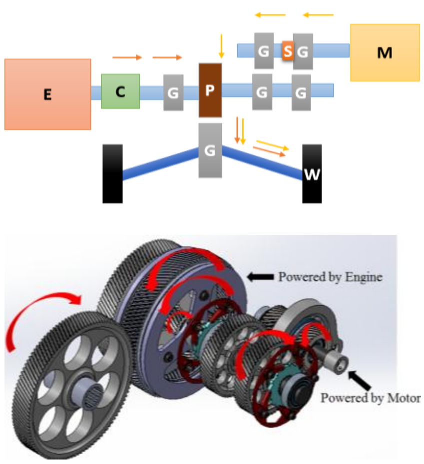

Fig. 5: Parallel mode

2. Design calculations

Planetary gear set, compound gears, ratchet & pawl and shifting mechanism form the main parts of the transmission system design. The fundamental stress equations according to American Gear Manufacturer's Association (AGMA) for gear design are as follows,

$$\sigma = W^t K_o K_\upsilon K_s \left(\frac{1}{bm_t}\right)\left(\frac{K_H K_B}{Y_J}\right) \quad (1)$$

$$\sigma_c = Z_E \sqrt{W^t K_o K_\upsilon K_s \left(\frac{K_H}{d_{w1} b}\right)\left(\frac{Z_R}{Z_I}\right)} \quad (2)$$

$$\sigma_{all} = \frac{\sigma_{FP} Y_N}{S_F Y_\theta Y_Z} \quad (3)$$

$$\sigma_{c,all} = \frac{\sigma_{HP} Z_N Z_W}{S_H Y_\theta Y_Z} \quad (4)$$

Where σ and σ_c are the applied bending and pitting stress respectively. σ_{all} and $\sigma_{c,all}$ are the allowable bending and pitting stress respectively. When analysing gear teeth, after the bending and contact stress values are found, they are compared with allowable values of stress to make sure that the design is satisfactory. If SF > 1 and SH > 1 (factor of safety) then the design is safe. From literature survey it has been observed that the sun gear has to be stronger than the planet and ring gear. Hence EN-24 material is chosen for sun gear and EN-19 material is chosen for planet and ring gear. Ring gear (and final drive gear) has not been included in the calculations since it should only be modelled accordingly after sun and planets have been designed. Compound gears have been designed according to reduction required. From Table 1, it can be observed that since all the factor of safety (FOS) values are above 1, the design is safe.

Table 1: Gears with final material selection and FOS values

Gear	Material	FOS in bending	FOS in pitting
Sun	EN-24	1.53	1.02
Planet	EN-19	1.99	1.19
Reduction 1: Engine gear	EN-24	1.06	1.00
Reduction 1: Mating with Engine gear	EN-24	1.02	1.92
Reduction 2: Shifter Gear	AISI 4140 R	2.54	1.06
Reduction 2: Mating with shifter gear	AISI 4140 R	2.54	1.51
Shifting gear	AISI 4140 R	2.5	1.06

Two ratchet and pawl systems, one for planetary carrier hollow shaft and another for sun solid shaft are designed. Iterations are carried out for different modules and number of teeth on ratchet. Since the shaft diameter is 32mm on hollow shaft, values are chosen from the area as indicated in Fig. 6, for the design to be safe. Lesser number of teeth implies that there will be greater allowed rotation before the ratchet and pawl mechanism actuates and as module increases so does the FOS. The design constraint here is the inner diameter of the ratchet has to be spliced to the shaft. Hence, the graph for internal diameter vs. module for various numbers of teeth was plotted in Fig. 6. A module of 7 for hollow shaft and 4 for sun shaft was selected. The number of teeth was selected such that symmetry was maintained and the bending loads on the shafts were avoided. Other dimensions are calculated using a standard design data handbook for ratchet and pawl design. Thickness value of designed ratchet and pawl are varied and analysed such that the FOS value to reach > 1.

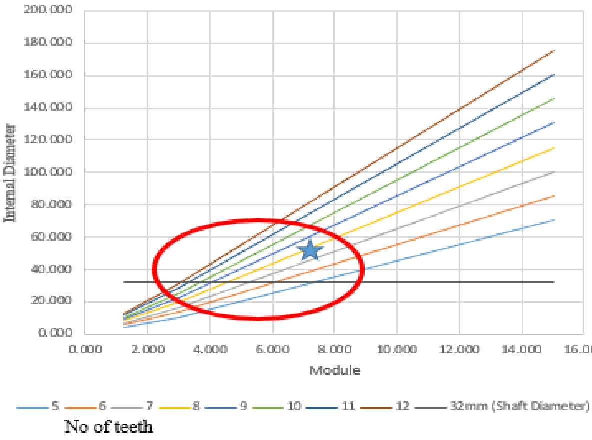

Fig. 6: Graph for selection of ratchet's module

In the shifting mechanism, friction and block release torques act on this part, Friction torque (T_F) is the torque caused by the frictional force that occurs when synchronizer and shifter gear rub against each other. Blocking release torque (T_Z) is the torque that blocks the shifting of sleeve. For blocking safety it is necessary that $T_F > T_Z$ as demonstrated in the following equations,

$$T_F = n_c \mu d_m \left(\frac{F_a}{2\sin\propto}\right) \quad (5)$$

$$T_F = 100*0.11*0.0403\left(\frac{100}{2\sin(6^o)}\right) = 63.61Nm \quad (6)$$

$$T_Z = F_a \frac{d_D}{2}\left(\frac{\cos(\beta/2) - \mu_D \sin(\beta/2)}{\sin(\beta/2) + \mu_D \cos(\beta/2)}\right) \quad (7)$$

$$T_Z = 100 * \frac{0.0405}{2}\left(\frac{\cos(60/2) - 0.11\sin(60/2)}{\sin(60/2) + 0.11\cos(60/2)}\right) \quad (8)$$

$$T_Z = 3.065Nm$$

Four kinds of shafts are used for this application: A left and right hollow shaft, sun solid shaft and motor shaft totalling to two hollow and two solid shafts. Based on the appropriate positioning of bearings, gears and ratchet on the shaft and their weights, the forces and torque that they produce on the shaft are calculated. The shaft material is chosen based on the analysis to find out the maximum equivalent stress that will be acting on the shaft. Two conditions are considered - (1) when the shaft is engaged and (2) when shaft is disengaged due to ratchet and pawl. Diameters for each of the shafts are assumed. They are changed if the final maximum torque calculated is higher than the material properties. Calculations are done as per American Society of Mechanical Engineers (ASME) standards. An example of sun shaft is shown in Fig. 7. The mass of gears, radial and tangential forces transferred by them are indicated.

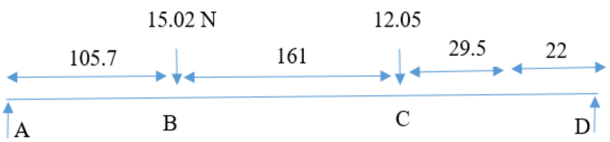

Fig. 7: Line diagram of sun shaft

The shaft diameter can be deduced on the basis of maximum shear stress theory as follows,

$$d_o = \left(\frac{16\sqrt{(C_m M)^2 + (C_t T)^2}}{\pi \tau_{\max} \left(1 - K^4\right)} \right)^{1/3} \quad (9)$$

The designed dimensions of the shaft and selected material for the key components of the transmission system are given in Table 2.

Table 2: Final shaft dimensions and material values chosen

Shaft	Diameter (mm)	τ_{max}(MPa)	τ_{max} (MPa)	Material
Sun solid	25	12.6	97.13	C-45 steel
Hollow planetary carrier	OD- 32, ID-28	43.77	8.26	C-40 steel
Motor shaft	17	80.014	-	C-40 steel

3. Modelling

Table 3 lists a total of 56 components which make up the system as modelled in SOLIDWORKS. Gears have a 30° helix and are of 20° stub tooth involutes system with module of 1.75. Sun gear, three planet gears and ring gear are assembled and a planetary carrier enclosing the system which is able to confine any degrees of movement of its components is modelled. Thrust bearings are fixed into an adapter which is fixed onto the surface of the ring gear to confine axial movement of it. Shafts are modelled with splines present on them (to mount the gears) and shouldered for positioning the bearing with grooves for circlip positioning to restrict axial movement. To provide shifting between the modes a support system is built for the switching. Two shifter gears are mounted on deep groove ball bearings which are in constant mesh with their mating gears. Pillow ball bearing having 14° angular movement is chosen and the system is so adjust that the shifting between two modes is seamless. Ratchet and pawl assembly is put together keeping packaging in mind and to locate mounting of the carrier. Gear box is the support structure that encloses and supports the transmission system by taking loads of varying magnitudes. Design details of modelled components are shown in Figs. 8 to 10.

Table 3: List of number of components designed (56 off)

Components	Qty	Components	Qty
Gears	12	Angle restrictor	1
Shaft	4	Restrictor plate	2
Planetary carrier	2	Sleeve	1
Thrust bearing adapter	2	Hub	1
Planetary Adapter	6	Synchronizer	2
Rosette weld part	2	Coupling	2
Gearbox	4	Ratchet	2
Shifter ring	1	Pawl	9
Shifter rod	1	Carrier	2

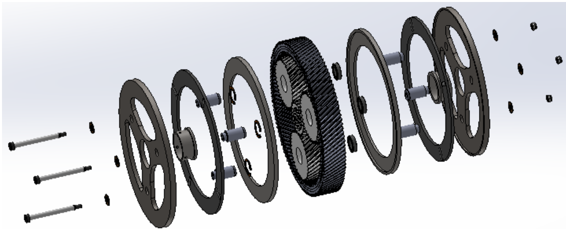

Fig. 8(a): Exploded view of planetary gear set

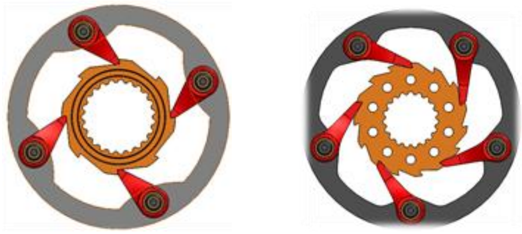

Fig. 8(b): Two kinds of ratchet and pawl assembly

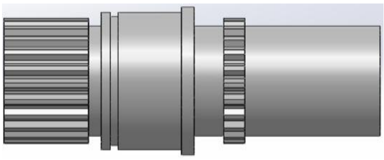

Fig. 9(a): Model of hollow shaft

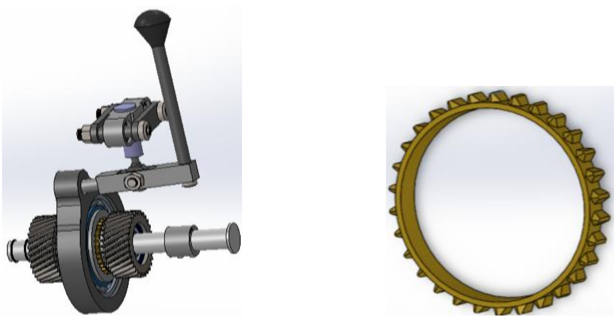

Fig. 9(b): Shifting assembly and synchronizer

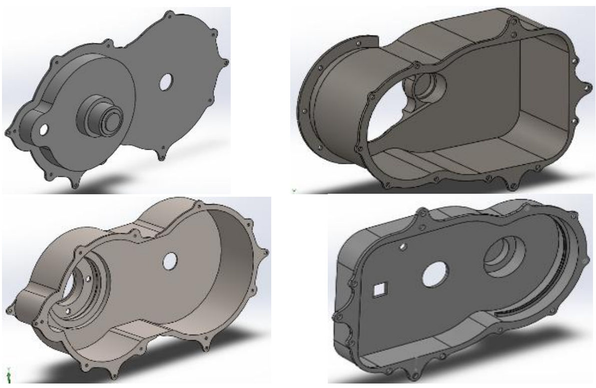

Fig. 10: Model of gearbox parts

4. Analysis

Finite Element Analysis (FEA) is carried out using ANSYS to verify the design. The simulated structural, modal analysis and harmonic response are shown in Figs. 11 to 15. Planetary gear set is analysed as a whole during engine only mode. Planetary carrier is given the required torque and the sun gear is fixed along with the external teeth of ring gear. In motor only mode, the sun gear is given the required torque and the planetary carrier is fixed. In parallel mode, both sun and planetary carrier are given torque while fixing external teeth of ring gear. In the analysis of reduction gears, the internal splines' part is fixed. Tangential and radial forces are applied to a tooth. Stopping and impact forces are applied on ratchet, pawl and carrier. In the switching mechanism, as the shifting gear and synchronizer are only the important parts, they are analysed. Since the loads on the gearbox is sinusoidal in nature, modal and harmonic response analyses are carried out by taking into account of the unbalanced vibration (i.e. breaking of a tooth on gear) which are then compared with the system's natural frequency to check the resonance.

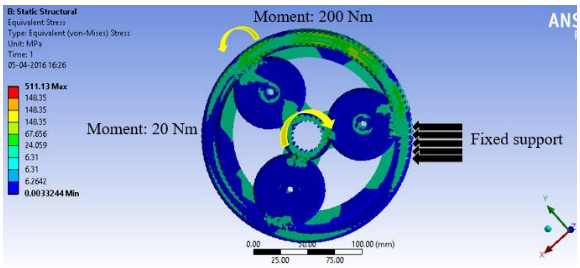

Fig. 11(a): Von-mises stress in parallel mode

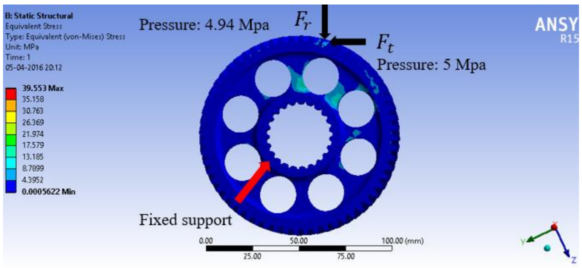

Fig. 11(b): Reduction gear analysis

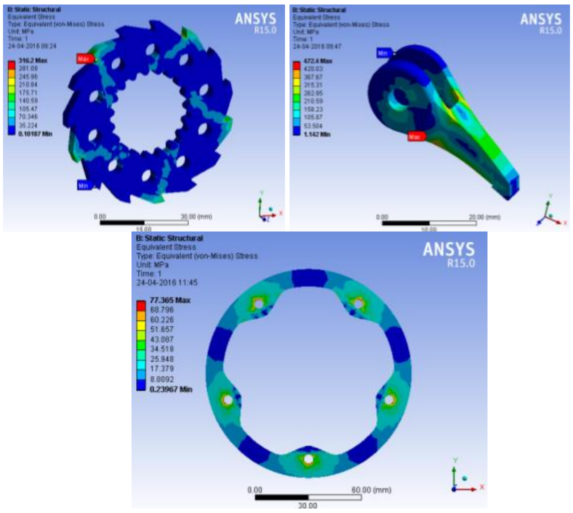

Fig. 12: Von-mises stress in sun shaft – Ratchet (top left), Pawl (top right) and carrier (bottom)

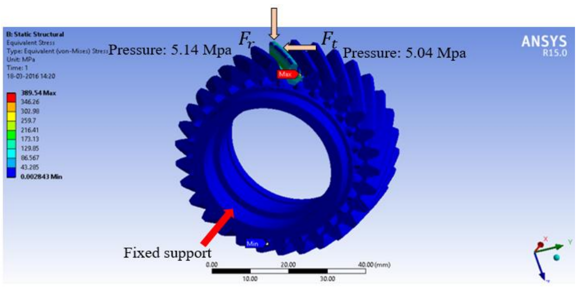

Fig. 13(a): Von-mises stress in shifter gear

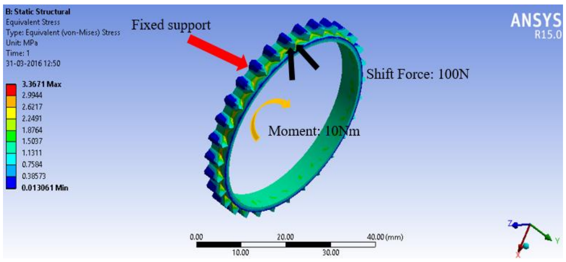

Fig. 13(b): Equivalent Von-mises stress in synchronizer

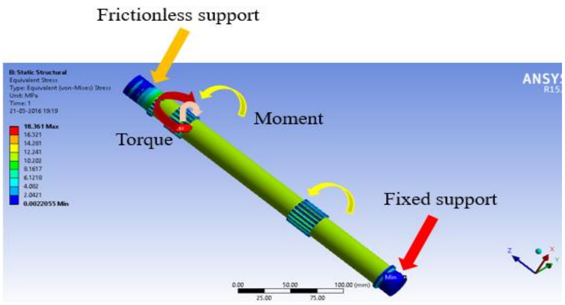

Fig. 14(a): Equivalent Von-mises stress in sun shaft when engaged

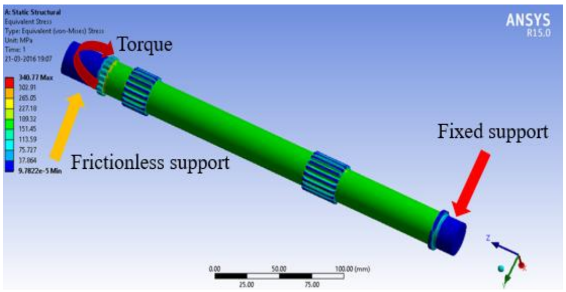

Fig. 14(b): Von-mises stress in sun shaft when dis-engaged

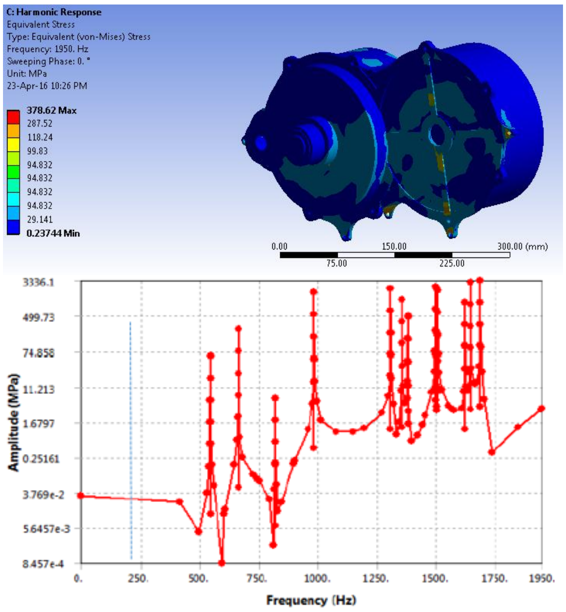

Fig. 15: Harmonic response analysis of Gearbox I & II

5. Results and discussions

Calculated theoretical acceleration results for each mode are given in Tables 4 to 6. In the parallel mode, the time taken was 7.3606s as compared to 10.59s in engine only mode to complete a distance of 0-100m. The motor was designed to be operated from 20-60kmph (for brake and go conditions, maximum efficiency). The time taken for acceleration was 13.25s.

Table 4: Acceleration calculation for engine only mode

S kmph	R 10^{-2}	F' +D N	W rpm	GR	CVT	ω rad/s	E rpm	P HP	F N	F- N	Inst a m/s^2	Avg a m/s^2	D m	T s
5	1	34	52	12	4	5	620	2	967	933	3	2	2	1
10	1	34	104	12	4	11	1241	2	483	449	1	2	2	1
15	1	36	156	12	4	16	1861	2	322	286	1	1	6	1
20	1	38	208	12	4	22	2482	2	242	204	1	1	11	2
25	1	41	260	12	4	27	3102	8	735	694	2	1	7	1
30	1	44	312	12	4	33	3723	8	612	569	2	2	6	1
35	1	48	364	12	4	38	4343	10	663	615	2	2	7	1
40	1	52	416	12	4	44	4963	11	641	589	2	2	8	1
45	1	57	468	12	4	49	5584	12	623	566	2	2	10	1
50	1	62	520	12	4	54	6204	14	657	595	2	2	10	1
55	1	69	572	12	4	60	6691	16	686	617	2	2	11	1
60	1	75	624	11	4	65	6691	16	628	553	2	2	12	1
65	1	83	676	10	3	71	6691	16	580	497	2	2	15	1
70	1	90	728	10	3	76	6691	16	539	448	1	2	18	1
75	1	99	780	9	3	82	6691	16	503	404	1	1	21	1
80	2	108	832	8	3	87	6691	16	471	363	1	1	25	1
85	2	118	884	8	2	93	6691	16	444	326	1	1	29	1
90	2	128	936	7	2	98	6691	16	419	291	1	1	-	-

Table 5: Acceleration calculation for motor only mode

S kmph	R 10^{-3}	F' + D N	W rpm	GR	ω rad/s	M RPM	F N	F- N	Inst a m/s^2	Avg a m/s^2	D m	T s (10^{-2})
20	12	38	208	8	22	1591	648	610	2	1	2	27
25	12	41	260	8	27	1989	518	478	2	2	1	17
30	12	44	312	8	33	2386	432	388	1	1	2	21
35	12	48	364	8	38	2784	370	323	1	1	3	25
40	12	52	416	8	44	3182	324	272	1	1	3	30
45	13	57	468	8	49	3579	288	231	1	1	4	35
50	13	62	520	8	54	3977	259	197	1	1	6	42
55	13	69	572	8	60	4375	236	167	1	1	8	49
60	13	75	624	8	65	4773	216	141	0	0	10	58
65	14	83	676	8	71	5170	199	117	0	0	13	70
70	14	90	728	8	76	5568	185	95	0	0	-	-

Table 6: Acceleration calculation for parallel mode

S kmph	R 10^{-2}	F' + D N	W rpm	GR	CVT	ω rad/s	E rpm	P HP	F N	F- N	Inst a m/s^2	Avg a m/s^2	D m	T s
5	1	34	52	12	4	5	620	6	2900	2867	10	5	1	0
10	1	34	104	12	4	11	1241	6	1450	1416	5	7	1	0
15	1	36	156	12	4	16	1861	6	967	931	3	4	2	0
20	1	38	208	12	4	22	2482	6	725	687	2	3	3	1
25	1	41	260	12	4	27	3102	8	773	733	2	2	4	1
30	1	44	312	12	4	33	3723	12	967	923	3	3	5	1
35	1	48	364	12	4	38	4343	16	1105	1057	4	3	4	0
40	1	52	416	12	4	44	4963	18	1088	1036	3	3	5	0
45	1	57	468	12	4	49	5584	20	1074	1017	3	3	5	0
50	1	62	520	12	4	54	6204	21	1015	953	3	3	6	0
55	1	69	572	12	4	60	6691	22	967	898	3	3	7	0
60	1	75	624	11	4	65	6691	22	886	811	3	3	8	0
65	1	83	676	10	3	71	6691	22	818	735	2	3	10	1
70	1	90	728	10	3	76	6691	22	760	669	2	2	12	1
75	1	99	780	9	3	82	6691	22	709	610	2	2	14	1
80	2	108	832	8	3	87	6691	22	665	557	2	2	16	1
85	2	118	884	8	2	93	6691	22	626	508	2	2	19	1
90	2	128	936	7	2	98	6691	22	591	463	2	2	-	-

The top speed achieved by the engine for the parallel mode is detailed as follows:

- Torque given by motor after reduction = 72 Nm.
- Torque required = 271.63 Nm.
- Torque to be given by engine = 271.63-72 = 199.63 Nm. This torque after reduction = 15 Nm.
- From engine performance graph, required rpm to be set on engine = 3700 rpm
- Speed on ring gear is calculated as below; N = 1793.055 rpm, ω = 187.67 rad/s

$$N_s = \left(1 + \frac{R}{S}\right) N_c - \frac{R}{S} N_r$$

- Power given by engine at 3700 rpm = 8.206 kW. For this power and speed on the ring gear, the toque achievable is calculated as,

$$T = \frac{P}{\omega} = \frac{(4+8.206)*1000}{187.67} = 65.04 Nm$$

- The top speed as achieved by the engine is calculated from the following,

$$F = cmg + \frac{1}{2}\rho A C_d v^2$$

Substituting F = 255.058N, ν = 40.15 m/s, the top speed in parallel mode is 144.53 kmph.

The power input and output values are given in Table 7.

Table 7: Power input and output values for different modes

Mode	Power input, kW	Power output, kW
Engine only with regeneration mode	14	12.47
Motor only mode	4	3.768
Parallel mode	12.206	11

6. Conclusions

Transmission system for a hybrid vehicle can be completely mechanical as opposed to having a core electrical drive. A cost-effective power-split hybrid transmission is developed. A single motor and an IC engine system can be used to replace two motors for a powerful IC engine giving high fuel-economy, reduced maintenance while delivering similar power output. It can be observed that the best results are for parallel mode making it the best mode out of the studied engine-only with regeneration mode, motor only mode and parallel mode. Final top speed calculation for parallel mode yielded a velocity of 40.15m/s (144.53kmph). Further this technology has been filed for patent.

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